Appearance Probability Model of Electron Anti-neutrinos Accounting for Different Reactor Distances

Maria Veronica Prado, Advisors: Dr. Glenn Horton-Smith & Dr. Larry Weaver

REU in Physics, Department of Physics, Kansas State University

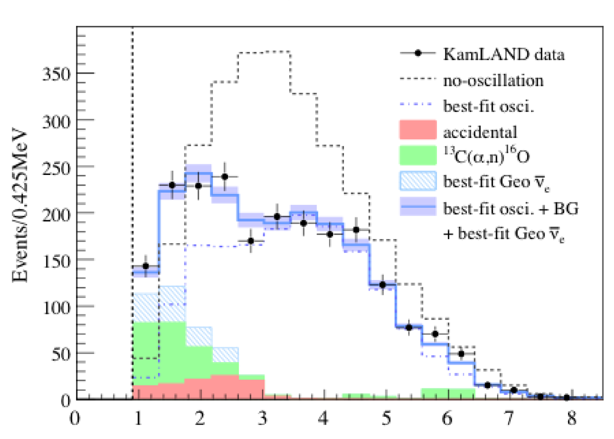
Abstract

The KamLAND experiment that had 56 reactor sources at multiple distances provided an appearance probability vs $L_0/E\nu$ graph based on one average reactor distance. This research addresses a way to find a close approximation of the appearance probability by doing a change of variables to account for the different reactor distances. The appearance probability is then able to be calculated without any assumptions of neutrino oscillations. With the data obtained from the experiment at KamLAND, it was found that using the method described here proves that neutrinos do not have a constant appearance probability. Furthermore, the research shows an empirical appearance probability as a function of L/E_ν with estimated correlated error.

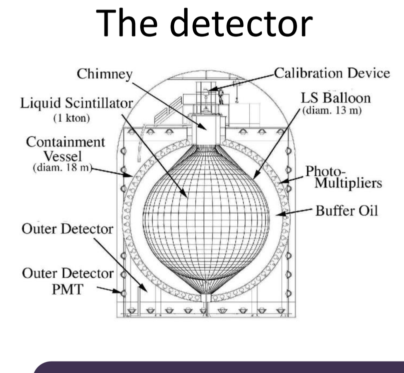
KamLAND Experiment

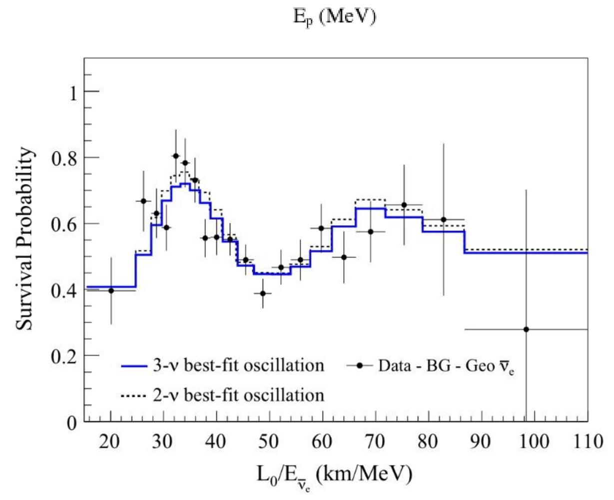
The KamLAND experiment, with its base location on the island of Honshu, Japan, consisted of 56 nuclear reactor sources at different distances containing Uranium 235 and 238, and Plutonium 239 and 241. Spontaneous fission within the reactors produced neutrons that experienced beta decay creating certain anti-neutrinos that eventually reached one detector. From the anti-neutrinos that arrived at the detector, some of them would interact with a proton found in the Liquid Scintillator and be detected by the Photo-Multiplier Tubes.

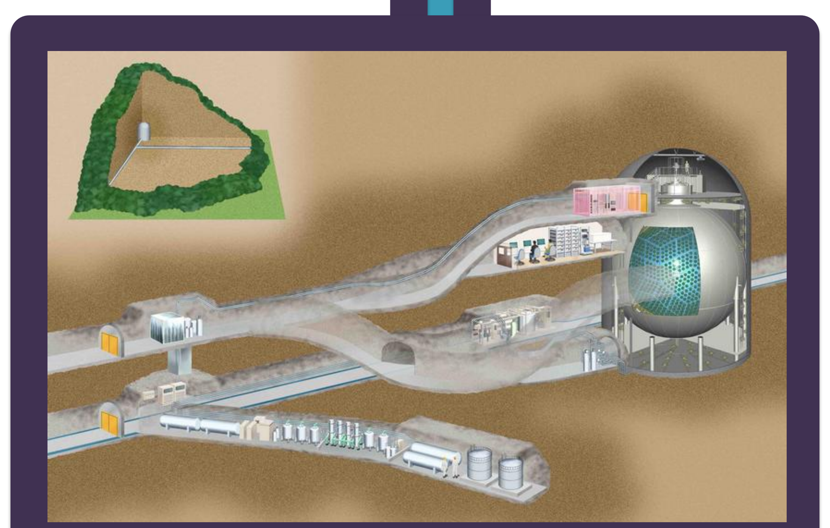
The experiment measured the number of counts of each prompt energy detected from the incoming anti-neutrinos, and it was primarily looking to figure out an appearance probability for these anti-neutrinos to, then, be able to decipher a value for delta m^2 and theta 12. The way the appearance probability was determined, however, was by calculating an average reactor distance, L_0.



The detector







From these two graphs, KamLAND chose values for both delta m^2 and theta 12 that made their theoretical equation, which assumed that neutrinos oscillate, fit the curves the best.

Our Research

This research develops an empirical approximation of the appearance probability without assuming neutrino (or anti-neutrino) oscillations and takes into account most of the reactor distances.

Incorporating Multiple Reactor Distances

Number of Ep Counts equation:

$$N(E_1 < E_p < E_2) = \sum_{i=1}^{M} \int_{E_1}^{E_2} \int_{1.8}^{10} \frac{S_i(E_\nu)}{4\pi L_i^2} \sigma(E_\nu) T(E_p, E_\nu) P_\nu\left(\frac{L_i}{E_\nu}\right) dE_\nu dE_p$$

After a change of variables:

$$\underbrace{N(E_1 < E_p < E_2)}_{N} = \underbrace{\sum_{i=1}^{M} \int_{E_1}^{E_2} \int_{l=0}^{l=\infty} dl dE_p \frac{1}{4\pi L_i l^2} S_i\left(\frac{L_i}{l}\right) \sigma\left(\frac{L_i}{l}\right) T\left(E_p, \frac{L_i}{l}\right)}_{Q} \underbrace{P_\nu(l)}_{P}$$

where, $$l = \frac{L_i}{E_\nu}$$

$$N(E_p) = Q(E_p, l) P(l)$$

what we want to find empirically

Minimizing chi square, we get:

$$\boxed{(Q^T V^{-1} Q)^{-1} Q^T V^{-1} N = P}$$

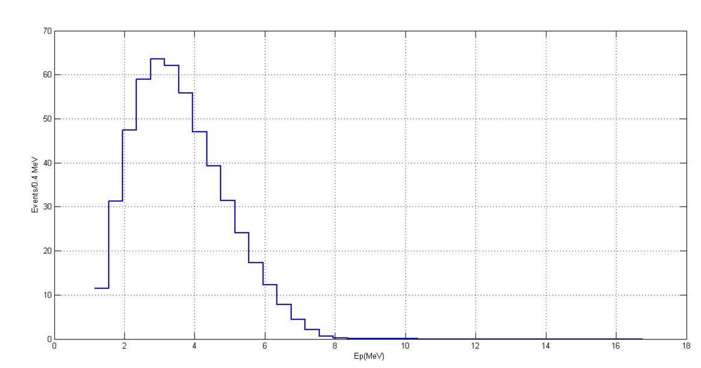
$$V^{-1} = \begin{pmatrix} \frac{1}{\sigma_1^2} & 0 & 0 & \cdots \\ 0 & \frac{1}{\sigma_2^2} & 0 & \cdots \\ 0 & 0 & \frac{1}{\sigma_3^2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Testing our Method

Forming the Q matrix:

- Test for ℓ values ranging from 9-550 km per MeV
- If the Enu lies between 1.8 MeV-10 MeV, then the values are plugged into the Q equation
- If the Enu lies outside of that range, it does not contribute to the detector, so zero is inputted for that matrix element
- Obtain a different Q matrix for each reactor
- Superpose all the Q matrices

Ex:
$$\begin{pmatrix} N(E_{p1}) \\ N(E_{p2}) \\ N(E_{p3}) \\ N(E_{p1}) \end{pmatrix} = \begin{pmatrix} Q(E_{p1},l_1) & Q(E_{p1},l_2) & Q(E_{p1},l_3) \\ Q(E_{p2},l_1) & Q(E_{p2},l_2) & Q(E_{p2},l_3) \\ Q(E_{p3},l_1) & Q(E_{p3},l_2) & Q(E_{p3},l_3) \\ Q(E_{p4},l_1) & Q(E_{p4},l_2) & Q(E_{p4},l_3) \end{pmatrix} \begin{pmatrix} P(l_1) \\ P(l_2) \\ P(l_3) \end{pmatrix}$$



Our 'no oscillations' graph (without taking into account certain small factors)

Binning:

Why bin the Eps?
- The greater the counts per bin, the smaller the relative error

Why bin the ℓ's?
- More functions than unknowns
- A higher sum in each ℓ column will provide for a smaller error

$$N_0 = Q_0 P_0$$

$$(\underbrace{\boxed{Q_0'^T V^{-1} Q_0'}}_{C})^{-1} \underbrace{Q_0'^T V^{-1}}_{Y} N = P$$

$$V^{-1} = \begin{pmatrix} \frac{1}{N'_{0_1}} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{N'_{0_2}} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{N'_{0_3}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & & \frac{1}{N'_{0_{17}}} \end{pmatrix}$$

- Q_0 = 76x1083 Q
- P_0 = 1083x1 (no oscillations)
- N_0 = 76x1
- Q'_0 = 17x11 binned Q_0
- N'_0 = 17x1 binned N_0
- P_1 = 1083x1 (using oscillation formula)
- N_1 = 76x1 (test N or real events data N)
- N'_1 = 17x1 binned N_1

Since,
$$C = C^T$$
$$C = RDR^T$$
$$C^{-1} = R^{T^{-1}} D^{-1} R^{-1}$$
$$\tilde{C}^{-1} = R^{T^{-1}} \tilde{D}^{-1} R^{-1}$$

Has the smallest eigenvalue element equal to zero

$$N_{1true} = Q_0 P_1$$
$$N'_{1observed} = N'_{1true} + \eta_{noise}$$
$$\boxed{\tilde{C}^{-1} Y N'_{1observed} \approx P_1}$$

Where R is a matrix containing the orthonormal eigenvectors for each eigenvalue, and D is a diagonal matrix containing all the eigenvalues of C

Why omit the smallest eigenvalue?

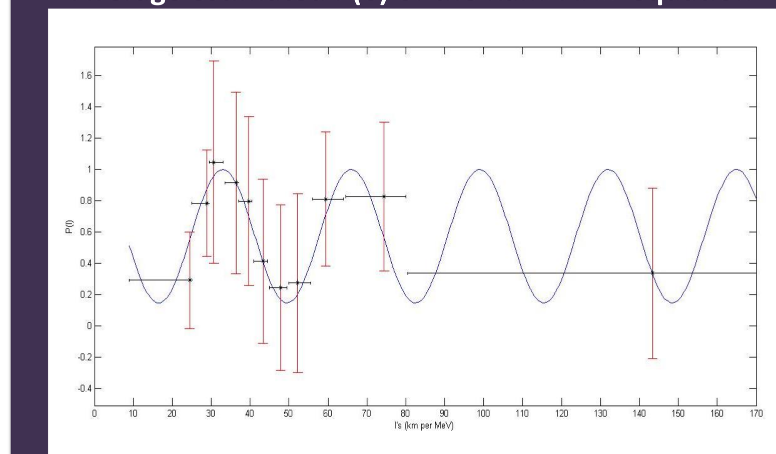
$$P_1 \approx \tilde{C}^{-1} Y (N'_{1true} + \eta_{noise})$$ Contains inverse eigenvalues

$$P_1 \approx \tilde{C}^{-1} Y N'_{1true} + \boxed{\tilde{C}^{-1} Y \eta_{noise}}$$ The smaller the eigenvalue, the more noise error it contributes

Averaging 1000 P(ℓ)s:

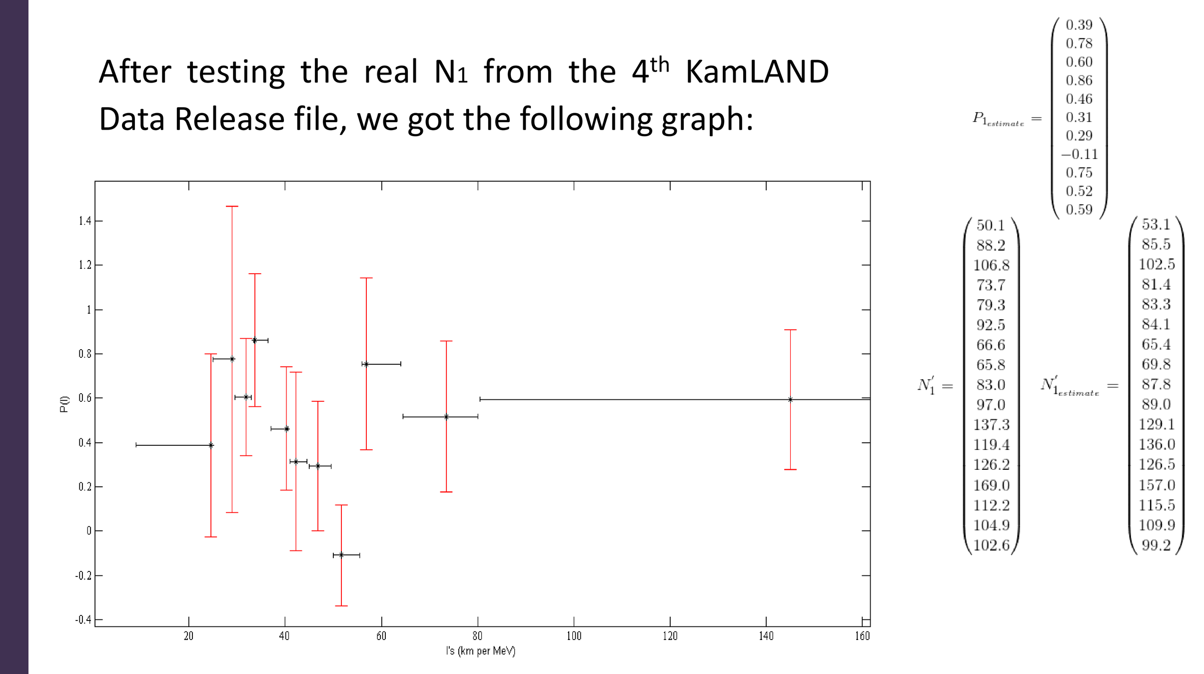
- Create N'_1 with an oscillating P_1
- Add randomized background noise to N'_1
- Create 1000 different P(ℓ)s, each using a different randomized N'_1
- Find the average P(ℓ) and its standard deviation to obtain different error bars for each P(ℓ) entry

Average estimated P(ℓ) of the KamLAND equation

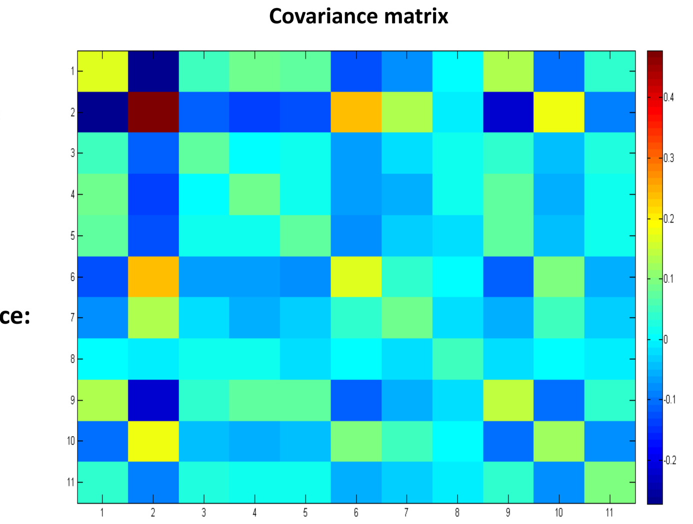


Results

After testing the real N_1 from the 4th KamLAND Data Release file, we got the following graph:



Covariance matrix



Chi square of the N between our estimate and the N observed:
6.68

Chi square of the P between our estimate and the closest straight line of 0.44 without taking into account covariance:
9.65

The above chi square with covariance:
67.95

Why this is Important

- Prove neutrino oscillations and KamLAND's conclusions empirically
- Gain knowledge about how neutrinos behave, which could lead to a better understanding of dark matter
- Gain knowledge about neutrinos to be able to control nuclear reactors efficiently by monitoring neutrinos that leave

Conclusions

After KamLAND's counts data was run through our method, we obtained appearance probabilities for 11 values of L/Eν without assuming an average L. Even though the errors appear large, the chi square accounting for covariance shows that not even the best fitted horizontal line of P(ℓ) = 0.44 will fit the data obtained due to correlations between the data points.

Using the appearance probability graph obtained in this research, the predicted N matched KamLAND's observed N based on the chi square.

We intend to apply the methods learned and used in this research to current and future experiments.

Acknowledgements

We acknowledge funding from the National Science Foundation through grant number PHY-1157044. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

A special thanks to Dr. Weaver, Dr. Horton-Smith, and the Kansas State University HEP Department.

References:

- KamLAND 4th Data Release File: http://www.awa.tohoku.ac.jp/KamLAND/4th_result_data_release/4th_result_data_release.html
- KamLAND Probability graph: http://arxiv.org/pdf/1009.4771v2.pdf
- Detector picture and detector diagram: http://kamland.lbl.gov/Pictures/kamland-ill.html