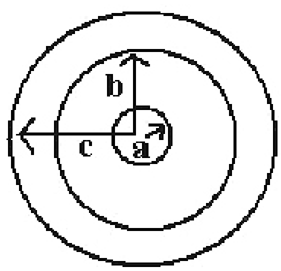
Electricity & Magnetism
Study Questions for the Spring 2022 Department Exam
December 14, 2021

1. a. Find the capacitance of a spherical capacitor with inner radius ℓ_i and outer radius ℓ_0 filled with dielectric of permittivity ε .
   b. Find the capacitance if the dielectric fills only the lower half of the capacitor.

2. A 0.5 ampere electron beam is accelerated through a potential of 5000 Volts, after which it becomes an (infinitely) long cylindrically symmetric pencil with uniform current density across a 0.3 mm radius, with no current outside this radius. The beam is surrounded by a long concentric tube of 3 mm radius held at ground potential.
   a. Calculate the charge density within the beam.
   b. Calculate the radial field and potential as a function of distance from the beam axis (r), both inside and outside the beam.
   c. Sketch E and V versus r.
   d. Evaluate V on the axis of the beam.

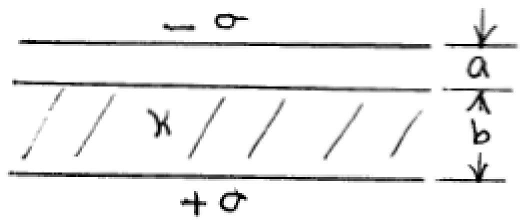
3. A long co-axial cable (shown in cross section) has uniform current 'I' flowing in the center conductor into the paper and the same current 'I' flowing in the outer cylinder out of the paper. Find the magnetic field in the following regions - (i) $r<a$ (ii) $a<r<b$ (iii) $b<r<c$ (iv) $r>c$ where 'r' is the distance of any point (in the specified regions) from the center of the co-axial cable.



4. For the Yukawa potential $\phi(r) = g^2 e^{-mr}/r$, find the electric field, and the charge distribution that produces this potential.

5. <u>Give short answers</u>.
   a. If one wants to define uniquely the electric field $\vec{E}$ in a given region, is it enough to specify divergence of $\vec{E}$?
   b. How will you express mathematically the law of conservation of charges?
   c. What is the physical significance of the Maxwell's equation
   $$\nabla \cdot \vec{B} = 0.$$
   d. If the magnetic vector potential is zero at the center of a current carrying loop, then does it mean that $\vec{B}$ is zero at that point?
   e. Are advanced potentials physically acceptable solutions to Maxwell's equations?
   f. Which of the four Maxwell's equations is the consequence of Faraday's law of induction?
   g. State clearly the Coulomb gauge and the Lorentz gauge.
   h. What is a plasma frequency in a medium of free charges?

6. Two large conducting parallel plates are separated by a +b and are oppositely charged with $+\sigma$ and $-\sigma$ surface charge densities. On the lower plate resides an insulator with dielectric constant κ, and thickness b. The space of thickness a is filled with air with $\kappa=1$.
   a. What is the electric field in the air space?
   b. What is the electric field in the insulator?
   c. What is the surface charge density on the top of the insulator?
   d. What is the voltage between the plates?



7. Suppose that one measures the electrical conductivity of the following materials at room temperature:
   - high purity copper
   - n-type germanium
   - niobium

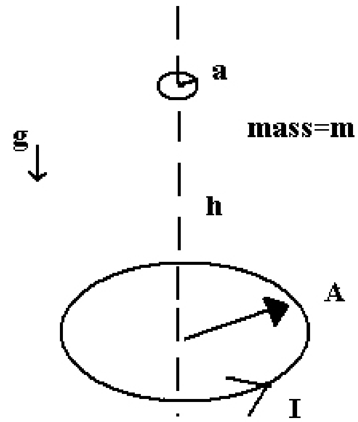
   One then plunges each of these samples into helium (4K) and repeats the measurement. How much does the conductivity of each material change? (qualitative answer) In which direction? Why?

8. A plane electromagnetic wave of frequency ω is incident from vacuum onto a slab of dielectric material with permittivity ε, thickness d, at normal incidence. Behind the dielectric is a perfect conductor. We are interested in the reflectivity properties of this arrangement. Let E_1 be the incident electric field, and E'_1 be the reflected electric field, both measured in the vacuum at the vacuum-dielectric boundary ($z = 0$). The dielectric-conductor boundary is at $z = d$.
   a. Give space- and time-dependent expressions for the electric and magnetic fields in the vacuum and in the dielectric.
   b. Apply appropriate boundary conditions to determine the ratio of amplitudes, E'_1/E_1.
   c. What is the resulting reflection coefficient?

9. A plane wave is polarized in the +x direction and travelling in the +z direction. The wave is reflected from the xy plane at $z = 0$, where the index of refraction is n_1 for $z < 0$ and n_2 for $z > 0$.
   a. Write down the E and B fields for the incident, the reflected and the transmitted wave.
   b. Calculate the transmission and reflection coefficients.

10. A thin uniform metal disk lies on an infinite conducting plane. A uniform gravitational field is oriented normal to the plane. Charge is slowly added to this initially uncharged system. At what value of surface charge density will the disk leave the plane?

11. The small loop of wire (radius a, resistance R) falls under gravity towards the larger loop (radius A), which has a constant current I? The small loop is constrained to move along the axis of the large loop and remain parallel to the large loop.

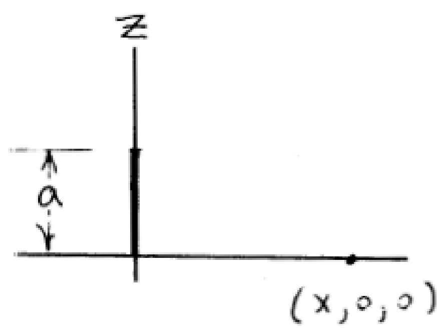


a. Explain (in words) what happens to the small loop.
b. For a particular height h and velocity v, what is the induced emf in the small loop? Assume a<<A.
c. What is the electromagnetic force acting on the small loop?

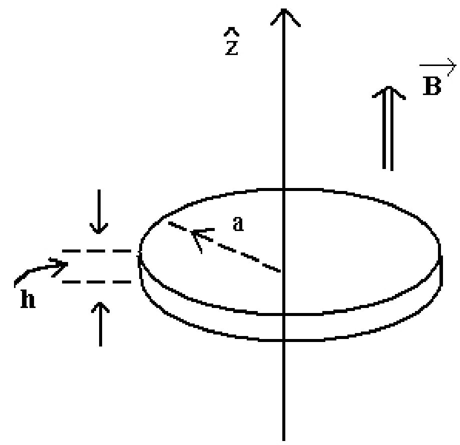
12. A uniformly charged rod of length a resides along the z-axis as drawn. The linear charge density is λ. The voltage potential at (x, 0, 0) is

$$V = \frac{1}{4\pi\varepsilon_o}\int_o^a \frac{\lambda\, dz}{\sqrt{z^2 + x^2}}$$

a. Expand the denominator of this expression for x >>z to two nonzero terms.
b. Substitute this expansion into the expression for V, work the two integrals.
c. Identify the two results as monopole, dipole, quadrupole, octapole or whatever and give an expression for the two moments.



13. A thin non-magnetic, conducting disk of thickness h, radius a, and conductivity σ is placed in a region where the net, spatially uniform alternating magnetic field $\vec{B} = \hat{z} B_o \sin \omega t$ is parallel to the z axis, as shown in the figure.



a. Find the induced current *density* as a function of radial distance from the axis of the disk.
b. What is the direction of this current at any instant in the first quarter of the period $\pi/(2\omega)$?
c. Find the total induced current at any instant in the above period.
d. Calculate the average Joule heating *i.e.*, the electromagnetic energy that is converted to heat.

14. An off-centered hole of radius *a* is bored parallel to the axis of an infinite-length right circular cylinder of radius *b* (and contained within the circumference of the cylinder). The two axes are a distance *d* apart. A current *I* flows in the solid cylinder. Compute the magnetic field at the center of the hole.

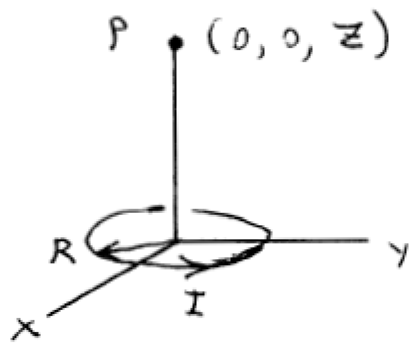
15. A (non-radiating) particle of mass m, charge q, moves under the influence of a uniform electric field $\vec{E} = E\hat{y}$. The particle has initial momentum $\vec{P}_o = P_o\hat{x}$. Using *relativistic* dynamics, determine,
    a. the kinetic energy as a function of time,
    b. the position of the particle as a function of time,
    c. the shape of the path followed by the particle.

16. Find the self-inductance per unit length of a long coaxial cable for which the inner conductor is a solid cylinder of radius a and the outer conductor has radius b. Let ℓ be the length of the cable. Assume that the current in the inner cable is uniformly distributed over the cross section.

17. A conducting sphere of radius *a* and total charge Q is surrounded by a spherical shell of dielectric material (with permittivity ε) of inner radius *a* and outer radius *b*. Find the electrostatic energy of the system.

18. An infinite uniform plane current sheet has a surface current density $\vec{K} = K\hat{x}$. Use the integral form of Amphere's Law to find the magnetic induction. Make a sketch of your line integral indicating the direction of integration and the directions of $\vec{B}$. What is the difference of $\vec{B}$ for x > 0 and x < 0?

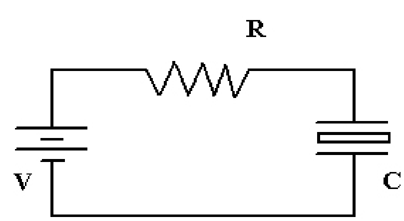
19. A loop of wire of radius R lies in the xy plane centered on the origin carrying a current I as drawn.
   a. Find the magnetic field at point P at (0, 0, z).
   b. Expand your result to two terms for z >> R. Identify the poles.



20. A magnetically "hard" material in the shape of a right circular cylinder of radius *a* and length L has magnetization M uniform throughout its volume and parallel to its axis. Find $H_Z(z)$ on the axis of the magnet by treating it as two sheets of magnetic charge located at z = +L/2 and z = -L/2.

21. A cylindrical wire of length *l,* radius a, and conductivity σ has a uniformly distributed current density J flowing through it. Evaluate the **E** and **H** fields to get the Poynting flux **S** through a closed surface just outside the conductor. Show that it is exactly equal to the heat produced by the current.

22. A battery with emf V, a resistor with resistance R, and a capacitor with initial capacitance C have been connected in series as shown for a very long time. The dielectric, with permittivity ε, occupies 1/2 the gap in the parallel plate capacitor. Now suppose that the dielectric is removed very quickly, in a time short compared to any relevant time constants. The new capacitance is C'.
   a. Give an explanation of what happens to each circuit element. After a very long time, a new equilibrium is reached. In terms of C, C', R and V, give expressions for
   b. the net change in energy in the capacitor,
   c. the net change in energy in the battery,
   d. the total energy dissipated in the resistor,
   e. and the amount of work that was done to remove the dielectric.
   f. In terms of ε, what is the ratio C/C'?



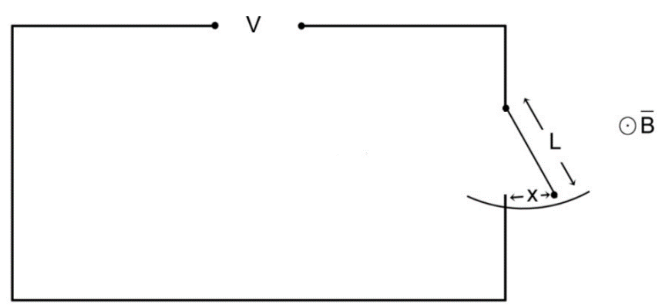
23. The intensity of sunlight at the earth's surface is 1.2×10^6 erg $cm^{-2}s^{-1}$.

    a. Find the electric field of this radiation at the earth's surface in units of V m^{-1}.
    b. Find the radiation pressure (in dyne cm^{-2}) if the sunlight is fully reflected at normal incidence.
    c. Find the radiation pressure if the sunlight is absorbed without reflection.

24. A copper disk of radius 5 cm rotates at 20 revolutions per second, in a magnetic field B=0.5 T perpendicular to the disk. The rim and center are connected electrically by a fixed wire with sliding contacts. The total resistance is $10\,\Omega$. Calculate the induced current.

25. A cylinder with length l and radius r_0 contains a uniform volume charge density ρ_0. When set rotating at angular speed ω around it axis, how large is its magnetic dipole moment?

26. As shown in the figure, a wire pendulum of length L performs small oscillations with velocity $\dot{x} = \omega D\cos(\omega t)$ where D is the maximum horizontal deflection of the pendulum. A constant magnetic field of amplitude B points out of the plane of oscillation of the pendulum. What is the induced voltage V?



27. Most of the electromagnetic energy in the Universe is in the cosmic microwave background radiation, a remnant of the Big Bang. This radiation was discovered by A. Penzias and R. Wilson in 1965, by observations with a radio telescope. The radiation is electromagnetic waves with wavelengths around 1.1mm. The energy density is 4.0×10^{-14}J/m^3. (This is 2.5×10^5eV/m^3, half the rest energy of an electron in each cubic meter of the Universe.)

    a. What is the RMS electric field strength of the cosmic microwave background radiation?
    b. How far from a 1000 W transmitter would you have to go to have the same field strength? Assume the power from the transmitter is isotropic.

28. Linearly polarized light of wavelength $\lambda = 800\text{nm}$ is incident on a crystal of birefringent material of indices of refraction $n_1 = 1.66$ and $n_2 = 1.49$. The emerging light is circularly polarized.
    a. What is the minimum thickness of the crystal?
    b. Suppose the wavelength of the incident light is decreased until linearly polarized light emerges from the crystal. At what wavelength will this occur?

29. A linear molecule with a permanent electric dipole moment p_o and moment of inertia I (for example, HCl) is placed in a uniform electric field E.
    a. Make a sketch showing the charges that make up the dipole, the dipole moment and the electric field when the dipole is in its equilibrium orientation.
    b. Derive an expression for the frequency ω_o of small amplitude oscillations about the equilibrium orientation.
    c. Describe the polarization and power of the radiation produced by this oscillating dipole, assuming it has been initially displaced by angle $\theta << 90$ degrees from its equilibrium orientation. Make a sketch of the angular distribution of this power
    d. Over what time scale will it continue radiating appreciably, and therefore what range of frequencies are emitted.

30. A point dipole sits at the origin of a spherical coordinate system. It points in the z-direction and has a dipole moment $\vec{m}_1 = m\hat{z}$. A second dipole sits at a point $(\mathbf{r},\theta,\phi) = (\mathbf{d},90°,0°)$. It also has a dipole moment $\vec{m}_1 = m\hat{z}$.
    a. What is the vector potential due to the dipole located at the origin?
    b. What is the magnetic induction due to the dipole located at the origin?
    c. What is the value of $\vec{\mathbf{B}}$ at the position of the second dipole?
    d. What is the magnitude of the magnetic dipole-dipole interaction energy for these two dipoles?
    e. What are the translational force and torque on the second dipole?