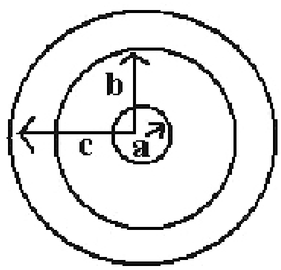
Electricity & Magnetism
Study Questions for the Fall 2022 Department Exam
May 18, 2022

1. a. Find the capacitance of a spherical capacitor with inner radius ℓ_i and outer radius ℓ_0 filled with dielectric of permittivity ε .
   b. Find the capacitance if the dielectric fills only the lower half of the capacitor.

2. A 0.5 ampere electron beam is accelerated through a potential of 5000 Volts, after which it becomes an (infinitely) long cylindrically symmetric pencil with uniform current density across a 0.3 mm radius, with no current outside this radius. The beam is surrounded by a long concentric tube of 3 mm radius held at ground potential.
   a. Calculate the charge density within the beam.
   b. Calculate the radial field and potential as a function of distance from the beam axis (r), both inside and outside the beam.
   c. Sketch E and V versus r.
   d. Evaluate V on the axis of the beam.

3. A long co-axial cable (shown in cross section) has uniform current 'I' flowing in the center conductor into the paper and the same current 'I' flowing in the outer cylinder out of the paper. Find the magnetic field in the following regions - (i) $r<a$ (ii) $a<r<b$ (iii) $b<r<c$ (iv) $r>c$ where 'r' is the distance of any point (in the specified regions) from the center of the co-axial cable.

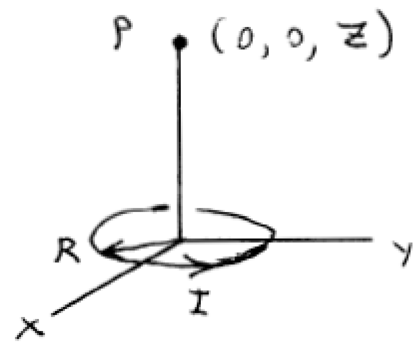


4. For the Yukawa potential $\phi(r) = g^2 e^{-mr}/r$, find the electric field, and the charge distribution that produces this potential.

5. A plane wave is polarized in the +x direction and travelling in the +z direction. The wave is reflected from the xy plane at $z = 0$, where the index of refraction is n_1 for $z < 0$ and n_2 for $z > 0$.
   a. Write down the E and B fields for the incident, the reflected and the transmitted wave.
   b. Calculate the transmission and reflection coefficients.

6. A (non-radiating) particle of mass m, charge q, moves under the influence of a uniform electric field $\vec{E} = E\hat{y}$. The particle has initial momentum $\vec{P}_o = P_o\hat{x}$. Using *relativistic* dynamics, determine,
   a. the kinetic energy as a function of time,
   b. the position of the particle as a function of time,
   c. the shape of the path followed by the particle.

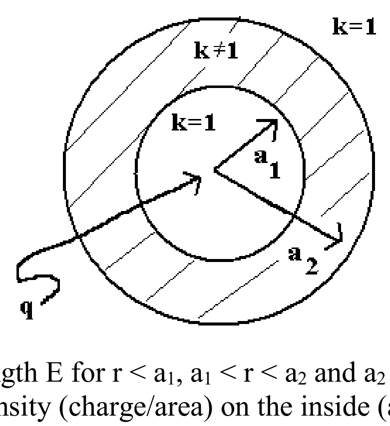
7. A loop of wire of radius R lies in the xy plane centered on the origin carrying a current I as drawn.
   a. Find the magnetic field at point P at (0, 0, z).
   b. Expand your result to two terms for z >> R. Identify the poles.



8. A cylindrical wire of length *l,* radius a, and conductivity σ has a uniformly distributed current density J flowing through it. Evaluate the **E** and **H** fields to get the Poynting flux **S** through a closed surface just outside the conductor. Show that it is exactly equal to the heat produced by the current.

9. Answer briefly the following questions.
   a. What is the physical interpretation of the surface integral of the Poynting vector over a closed surface?
   b. Discuss how magnetization current arises in a magnetic material.
   c. Why can there be no static fields inside a perfect conductor?
   d. Are all Maxwell's equations independent?
   e State the boundary conditions for the electric and magnetic field components at the interface between two dielectric media.

10. A hollow dielectric sphere is centered on the origin. It has dielectric constant $K \neq 1$. Its inner radius is a_1 and its outer radius is a_2. It is uncharged. Now a point charge $q > 0$ is placed at the origin.



   a. Find the electric field strength E for $r < a_1$, $a_1 < r < a_2$ and $a_2 < r$.
   b. Find the surface charge density (charge/area) on the inside (at $r = a_1$) and on the outside (at $r = a_2$).
   c. Now the situation is changed. A thin conducting coating is applied to the outside of the sphere, and this surface is maintained at $+V_0$ volts relative to infinity. Find E(r) for $r > a_2$.

11. In free space an electromagnetic wave has its electric field vector given by

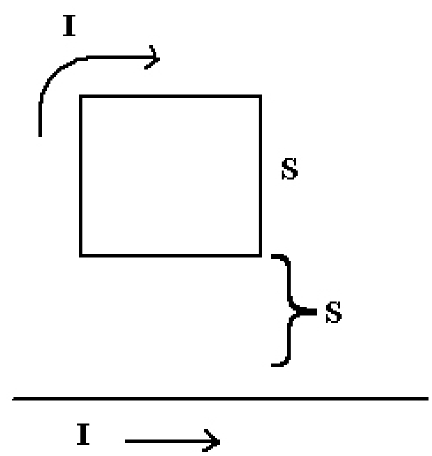
$$\vec{E}(x, y, z, t) = E_o \hat{x} \cos(kz - \omega t) + E_o \hat{y} \sin(kz - \omega t)$$

   a. Describe this wave: polarization? direction of propagation? intensity?
   b. Write an equation for $\vec{B}$ in this electromagnetic wave.
   c. What is E_0 if the wavelength is 4000 Å and the intensity in the beam is 10^5 W/m^2?

12. A cylindrical capacitor consists of two long, concentric tubes of sheet metals of radii R_1 and R_2, respectively. The space between the tubes is filled with a dielectric of constant ε.
   a. Find the capacitance of this capacitor.
   b. Suppose the potential difference between the shells is V_o, find the electrostatic energy.

13. A sphere of radius R carries charge Q distributed uniformly over its surface. The sphere is rotated at a constant angular velocity ω around the z-axis. Calculate the magnetic dipole moment. Find the magnetic field at the point (x,0,0) where x>>R.

14. Find the force acting on a square loop placed as shown in the figure. Indicate if the force is attractive or repulsive.



15. A plane electromagnetic wave with frequency ω travels in the z-direction in a medium with a real dielectric constant ε and a real conductivity σ. The wave is plane polarized in the x-direction.
   a. What is the complex wavevector $\vec{k}$?
   b. Give expressions for $\vec{E}(\vec{r}, t)$ and $\vec{B}(\vec{r}, t)$.
   c. Over what distance ℓ is half of the wave's energy density extinguished?

16. A copper wire with a diameter of 2 mm has a resistance of 1.6 ohm for every 300 meters of length. It is carrying a current of 25 Amps. Assuming that this DC current density is uniformly distributed over the cross section,

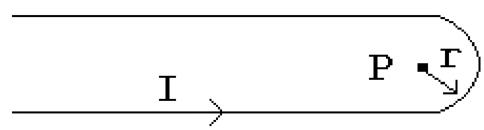
a. What is the numerical value of the resistivity ρ of the copper from the information given?

b. Determine the electric field vector **E**, the magnetic induction vector **B**, and the Poynting vector **S** at the surface of the wire. (Be sure to state their directions and actual numerical values. If you need it, $\mu_o / 4\pi = 10^{-7}$, $1/4\pi\varepsilon_o = 9 \times 10^9$ in MKS units.)

c. How much energy (in joules/meter) is stored in the electric field and in the magnetic field within the wire?

d. Suppose the current is increasing at the rate of 1 Amp/s. What statement can be made about the radial dependence of the induced electric field that results within the wire?

17. A long wire is bent into the hairpin-like shape shown in the figure. Find an exact expression for the magnetic field at the point P which lies at the center of the half-circle:



18. A thin grounded metal foil is in the plane (x,5,z). A +2 μC charge is at (2,2,3).

a. What is the potential (in Volts) at (4,4,2)?

b. What is the surface charge density at (4,5,2)?

c. What is the potential at (4,7,0)?

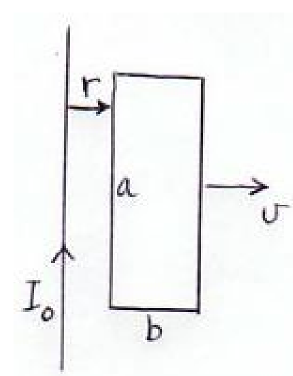
19. A circular loop of wire with radius a and electrical resistance R lies in the xy plane. A uniform magnetic field is turned on a time t=0; for t>0 the field is

$$B(t) = \frac{B_0}{\sqrt{2}}\left(\hat{j}+\hat{k}\right)\left[1-e^{-\lambda t}\right]$$

a. Determine the current I(t) induced in the loop.

b. Sketch a graph of I(t) versus t.

20. A rectangular coil of *N* turns and size $a \times b$ is near a long straight wire carrying a steady current I_0. The coil has total resistance *R*, and is being pulled away from the wire at a constant speed *v*.



a. Find the induced current in the coil as a function of the instantaneous coil-wire separation *r*.

b. What instantaneous force *F* must be applied, also as a function of the coil-wire separation *r*?

c. What total work input is required to remove the coil all the way to $r \to \infty$.