Amplitude ambiguities in second-harmonic generation frequency-resolved optical gating

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We construct field shapes with distinct amplitude profiles that have nearly identical second-harmonic generation frequency-resolved optical gating (SHG FROG) traces. Although such fields are not true mathematical ambiguities, they result in experimentally indistinguishable FROG traces. These fields are neither time-reversed copies nor pulselets with a mere relative phase difference, which are well known nontrivial ambiguities for SHG FROG. We also show that for certain example fields, second-order interferometric autocorrelation is more sensitive to the pulse shape than is SHG FROG.

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Frequency-resolved optical gating (FROG) [1] is one of the most commonly used self-referenced characterization techniques for ultrashort laser pulses. The FROG technique is very versatile and can be implemented by using various nonlinear mechanisms. Second-harmonic generation (SHG) FROG [2] uses the second-order $\chi^{(2)}$ nonlinearity and is considered to be one of the most sensitive [3] versions among all the self-referenced FROG geometries. Consequently, SHG FROG is a widely used technique. It can be implemented via scanning or in a single shot. GRENOUILLIE [4] is a simple nonscanning SHG FROG scheme. Several of these SHG FROG implementations are available as commercial instruments as well. The FROG traces in all of these methods, except for the recent interferometric SHG FROG [5], are described by the equation,

$$I^{\mathrm{FROG}}(\Omega,\tau) = \left| \int E(t)E(t-\tau)e^{i\Omega t}\mathrm{d}t \right|^2. \quad (1)$$

It is well known that this equation has four ambiguities, namely, trivial time shift, absolute phase, nontrivial time direction, and relative phase ambiguities. The time shift ambiguity refers to the ambiguity in the absolute arrival time of the pulse. The SHG FROG trace is symmetric about $\tau=0$ along the time axis; therefore the time direction cannot be determined. The first three ambiguities [6] are widely known and are related to the symmetries in the FROG trace. The measurement of the absolute time of arrival is physically irrelevant, while the time direction can be determined by an additional measurement [7]. Recently, yet another ambiguity was found [8]. When the original field is made of well-separated multiple peaks in either the time or the frequency domain, the relative phase between the two peaks cannot be exactly determined from the SHG FROG traces. For time domain peaks there is a relative phase ambiguity of $\pm\pi$, while the spectral peaks can have an arbitrary relative phase shift, although SHG FROG correctly characterizes the individual peaks within [8]. It is generally accepted that apart from these ambiguities FROG traces are essentially unambiguous [9].

In this Letter we describe an amplitude "ambiguity" by constructing pairs of pulse shapes that have nearly identical SHG FROG traces. Strictly speaking, this is not a true mathematical ambiguity. However, in practice, the maximum difference between two normalized FROG traces (Δ_m) can be very small, typically 10^{-5} or smaller. For these pulses, the normalized root-mean-squared difference between FROG traces (Δ_e), calculated similar to the FROG reconstruction error [7], is less than 10^{-9}, which suggests that the pulses are experimentally indistinguishable.

The main idea behind our approach is summarized by the example field profiles shown in Figs. 1(a) and 1(b). Consider a complex field envelope, $E_a(t)$, that comprises two pulselets in time that are identical (both phase and amplitude profiles) except for their relative peak magnitudes. We specifically choose the

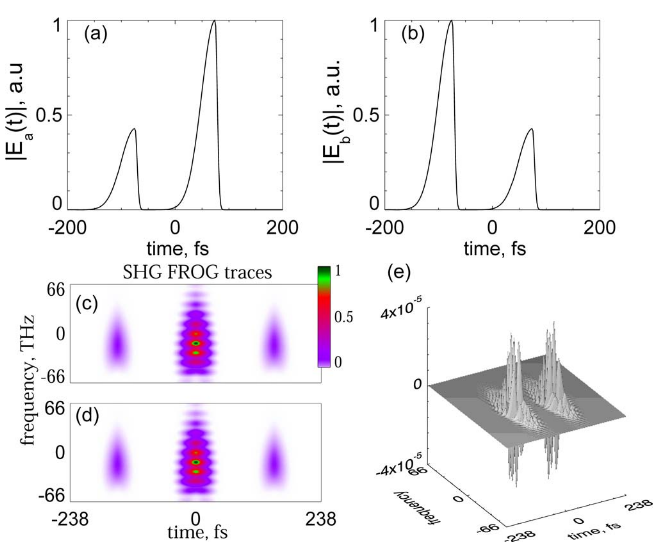

Fig. 1. (Color online) Example 1: (a), (b) two distinct electric field envelopes and (c), (d) the corresponding SHG-FROG traces. (e) Normalized difference between the two FROG traces. For the two pulses, $\Delta_m=0.004\%$ and $\Delta_e=3.8\times10^{-10}$.

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individual pulselet to be temporally asymmetric, so that the fields we construct are not time-reversed replicas of each other. From this field, we construct another field, $E_b(t)$, by simply interchanging the positions of the two pulses, while keeping the relative time delay constant. We will show that, when the pulselets are well separated in time, their SHG FROG traces are nearly indistinguishable. Obviously the two fields are very different and are not time-reversed copies of each other. For simplicity, we set the relative phase between the two pulselets within each field to be zero, so that the phase ambiguity described earlier [8] does not arise. The two fields have the same spectral magnitude and have no separate spectral peaks as in [8]. The complex (carrier removed) electric field of the two fields, $E_{a,b}(t)$, can be represented as

$$E_{a,b}(t) = S_{a_1,b_1}E_0(t-T) + S_{a_2,b_2}E_0(t+T). \tag{2}$$

Here the amplitude scaling factors (real) are related by $S_{a_1}=S_{b_2}$ and $S_{a_2}=S_{b_1}$. The distance between the pulselets within each field is $2T$. We denote the pulselets $E_0(t\pm T)$ by $E_\pm(t)$ and define a frequency-resolved cross correlation, $f_{xy}(\Omega,\tau)=\int E_x(t)E_y(t-\tau)e^{i\Omega t}\mathrm{d}t$, for convenience. Substituting Eq. (2) into Eq. (1), the FROG traces for the two fields are

$$I_x^{\mathrm{FROG}} = |S_{x_1}^2 f_{--} + S_{x_2}^2 f_{++} + S_{x_1}S_{x_2}(f_{+-}+f_{-+})|^2. \tag{3}$$

Here the subscript x is either a or b, and it refers to one of the two fields. All the terms in Eq. (3) are a function of Ω and τ. Using the time-shift-invariance property of SHG FROG traces, we can derive the following equalities among various terms:

$$|f_{++}(\Omega,\tau)| = |f_{--}(\Omega,\tau)| = |f_{00}(\Omega,\tau)|, \tag{4}$$

$$|f_{\pm\mp}(\Omega,\tau)| = |f_{00}(\Omega,\tau\pm T)|. \tag{5}$$

The first two correlations of Eq. (3) have the same magnitude and are centered on the time axis. While the last two correlations have the same magitude, they are temporally shifted and symmetrically located about the center. If these four correlations were real, then they would result in identical FROG traces. However, they are complex and result in interference between various terms in the FROG traces. Therefore the FROG traces, in general, are always different, that is, $I_a^{\mathrm{FROG}}-I_b^{\mathrm{FROG}}$ is not equal to zero.

An interesting observation follows from Eq. (3) by using Eq. (4) and (5). Between the two FROG traces we can show that $|S_{a_1}^2 f_{--}+S_{a_2}^2 f_{++}|^2=|S_{b_1}^2 f_{--}+S_{b_2}^2 f_{++}|^2$ and $|S_{a_1}S_{a_2}(f_{+-}+f_{-+})|^2=|S_{b_1}S_{b_2}(f_{+-}+f_{-+})|^2$. Therefore the two FROG traces only differ because of the cross terms, $(S_{x_1}^2 f_{--}+S_{x_2}^2 f_{++})\times S_{a_1}S_{a_2}(f_{+-}+f_{-+})+\mathrm{c.c.}$ These cross terms are a product between terms that are centered at $\tau=0$ and $\tau=\pm T$. Obviously when the pulselets are sufficiently separated in time ($2T$ much greater than the pulse width of individual pulselet), the cross terms can be made negligible, and consequently the SHG FROG traces become nearly identical.

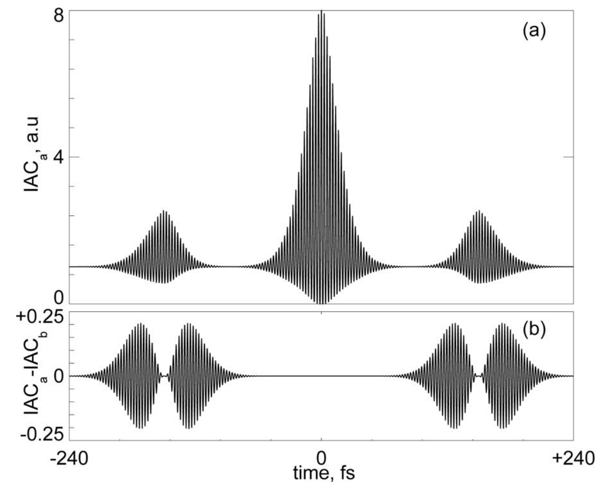

Fig. 2. (a) IAC trace for the first field, $E_a(t)$, in Example 1. (b) Difference between the IAC traces of $E_a(t)$ and $E_b(t)$ of Example 1. The maximum difference is 2.9%.

The FROG traces for the two example fields in Figs. 1(a) and 1(b) are plotted in Figs. 1(c) and 1(d). The pulselets in the example have a full width at half-maximum of 36.4 fs and temporal separation $2T=149.6$ fs. Each individual pulselet is an asymmetric Gaussian of the form $E_0(t)=e^{-t^2/\tau_g^2}e^{i\phi(t)}$, where $\tau_g=35.8$ fs for $t\leq 0$ and $\tau_g=5.38$ fs for $t>0$. The temporal phase is chosen to be a quadratic function, $\phi(t)=7.8\times t^2$ (t is in femtoseconds). Visually the two FROG traces look the same. The normalized difference between the two traces is shown in Fig. 1(e), where $\Delta_m\approx 10^{-5}$ and $\Delta_e\approx 10^{-9}$. We find that with decreasing temporal separation the difference between the FROG traces increases. When the separation is similar to the individual pulselet width, the increase in the difference between the FROG traces is no longer monotonic. Surprisingly, even when the pulselets overlap, the difference is only a couple of percent, still too small to be experimentally distinguished unless the FROG traces are very carefully measured.

We are interested to see whether the ambiguity described here exists in interferometric autocorrelations (IACs) [10] or modified spectrum autointerferometric correlation (MOSAIC) traces [11]. The IAC is made of three main envelopes. When Fourier transformed, the three envelopes are centered at $\omega=0$, ω_0, and $2\omega_0$ [10]. Here ω_0 is the central frequency of the optical field. The MOSAIC is derived from the IAC by using the envelopes centered at $\omega=0$ and $\omega=2\omega_0$. We found that the MOSAIC traces (hence envelopes centered at $\omega=0$ and $2\omega_0$) are susceptible to the ambiguity described in this Letter. However, the second correlation (arising from envelope centered at ω_0) is more robust and shows a significantly higher difference in comparison with the difference between the FROG traces. For the example field in Fig. 1(a), the IAC trace is plotted in Fig. 2(a). The difference between the IAC of the fields in Figs. 1(a) and 1(b) is plotted in Fig. 2(b). The difference is approximately 2.9% of the peak of the IAC traces. This is considerably higher than the difference between the FROG traces ($\Delta_m=0.004\%$) or MOSAIC traces

($6.5\times10^{-7}\%$). This suggests that for certain pulse shapes, IAC is more sensitive than the FROG traces, contrary to what is considered true in the literature. We note that the better sensitivity with IAC-based methods does not necessarily imply better reconstructions. Further, as the pulselets become closer, the difference between the FROG traces increases faster than difference between the IAC traces.

The pulse construction scheme described in Eq. (2) can be extended to include three pulselets instead of two. For example, we can add another pulselet, $S_0E_0(t)$, between the two pulselets in Eq. (2). The three pulselet pairs will also have nearly identical SHG FROG traces when the pulselets are sufficiently separated in time. The two and three pulselet pair methods can be generalized further to fields with an arbitrary number of even and odd pulselets. As an example, $E_{a,b}(t)=S_0E_0(t)+\Sigma_{i=0}^{n}[S_{a_i,b_i}E_0(t-T_i)+S_{a'_i,b'_i}E_0(t+T_i)]$ denotes the field pairs with $2n+1$ pulselets within. The pulselet amplitudes have to satisfy, $S_{a_i}=S_{b'_i}$ and $S_{a'_i}=S_{b_i}$.

It might seem that the ambiguity presented in this Letter may not be important because such special pulselets are relatively rare in practice or, even if they occur, one can tell what the other possible pulse shape should be. However, we find that this is not always true. By bringing the pulselets closer, complicated shaped fields with very distinct shapes can be constructed, whose FROG traces can differ by a mere percent or less. Especially when multiple pulselets are used, it can be hard to guess that the two pulses are in some way related. We present such an example in Fig. 3. The field shapes are constructed using three pulselets that are separated by $T=22$ fs. Obviously these fields are very different, and it is not evident that they are related. The SHG FROG traces are very similar, shown in Fig. 3(c) and 3(d). The difference between the two FROG traces [Fig. 3(e)] shows that $\Delta_m<2\%$ and $\Delta_e\approx10^{-6}$. Although it may be possible to notice such differences ($\Delta_m\approx2\%$) in carefully measured SHG FROG traces (since they have a good dynamic range [3]), it is important to note that Δ_e is a few orders of magnitude smaller than the best known experimental SHG FROG reconstruction error [7] (8×10^{-4}). This suggests that the SHG FROG reconstruction error, a measure of consistency, is not always a measure of accuracy.

In conclusion, we have presented an amplitude ambiguity with SHG FROG traces. It has come to our attention that such ambiguities were noticed during the use of multiple-pulse-based SHG FROG techniques [12,13]. The ambiguous fields are constructed by interchanging positions of identical pulselets within a field, in a certain order. When the pulselets are well separated, the FROG traces are nearly identical. Surprisingly for such fields, IAC traces are more sensitive than FROG traces. More complicated field shapes can be constructed when the separation is small. Some of these pulses can be discerned only by using very careful FROG measurements, and some may not be distinguishable, even with the current best signal-to-noise values.

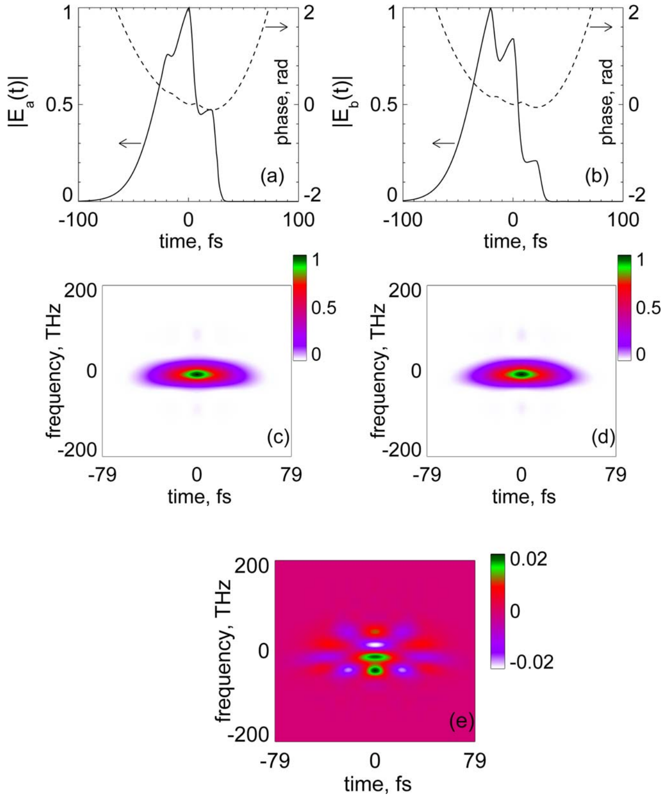

Fig. 3. (Color online) Example 2: (a), (b) magnitudes of electric field envelopes. (c), (d) SHG FROG traces for the fields in (a) and (b). (e) Normalized difference between the two FROG traces. For the two pulses, $\Delta_m=1.9\%$ and $\Delta_e=7\times10^{-6}$.

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