# Mathematatization:

## Student Resource Use in E&M 1

DYLAN E. MCKNIGHT

*ADVISOR: DR. ELEANOR C. SAYRE*

NSF Grants:
1430967, 1461251

# Purpose

Insight

- How upper division students think about the physical meaning of numbers.
- How to effectively teach complicated concepts such as the Taylor Series.

# Resources Framework

Resource: “Chunks of knowledge that students bring to bear on a situation.”[1]

- A Resource whose internal structure is explorable by the user is called a *concept.*[2]
- A Resource whose internal structure is no longer explorable is called a *primitive.*[2]

1 K. Black and M. Wittmann, presented at the Physics Education Research Conference 2009, Ann Arbor, Michigan, 2009, WWW Document, (http://www.compadre.org/Repository/document/ServeFile.cfm?ID=9455&DocID=1327).
2 E. Sayre, M. Wittmann, and J. Donovan, presented at the Physics Education Research Conference 2006, Syracuse, New York, 2007, WWW Document, (http://www.compadre.org/Repository/document/ServeFile.cfm?ID=5234&DocID=2130).

# Context

Junior Level Electricity and Magnetism I

16 Students

4 groups of 4 students

Groups collaborate to solve problems in class

Taylor Series covered during weeks 3 and 4

# Methodology

Progressive Refinement of Hypotheses[3] generates Emergent Claims

Video-based microanalysis of intra-group conversation

Data is taken from 2 of the 4 groups

3 R. Engle, F. Conant, and J. Greeno, 2007

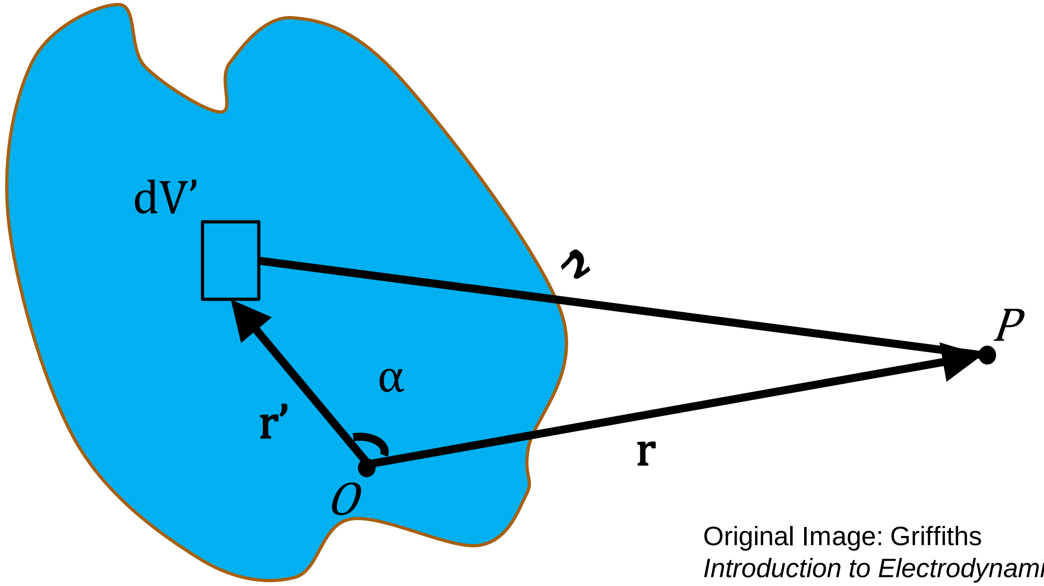
# Tasks

We observe students solving the following exercise:

Find the Multipole Expansion for the potential for *any* localized charge distribution. Write your answers in terms of powers of $\frac{1}{r}$.

The Taylor Series: $f(x) = \sum_{n=0}^{\infty} \frac{d^n f(a)}{dx^n} \frac{(x-a)^n}{n!}$

The Potential Function: $V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$




Original Image: Griffiths *Introduction to Electrodynamics* 4th Ed. Figure 3.28

# Solution

Write $\mathscr{r}$ in terms of **r** and **r’** using Law of Cosines.

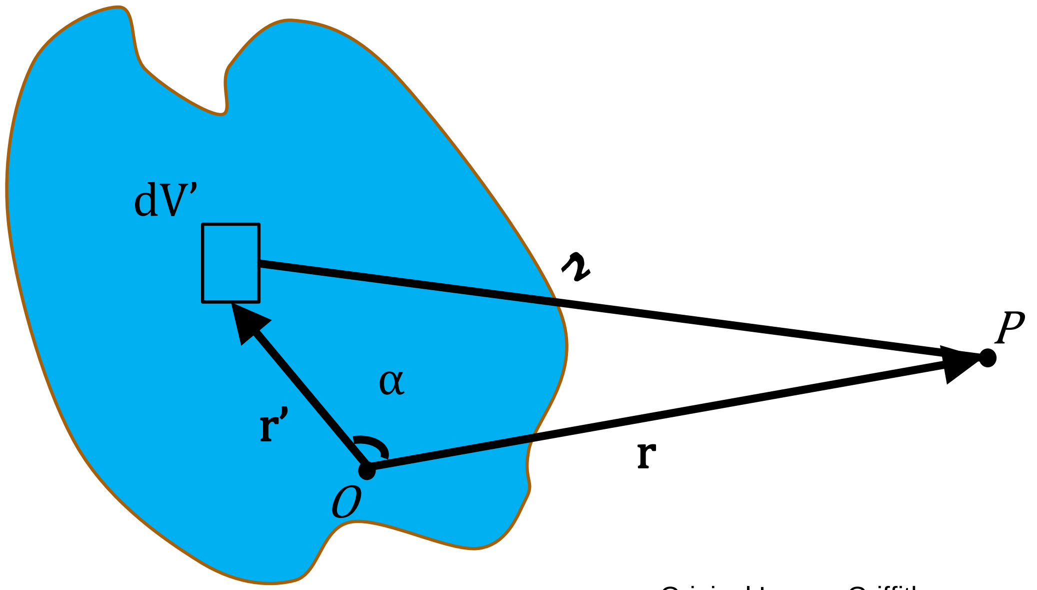
- $\mathscr{r}^2 = r^2 + (r')^2 - 2rr'\cos\alpha$

Factor a $r^2$ term out of the above expression.

- $\mathscr{r}^2 = r^2\left(1 + \left(\frac{r'}{r}\right)^2 - 2\frac{r'}{r}\cos\alpha\right)$

Set $\mathscr{r} = \sqrt{1+\epsilon}$, Where $\epsilon = \left(\frac{r'}{r}\right)\left(\frac{r'}{r} - 2\cos\alpha\right)$

- $\mathscr{r} = \frac{1}{r}(1+\epsilon)^{-1/2}$




Original Image: Griffiths *Introduction to Electrodynamics* 4th Ed. Figure 3.28

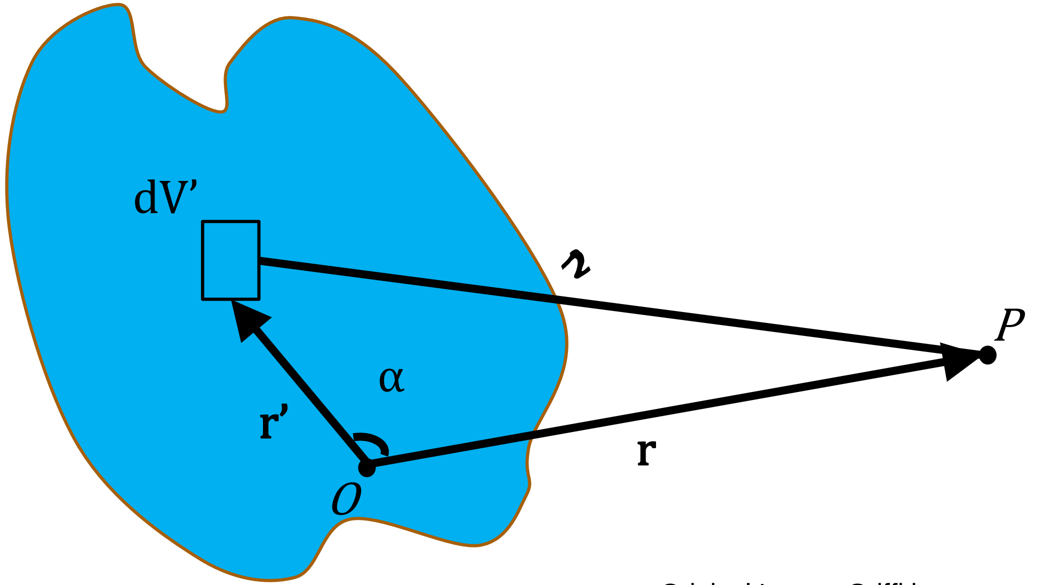
# Solution

Taylor Expand the expression $\frac{1}{r}(1+\epsilon)^{-1/2}$ with respect to $\epsilon$.

The Taylor Series: $f(x) = \sum_{n=0}^{\infty} \frac{d^n f(a)}{dx^n} \frac{(x-a)^n}{n!}$

Determine a

- a is related to the Potential as a function of r and r'.
- Assuming, $P$ is "far away," $\frac{r'}{r} \approx 0$, so we can take a $= 0$.
- $f(\epsilon) \approx \frac{1}{r}\left(1 - \frac{1}{2}\epsilon + \frac{3}{8}\epsilon^2\right)$




Original Image: Griffiths *Introduction to Electrodynamics* 4th Ed. Figure 3.28

# Observations

Students have to transform *a* from an arbitrary point to expand some function *f(x)* (Mathematics) to a value which has physical meaning (Physics).

This is exceedingly difficult for the students.

# Observations

Highly dependent on Instructor framing:

- **Instructor:** It is generally true that *a*=0.

- **Instructor**: What should we pick for *a*?

  **Adam**: 1

  **Ed**: 0

  **Bill**: -1

- Directly after discussion the Instructor asks "what should *a* be?" Ed responds "0."

- The Instructor asks "why should *a* be 0," Ed responds "because we never pick anything else."

# Observations

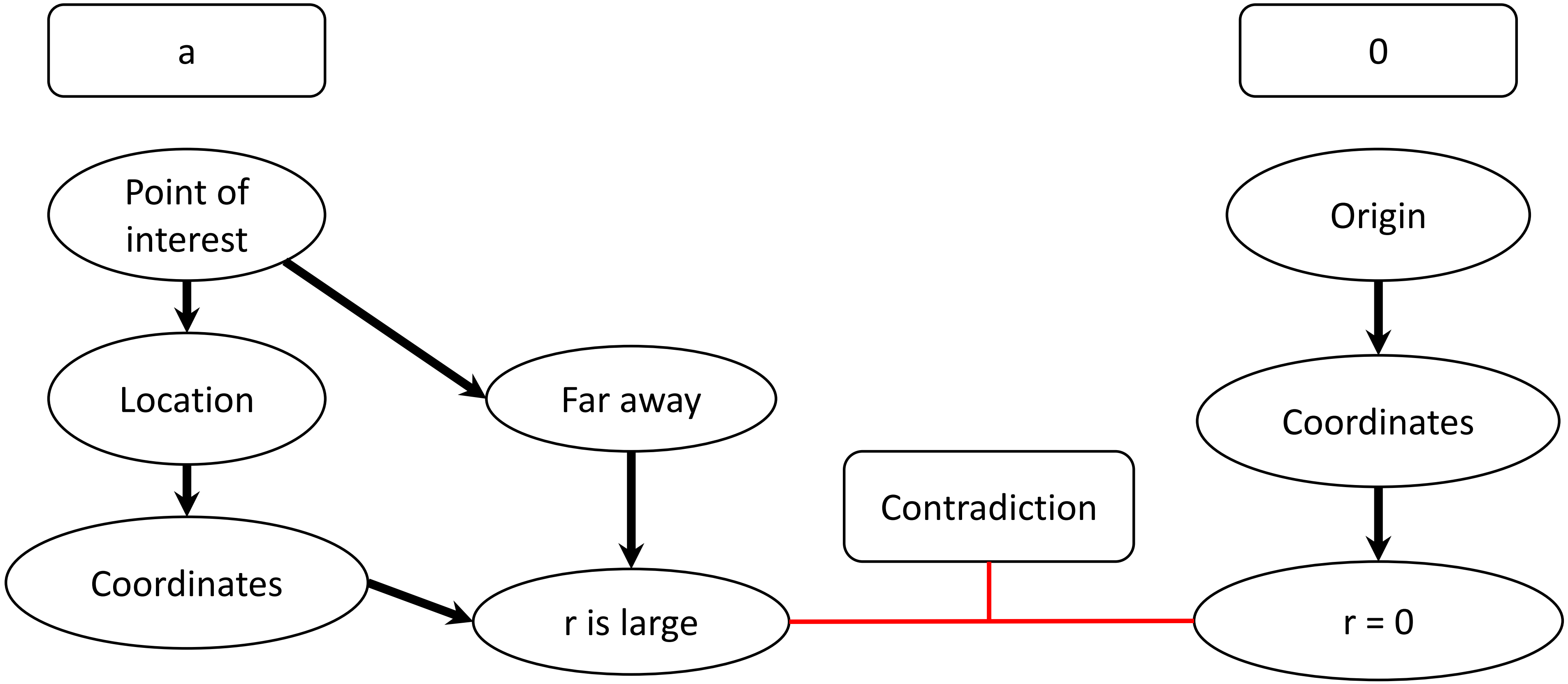
Students tend to rely heavily on prior experience when doing new Taylor Series.

- **Ed**: *a* was 0 here [Taylor Series quiz].
- **Jim**: This [Multipole Expansion] is different but I don't even understand what *a* is.
- **Ed: I** don't particularly understand why *a* was 0 is that one [Taylor Series quiz] either.

This suggests that the students are confused as to the physical meaning of *a*.

# Explanation: A Resource Graph

Observed students seem to have a thought process akin to:



# Potential Further Questions

When, Where, How should Taylor Series be presented in a Physics class and/or curriculum.

What are effective teaching strategies for framing this such that students understand *a* better?

Are there analogs to this in Chemistry and Engineering?

- If so, Where?
- Do they have similar or different resource graphs?