Modern Physics
Study Questions for the Spring 2022 Departmental Exam
December 14, 2021

1. The muon is an unstable particle that spontaneously decays into an electron and two neutrinos. If the number of muons at $t = 0$ is N_0, the number at time t is given by $N = N_0 e^{t/\tau}$, where τ is the mean lifetime, equal to 2.2 μs. Suppose the muons move at a speed of $0.95c$ and there are $5.0\ 10^4$ muons at $t = 0$.
   a. What is the observed lifetime of the muons?
   b. How many muons remain after traveling a distance of 3.0 km?.

2. X-ray photons of wavelength 0.02580 nm are incident on a target and the Compton-scattered photons are observed at 90^0.
   a. What is the wavelength of the scattered photons?
   b. What is the momentum of the incident photons? Of the scattered photons?
   c. What is the kinetic energy of the scattered electrons?
   d. What is the momentum (magnitude and direction) of the scattered electrons?

3. Recently, 2000 rubidium atoms $\left({}^{87}_{37}\mathrm{Rb}\right)$, which had been compressed to a density of 10^{13} atoms/cm^3, were observed to undergo a Bose-Einstein condensation as a function of temperature. This occurs when the de Broglie wavelengths overlap and the system forms a single macroscopic quantum state.
   a. Estimate the temperature at which Bose-Einstein condensation occurs for this system.
   b. The rubidium atoms are sufficiently far apart that they can be treated as non-interacting particles in a box. Estimate the transition energy from the ground state to the first excited state.

4. One of the strongest emission lines observed from distant galaxies comes from hydrogen and has a wavelength of 122 nm (in the ultraviolet region).
   a. How fast must a galaxy be moving away from us in order for that line to be observed in the visible region at 366 nm?
   b. What would be the wavelength of the line if that galaxy were moving toward us at the same speed?

5. The natural decay chain ${}^{238}_{92}U \rightarrow {}^{206}_{82}Pb$ includes several alpha and negative beta decays.
   a. How many decays of each type are there in this chain?
   b. The decay chain actually has several different branches, corresponding to different orders of the alpha and beta decays. Does the answer to (a) depend on which branch is taken?

6. If an electron, initially at rest, is accelerated through a potential difference of 180kV.
   a. Calculate the total relativistic energy of the electron.
   b. Calculate the relativistic mass of the electron.

7. A typical particle production process is

$$p + p \rightarrow p + p + \pi^0.$$

Mass of proton = 938 MeV/c^2; Mass of π^0 = 135 MeV/c^2

a. In a standard accelerator one of the initial protons would be at rest. In this case what is the minimum *laboratory* energy for the moving proton so that the reaction listed above occurs.

b. In colliding beam accelerators, such as the one proposed for the Superconducting Supercollider, the two initial particles are moving toward each other with equal speeds. Why do colliding beam accelerators have an energetic advantage over the type of accelerator described in part a of this question?

c. What energy would be required of *each proton* in a colliding beams accelerator to produce a pion?

8. A dust particle at rest relative to the sun is in equilibrium between radiation pressure and gravitational attraction. What is the size of the dust particle? (For simplicity you can assume that the dust particle is spherical.)

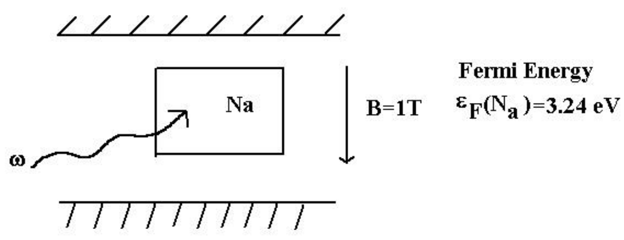
9. A pure single crystal of the metal sodium is placed in a static magnetic field of 1T as shown. The metal is illuminated by microwave energy of frequency ω. Excess microwave energy is absorbed at the cyclotron resonance frequency ω_c.

a. For an ideal free electron gas determine a value for ω_c.

b. Determine the characteristic size of the orbital motion.

c. Explain what is occurring at the resonance frequency.

d. Calcium has a similar atomic density as sodium but it has a valence of 2 while sodium has a valence of 1. Estimate the characteristic size for orbital motion for calcium.



10. When sodium metal is illuminated with light of wavelength 4.20 x 10^2 nm, the stopping potential is found to be 0.65 V; when the wavelength is changed to 3.10 x 10^2 nm, the stopping potential is 1.69V. Using *only these data* and the values of the speed of light and the electronic charge, find the work function of sodium and a value of Planck's constant.

11. A rocket with a proper length of 2000 m moves at a speed of 0.83c directly away from an observer on earth. An astronaut standing at the center of the rocket fires two electrons (rest mass 0.511 MeV/c^2) at a speed of 0.93c through a vacuum pipe; one electron is aimed toward the front of the rocket, the other toward the rear.
   a. In the astronaut's frame, which electron reaches its end of the rocket first? How much time elapses between the two collisions?
   b. In the earth observer's frame, which electron reaches its end of the rocket first? How much time elapses between the two collisions?
   c. What is the momentum of each electron in the astronaut's frame?
   d. What is the momentum of each electron in the earth observer's frame?

12. In a hot star, a multiply ionized atom with a single remaining electron produces a series of spectral lines as described by the Bohr model. The series corresponds to electronic transitions that terminate in the same final state. The longest and shortest wavelengths of the series are 63.3 nm and 22.8 nm, respectively.
   a. What is Z of the ion?
   b. Find the wavelengths of the next three spectral lines nearest to the line of longest wavelength.

13. The measured conductivity of copper at room temperature is $5.88\text{x}10^7\ \Omega^{-1}\text{m}^{-1}$, its Fermi energy is 7.03 eV, density is 8.96 gm/cm^3 and molar mass is 63.5 gm/mole.
   a. Calculate the density of free electrons in copper.
   b. Calculate the average time between collisions of the conduction electrons.
   c. Calculate the mean free path of electrons in copper.
   d. Calculate the smallest possible de Broglie wavelength of electrons in copper at T = 0K. How does this value compare with the atomic separation in copper?

14. Ordinary potassium contains 0.012 percent of the naturally occurring radioactive isotope 40K, which has a half-life of 1.3 x 109 years.
   a. What is the activity (decays/sec) of 1.0 kg of ordinary potassium?
   b. What would have been the fraction of 40K in ordinary potassium 4.5 109 years ago?

15. A linear symmetric triatomic molecule can have both rotational and vibrational energy levels.
   a. Calculate the energy levels for the rotational states and sketch.
   b. Sketch the normal vibrational modes of the molecule.
   c. If the molecule is CO_2 and has an internuclear distance of ~1.8Å, how many rotational states are excited at room temperature?.

16. Since the neutron has no charge, its mass must be found indirectly. From the data given below determine the mass difference m_n-m_H. It might not be necessary to use all of the information provided.
   a. $^2H + {}^2H \rightarrow {}^3H + {}^1H + 4.031$ MeV
   b. $^1H + n \rightarrow {}^2H + \gamma$, $E\gamma = 2.225$ MeV
   c. $^3H \rightarrow {}^3He + e + 0.0185$ MeV
   d. $^2H + {}^2H \rightarrow {}^3He + n + 3.267$ MeV

17. In a double-slit interference experiment using helium atoms, the beam has atoms of kinetic energy 0.020 eV. The distance from the double slits to the observation plane is 64 cm.
   a. What is the de Broglie wavelength of a helium atom with this kinetic energy?
   b. What separation between the double slits is needed to produce interferences fringes separated by 10 μm at the observation plane?

18. A 1.0 mg sample of uranium-238 emits 738 α-particles per minute. Based on this information, what is the half life of ${}^{238}_{92}U$?

19. Several spacecraft leave a space station at the same time. Relative to an observer on the station, A travels at 0.60c in the x direction, B at 0.50c in the y direction, C at 0.50c in the negative x direction, and D at 0.50c at 45º between the y and negative x directions. Find the velocity components, directions, and speeds of B, C, and D as observed from A.

20. The atomic number of Na is 11.
   a. Write down the electronic configuration for the ground state of the Na atom showing in standard notation the assignment of all electrons to various one-electron states.
   b. Give the standard spectroscopic notation for the ground state of the Na atom.
   c. The lowest frequency line in the absorption spectrum of Na is a doublet. What are the spectroscopic designations of the pair of energy levels to which the atom is excited as a result of this absorption process?
   d. Discuss the mechanism responsible for the splitting between this pair of energy level.
   e. Which of these levels lies lowest? Discuss the basis on which you base your choice.

21. A gaseous discharge tube emits radiation lines which are identified as the $3P_{1/2}$ - $3S_{1/2}$ and $3P_{3/2}$ - $3S_{1/2}$ transitions. If the discharge tube is placed in a uniform weak magnetic field of magnitude B, the transitions are observed to split into several spectra lines. Make a level diagram with and without the magnetic field, and label the final levels and the transitions. Calculate the Landé g factors, the level splittings, the resulting transition energies and the character of the radiation.

22. In HCl molecule a number of absorption lines with wave numbers (in cm^{-1}) 83.03, 103.73, 124.30, 145.03, 165.51, 185.86 have been observed.
   a. Are these vibrational or rotational transitions. Justify the answer.
   b. If these transitions represent vibrational transitions, determine the characteristic frequency and effective "spring" constant. If these transitions represent rotational transitions determine the J values, moment of inertia of HCl and separation distance between the nuclei.

23. A beam of protons, each with kinetic energy 40 MeV, approaches a step potential of 30 MeV.
   a. What fraction of the beam is reflected and transmitted?
   b. How does your answer change if the particles are electrons?

24. A helium-neon laser emits light with wavelength 6328 Å in a transition between two states of neon atoms.
   a. Why is helium needed in the laser? Why not just neon?
   b. What would be the ratio of population of the upper state relative to the lower state of the neon atoms if they were in thermal equilibrium at 300 K?

25. Consider a Si semiconductor, in which the dielectric constant $\kappa = 12$ and electron effective mass $m^* = 0.2m_e$.
   a. Calculate the binding energy of the extra electron of an impurity arsenic.
   b. Suppose the arsenic doping concentration is 10^{17} cm^{-3}, what fraction of these donor atoms would supply an electron to the conduction band at room temperature?

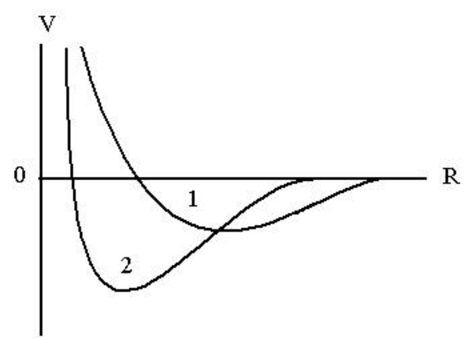
26. Three regions of scattered light are typically observed in Raman spectra.
   a. Explain what these three regions are. What is the physical reason for the appearance of inelastically-scattered components?
   b. Using the Boltzmann equation calculate the ratios of anti-Stokes to Stokes intensities at 20 °C and 40 °C for the Raman lines of CCl_4 molecule (218 cm^{-1} and 459 cm^{-1})

27. The sun is not only a strong source of photons, but is also a strong source of neutrinos.
   a. What is the basic process for energy generation in the sun? Why does this process generate neutrinos?
   b. The flux of radiant energy (photons) from the sun striking the earth's atmosphere is 1350 W/m^2. Estimate the number of solar neutrinos which strike a square meter of the earth's surface every second. Justify any assumptions or approximations used in your estimate. The binding energy of helium 4 is 7.2 MeV per nucleon, and 1 eV = 1.6×10^{-19} J.

28. A π^o-meson decays into two gamma ray photons with a lifetime of about 10^{-16} seconds. In the laboratory frame, the gamma ray photons have energies E_1 and E_2 and $\theta_{1,2}$ is the angle between their directions. In terms of E_1, E_2 and $\theta_{1,2}$, what is the rest mass of the π^0?

29. In a muonic hydrogen atom, the electron is replaced by a muon, which is 207 times heavier.
   a. What is the wavelength of the n=2 to n=1 transition in muonic hydrogen? In what region of the electromagnetic spectrum does this wavelength belong?
   b. How would the fine structure and Zeeman effects differ from those in ordinary atomic hydrogen?

30. The potential energies of two diatomic molecules of the same reduced mass are shown. From the graph determine which molecule has the larger



   a. inter-nuclear distance,
   b. rotational inertia (moment of inertia),
   c. separation between rotational energy levels,
   d. binding energy,
   e. zero-point energy,
   f. separation between low-lying vibrational states