**Modern Physics**
**Study Questions for the Fall 2022 Departmental Exam**
**August 2022**

1. The muon is an unstable particle that spontaneously decays into an electron and two neutrinos. If the number of muons at $t = 0$ is $N_0$, the number at time $t$ is given by $N = N_0 e^{t/\tau}$, where $\tau$ is the mean lifetime, equal to 2.2 μs. Suppose the muons move at a speed of $0.95c$ and there are $5.0\ 10^4$ muons at $t = 0$.
   a. What is the observed lifetime of the muons?
   b. How many muons remain after traveling a distance of 3.0 km?.

2 The reciprocal wavelength equivalent to the fundamental vibrational mode of a $Cl_2$ molecule, each of whose atoms has an atomic weight of 35, is 2940.8 $cm^{-1}$
   a. From this value, determine the corresponding reciprocal wavelength for $Cl_2$ in which one atom has atomic weight 35 and the other 37.
   b. Determine the separation of spectral lines due to this isotope effect in in electronvolts.

3. One of the strongest emission lines observed from distant galaxies comes from hydrogen and has a wavelength of 122 nm (in the ultraviolet region).
   a. How fast must a galaxy be moving away from us in order for that line to be observed in the visible region at 366 nm?
   b. What would be the wavelength of the line if that galaxy were moving toward us at the same speed?

4. The natural decay chain $^{238}_{92}U \rightarrow {}^{206}_{82}Pb$ includes several alpha and negative beta decays.
   a. How many decays of each type are there in this chain?
   b. The decay chain actually has several different branches, corresponding to different orders of the alpha and beta decays. Does the answer to (a) depend on which branch is taken?

5. A typical particle production process is

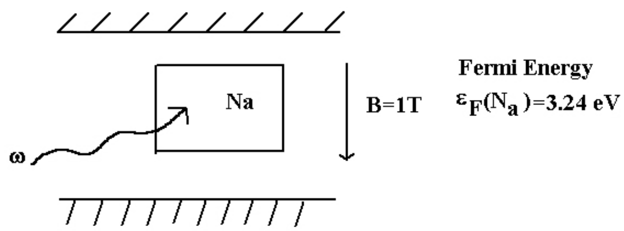
$$p + p \rightarrow p + p + \pi^0.$$

Mass of proton = 938 MeV/$c^2$; Mass of $\pi^0$ = 135 MeV/$c^2$

   a. In a standard accelerator one of the initial protons would be at rest. In this case what is the minimum *laboratory* energy for the moving proton so that the reaction listed above occurs.
   b. In colliding beam accelerators, such as the one proposed for the Superconducting Supercollider, the two initial particles are moving toward each other with equal speeds. Why do colliding beam accelerators have an energetic advantage over the type of accelerator described in part a of this question?
   c. What energy would be required of *each proton* in a colliding beams accelerator to produce a pion?

6. A slab of lead shielding 1.0 cm thick reduces the intensity of 15 MeV $\gamma$ rays by a factor of 2.
   a. By what factor will 5.0 cm of lead reduce the beam?
   b What is the interaction cross-section for 15 MeV $\gamma$-rays in lead nuclei? Lead has a density of 11.4 g/cm$^3$, and an atomic weight of 207 g/mol

7. A pure single crystal of the metal sodium is placed in a static magnetic field of 1T as shown. The metal is illuminated by microwave energy of frequency $\omega$. Excess microwave energy is absorbed at the cyclotron resonance frequency $\omega_c$.
   a. For an ideal free electron gas determine a value for $\omega_c$.
   b. Determine the characteristic size of the orbital motion.
   c. Explain what is occurring at the resonance frequency.
   d. Calcium has a similar atomic density as sodium but it has a valence of 2 while sodium has a valence of 1. Estimate the characteristic size for orbital motion for calcium. (6)




8. Ordinary potassium contains 0.012 percent of the naturally occurring radioactive isotope 40K, which has a half-life of 1.3 x 109 years.
   a. What is the activity (decays/sec) of 1.0 kg of ordinary potassium?
   b. What would have been the fraction of 40K in ordinary potassium 4.5 109 years ago?

9. Since the neutron has no charge, its mass must be found indirectly. From the data given below determine the mass difference $m_n$-$m_H$. It might not be necessary to use all of the information provided.
   a. $^2H + ^2H \rightarrow ^3H + ^1H + 4.031$ MeV
   b. $^1H + n \rightarrow ^2H + \gamma$, $E\gamma = 2.225$ MeV
   c. $^3H \rightarrow ^3He + e + 0.0185$ MeV
   d. $^2H + ^2H \rightarrow ^3He + n + 3.267$ MeV

10. In a double-slit interference experiment using helium atoms, the beam has atoms of kinetic energy 0.020 eV. The distance from the double slits to the observation plane is 64 cm.
   a. What is the de Broglie wavelength of a helium atom with this kinetic energy?
   b. What separation between the double slits is needed to produce interferences fringes separated by 10 $\mu$m at the observation plane?

11. A space rocket with a proper length of 2000 m moves at a speed of 0.85c directly away from an observer on earth. An astronaut standing at the center of the rocket simultaneously launches two electrons: one aimed toward the front of the rocket, the other toward the rear. The electrons move through a vacuum pipe at a speed of 0.95c; the rest mass of an electron is 0.511 $MeV/c^2$.

    a. In the astronaut's frame, which electron reaches its end of the rocket first? How much time elapses between the two collisions?

    b. In the earth observer's frame, which electron reaches its end of the rocket first? How much time elapses between the two collisions?

    c. What is the momentum of each electron in the astronaut's frame?

    d. What is the momentum of each electron in the earth observer's frame?

12. The atomic number of Na is 11.

    a. Write down the electronic configuration for the ground state of the Na atom showing in standard notation the assignment of all electrons to various one-electron states.

    b. Give the standard spectroscopic notation for the ground state of the Na atom.

    c. The lowest frequency line in the absorption spectrum of Na is a doublet. What are the spectroscopic designations of the pair of energy levels to which the atom is excited as a result of this absorption process?

    d. Discuss the mechanism responsible for the splitting between this pair of energy level.

    e. Which of these levels lies lowest? Discuss the basis on which you base your choice.

13. A beam of protons, each with kinetic energy 40 MeV, approaches a step potential of 30 MeV.

    a. What fraction of the beam is reflected and transmitted?

    b. How does your answer change if the particles are electrons?

14. A helium-neon laser emits light with wavelength 6328 Å in a transition between two states of neon atoms.

    a. Why is helium needed in the laser? Why not just neon?

    b. What would be the ratio of population of the upper state relative to the lower state of the neon atoms if they were in thermal equilibrium at 300 K?

15. Consider a Si semiconductor, in which the dielectric constant $\kappa = 12$ and electron effective mass $m^* = 0.2m_e$.

    a. Calculate the binding energy of the extra electron of an impurity arsenic.

    b. Suppose the arsenic doping concentration is $10^{17}$ $cm^{-3}$, what fraction of these donor atoms would supply an electron to the conduction band at room temperature?

16. The sun is not only a strong source of photons, but is also a strong source of neutrinos.

    a. What is the basic process for energy generation in the sun? Why does this process generate neutrinos?

    b. The flux of radiant energy (photons) from the sun striking the earth's atmosphere is 1350 $W/m^2$. Estimate the number of solar neutrinos which strike a square meter of the earth's surface every second. Justify any assumptions or approximations used in your estimate. The binding energy of helium 4 is 7.2 MeV per nucleon, and 1 eV = $1.6 \times 10^{-19}$ J.

17. A $K^o$ meson (mass 497.7 $MeV/c^2$) decays to a $\pi^+$, $\pi^-$ meson pair with a mean life of 0.89 x $10^{-10}$ s.

a. Which one of the fundamental interactions is responsible for this decay?

b. Suppose the $K^o$ has a kinetic energy of 276 MeV when it decays, and that the two $\pi$ mesons emerge at equal angles to the original $K^o$ direction. Find the kinetic energy of each $\pi$ meson and the opening angle between them. The mass of a $\pi$ meson is 139.6 $MeV/c^2$

18. In a muonic hydrogen atom, the electron is replaced by a muon, which is 207 times heavier.

a. What is the wavelength of the n=2 to n=1 transition in muonic hydrogen? In what region of the electromagnetic spectrum does this wavelength belong?

b. How would the fine structure and Zeeman effects differ from those in ordinary atomic hydrogen?

19. Three regions of scattered light are typically observed in Raman spectra.

a. Explain what these three regions are. What is the physical reason for the appearance of inelastically-scattered components?

b. Using the Boltzmann equation calculate the ratios of anti-Stokes to Stokes intensities at 20 °C and 40 °C for the Raman lines of $CCl_4$ molecule (218 $cm^{-1}$ and 459 $cm^{-1}$)

20. When sodium metal is illuminated with light of wavelength 4.20 x $10^2$ nm, the stopping potential is found to be 0.65 V; when the wavelength is changed to 3.10 x $10^2$ nm, the stopping potential is 1.69V. Using *only these data* and the values of the speed of light and the electronic charge, find the work function of sodium and a value of Planck's constant.