APPEARANCE PROBABILITY MODEL OF ELECTRON ANTI-NEUTRINOS ACCOUNTING FOR DIFFERENT REACTOR DISTANCES

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KamLAND Experiment

- 56 nuclear reactors and one detector
- Detector is located on the island of Honshu, Japan
- Each nuclear reactor contains Uranium 235 and 238 & Plutonium 239 and 241
- Fission occurs:

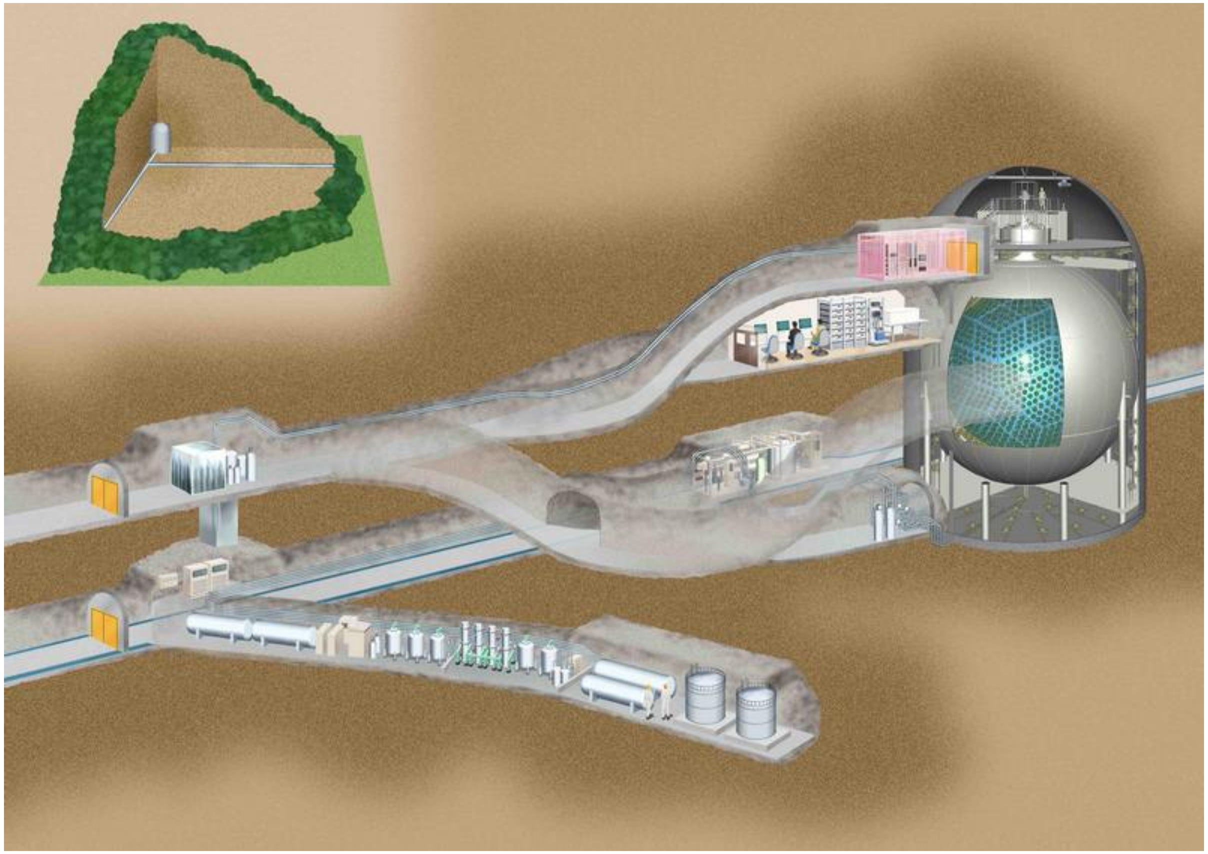
57.1% from U 235

7.8% from U 238

29.5% from Pu 239

5.6% from Pu 241

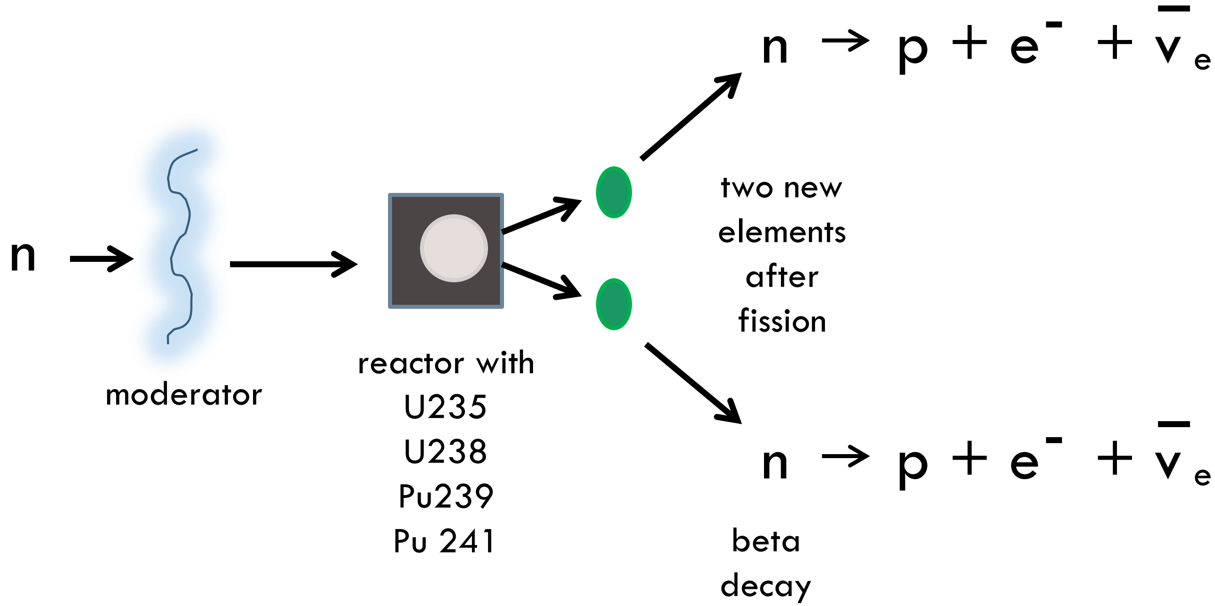
KamLAND Experiment



Source: http://kamland.lbl.gov/Pictures/kamland-ill.html

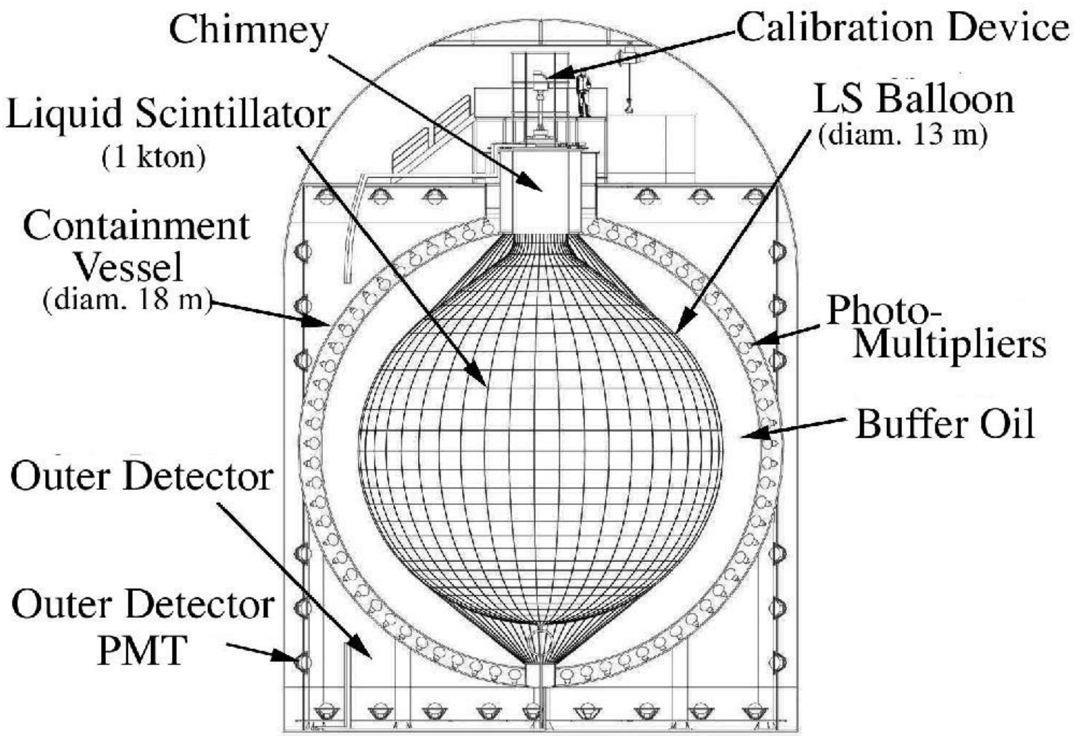
KamLAND Experiment

The reactors:



KamLAND Experiment

The detector:



Source: http://kamland.lbl.gov/Pictures/kamland-ill.html

KamLAND Experiment

- The Liquid Scintillator inside the detector contains $C_9 H_{12}$ (pseudocumene) and $C_{12} H_{26}$ (dodecane)
- Some of the anti-neutrinos coming from the reactors collide with protons found in these molecules
- Inverse beta decay

KamLAND Experiment

The detector:

Gamma photons: 10 keV
Optical photons: 1-3 eV

Positron moves through the LS losing KE as it ionizes atoms

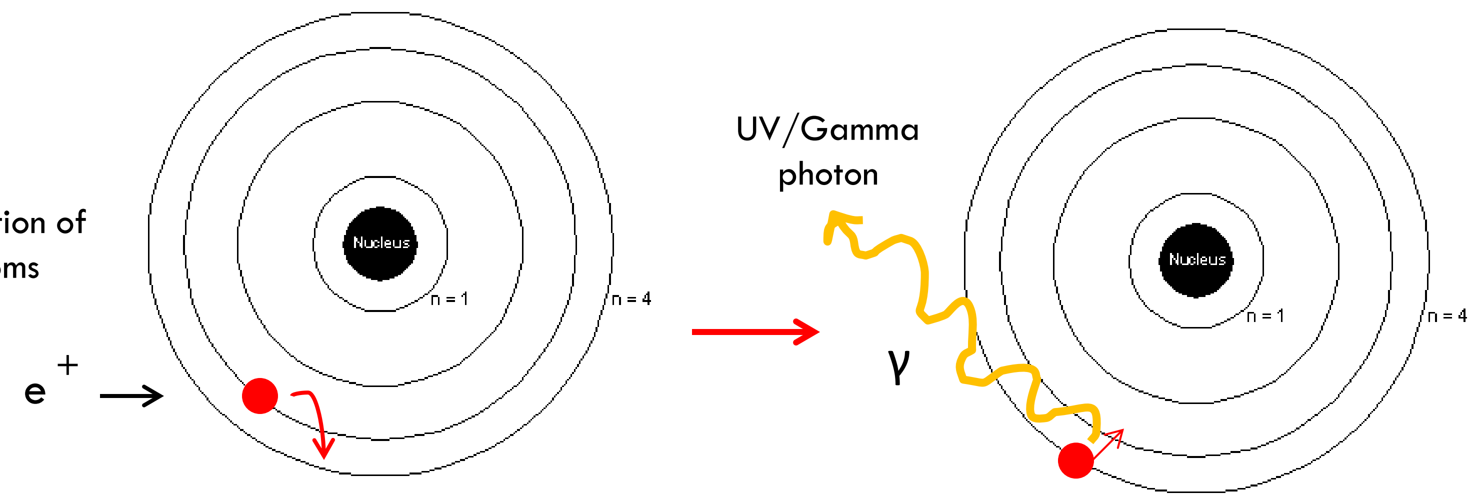
$$\bar{\nu}_e + p \rightarrow n + e^+$$

Inverse beta decay

Process A

unstable atom

Ionization of atoms

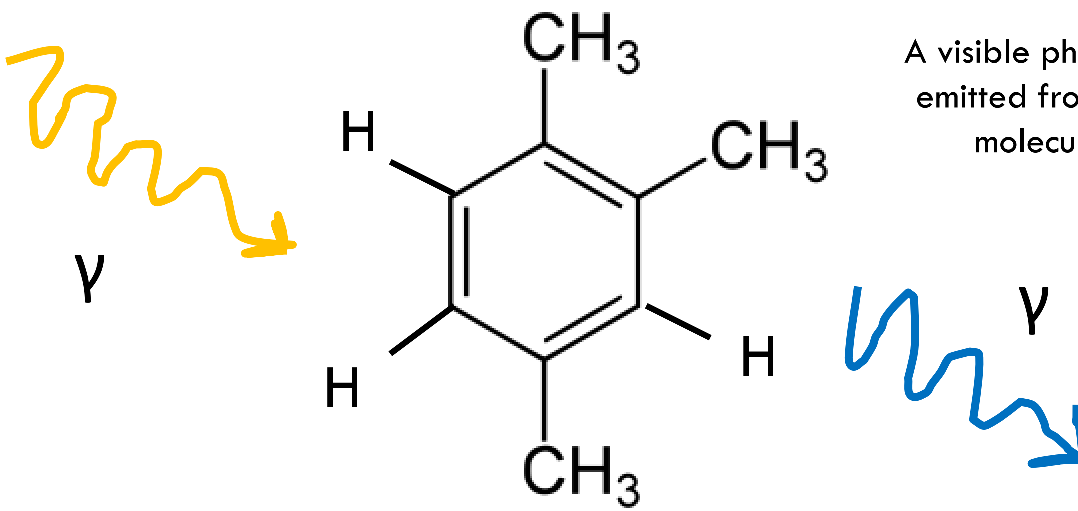


KamLAND Experiment

The detector:

Fluorescence

The UV photon hits one of the molecules and is absorbed



A visible photon is emitted from the molecule

Optical photon

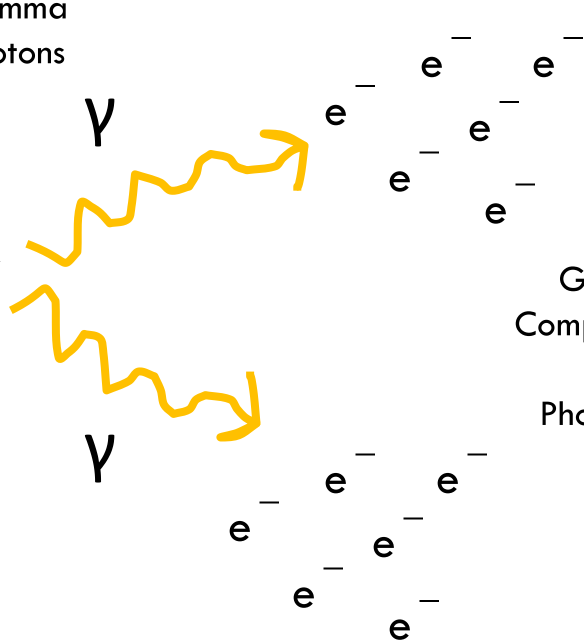
The visible photon is detected by all the photomultiplier tubes

KamLAND Experiment

Meanwhile still in the detector…

Gamma photons

$e^+ + e^- \longrightarrow$



The positron loses energy and comes to a stop where it annihilates with an electron

Gamma photons Compton scatter or go through the Photoelectric Effect

Go through Process A

DETECTED

Even though there are 3 separate signals in the PMTs, it is detected as only one

KamLAND Experiment

Simultaneously in the detector…

$$\bar{\nu}_e + p \rightarrow n + e^+$$

The neutron bounces off from the atoms in the LS and moves slower & slower until it is absorbed

Recoil

Go through
Process A

$$n + p \rightarrow {}^2H + \gamma$$

(Deuteron)

The neutron interacts with hydrogen (H) from the LS

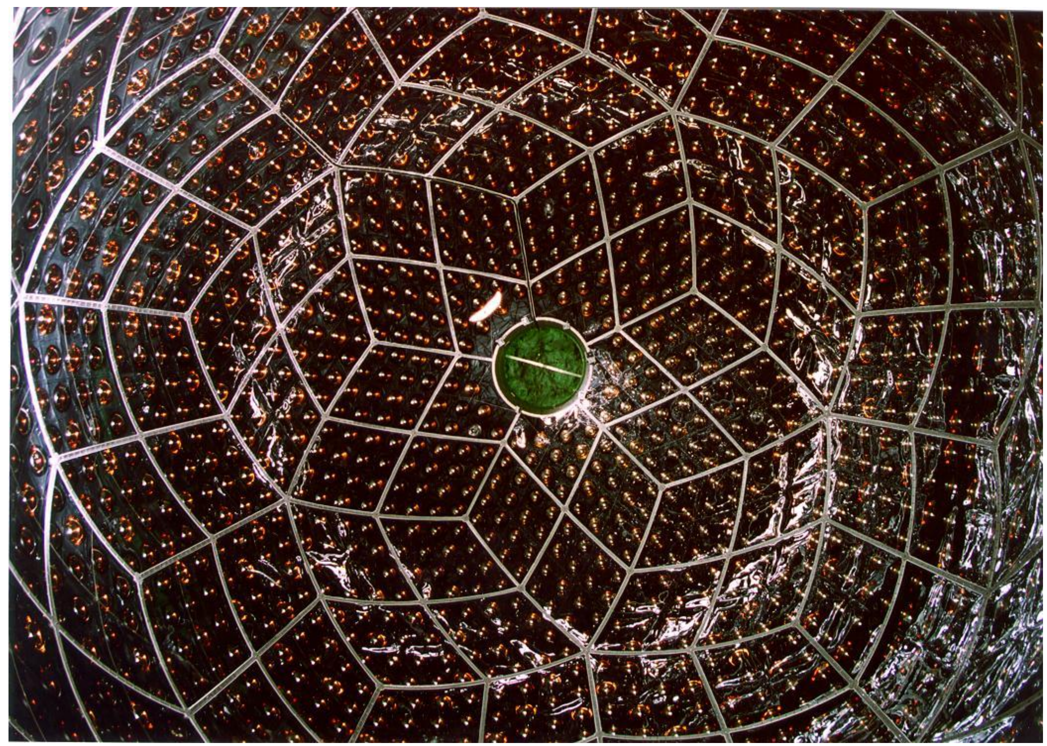
Or very rarely

$$n + p \rightarrow {}^{13}C + \gamma$$

The gamma photon compton scatters or goes through the photoelectric effect with the atoms in the LS. It produces a **detected** signal called the **delayed coincidence.**

KamLAND Experiment

The detector:

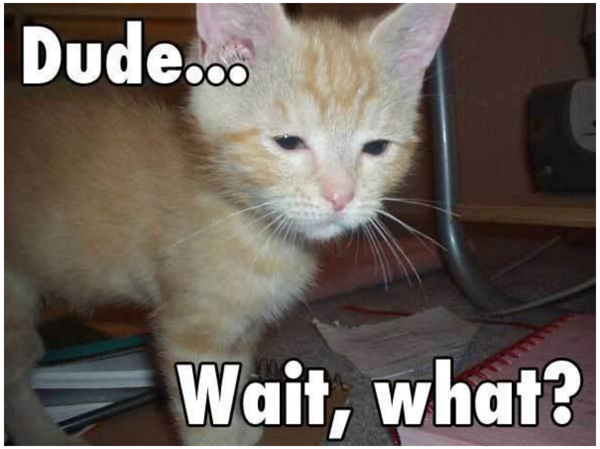


Source: kamland.lbl.gov/Pictures/picgallery.html

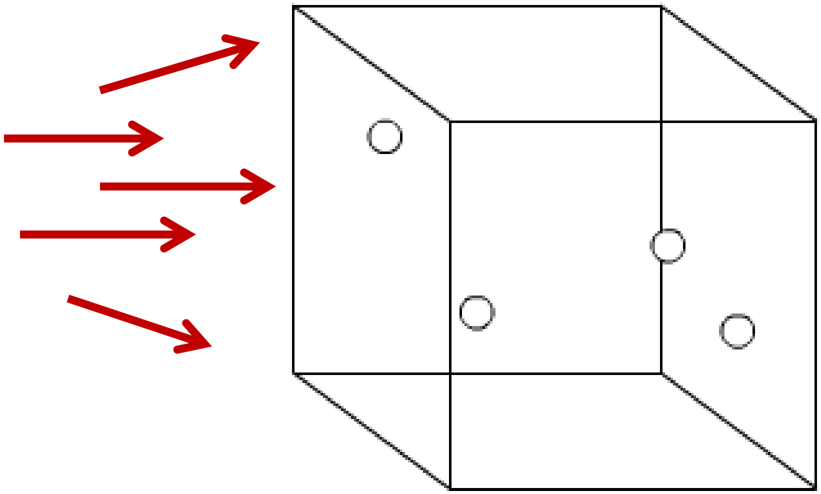
Number of Counts

Our main equation:

$$N(E_1 < E_p < E_2) = \sum_{1}^{M} \int_{E_1}^{E_2} \int_{1.8}^{10} \frac{S_i(E_\nu)}{4\pi L_i^2} \sigma(E_\nu) T(E_p, E_\nu) P_\nu(\frac{L_i}{E_\nu}) dE_\nu dE_p$$



A simple derivation



$$P_{hit} = \frac{N_t \sigma_t}{A}$$

$$Reactions = \frac{N_t \sigma_t N_{projectile}}{A}$$

$$Reactions = \frac{N_p \sigma_p N_\nu}{A}$$

Flux of the anti-neutrino

A simple derivation

$$N(E_1 < E_p < E_2) = \sum_{1}^{M} \int_{E_1}^{E_2} \int_{1.8}^{10} \frac{S_i(E_\nu)}{4\pi L_i^2} \sigma(E_\nu) T(E_p, E_\nu) P_\nu(\frac{L_i}{E_\nu}) dE_\nu dE_p$$

$$(Reactions) T(E_p, E_\nu) = Flux * \sigma_p N_p T(E_p, E_\nu)$$

where, $$Flux = \frac{S_i(E_\nu) P_\nu(\frac{L_i}{E_\nu})}{4\pi L_i^2}$$

Terms

$$N(E_1 < E_p < E_2) =$$

The number of counts at each energy prompt

$$\frac{S_i(E_\nu)}{4\pi L_i^2} P_\nu(\frac{L_i}{E_\nu}) =$$

The flux of anti-neutrinos expected at the detector

$$P_\nu(\frac{L_i}{E_\nu}) =$$

Probability that an electron anti-neutrino will stay an electron anti-neutrino by the time it reaches the detector

$$\sigma(E_\nu) =$$

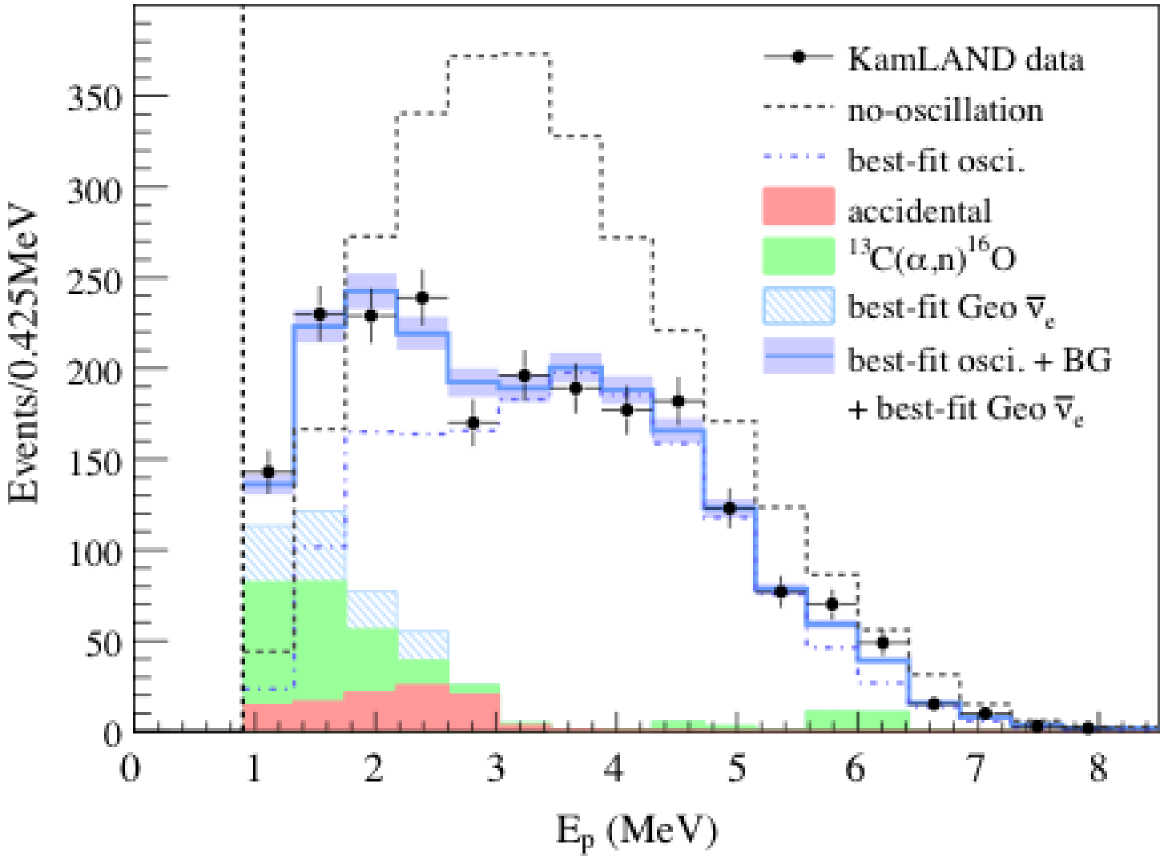
The cross section of one proton that could interact with the anti-neutrinos coming into the detector

$$\frac{S_i(E_\nu)}{4\pi L_i^2} \sigma(E_\nu) P_\nu(\frac{L_i}{E_\nu}) =$$

The number of reactions

$$T(E_p, E_\nu) =$$

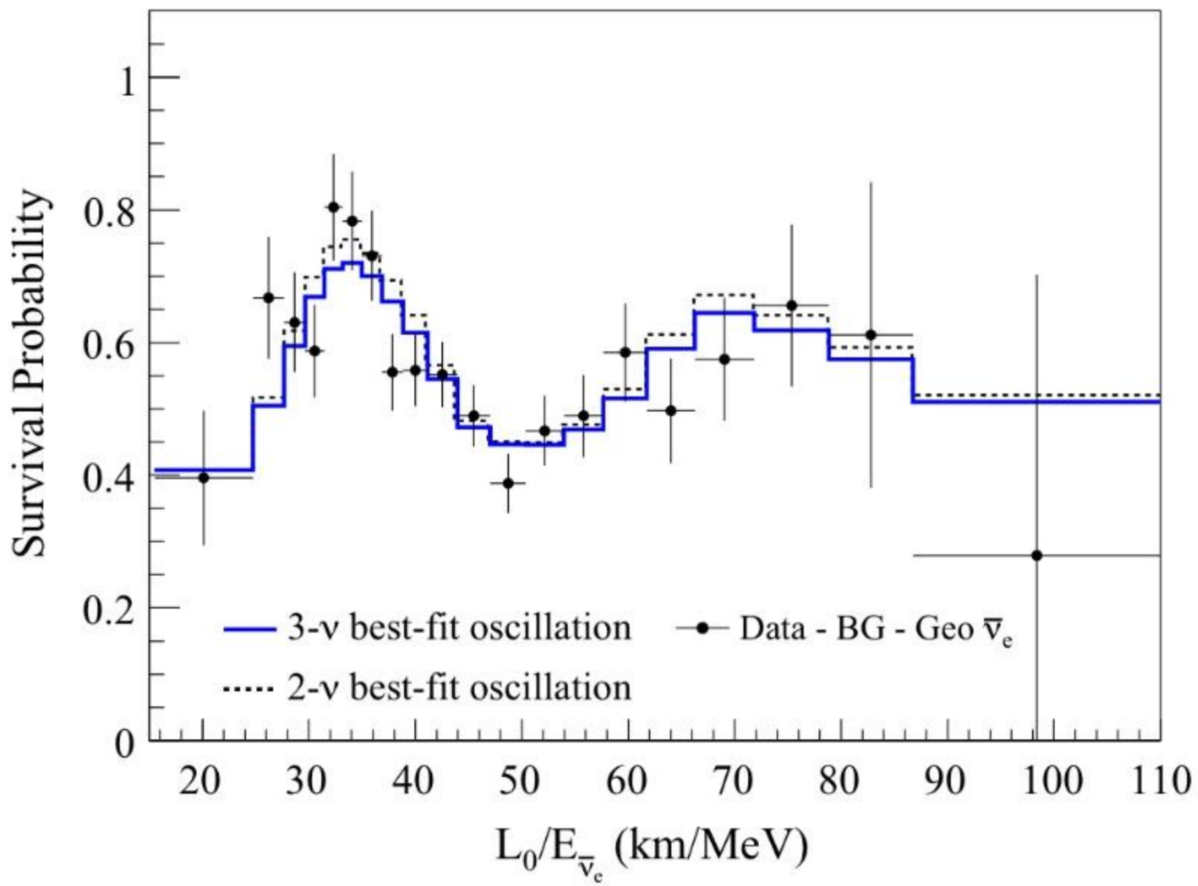
Probability of detecting a reaction from the reactions that have occurred (due to experimental error)



KamLAND events graph

Events graph with what was observed in the detector

http://www.awa.tohoku.ac.jp/KamLAND/4th_result_data_release/4th_result_data_release.html



KamLAND appearance probability graph

Appearance probability from an average reactor length

http://arxiv.org/pdf/1009.4771v2.pdf

Our theoretical research

$$N(E_1 < E_p < E_2) = \sum_{i=1}^{M} \int_{E_1}^{E_2} \int_{1.8}^{10} \frac{S_i(E_\nu)}{4\pi L_i^2} \sigma(E_\nu) T(E_p, E_\nu) P_\nu(\frac{L_i}{E_\nu}) dE_\nu dE_p$$

Make a change of variables: $$l = \frac{L_i}{E_\nu}$$

$$\underbrace{N(E_1 < E_p < E_2)}_{N} = \underbrace{\sum_{i=1}^{M} \int_{E_1}^{E_2} \int_{l=0}^{l=\infty} dl dE_p \frac{1}{4\pi L_i l^2} S_i(\frac{L_i}{l}) \sigma(\frac{L_i}{l}) T(E_p, \frac{L_i}{l})}_{Q} \underbrace{P_\nu(l)}_{P}$$

Our theoretical research

$$N(E_p) = Q(E_p, l)P(l)$$

For simplicity, we shall call this: $N = QP$ what we want to find empirically

We need to minimize χ^2 for N(Ep): $$\chi^2 = \sum_{i=1}^{n} \frac{(N_i - \bar{N})^2}{\sigma_i^2}$$

where, $\sigma = \sqrt{\bar{N}}$

★ N(Ep) has a Poisson distribution because of the rare amount of interactions at the detector

Our theoretical research

$$\chi^2 = (N - \bar{N})^T V^{-1} (N - \bar{N})$$

where, $$V^{-1} = \begin{pmatrix} \frac{1}{\sigma_1^2} & 0 & 0 \\ 0 & \frac{1}{\sigma_2^2} & 0 \\ 0 & 0 & \frac{1}{\sigma_3^2} \end{pmatrix}$$

By taking the derivative of χ^2 and setting it equal to zero, we get

transformation of the Q matrix

Our observed values

$$(Q^T V^{-1} Q)^{-1} Q^T V^{-1} N = P$$

Small proof

$$(Q^T V^{-1} Q)^{-1} Q^T V^{-1} N = P$$

$$(Q^{-1} V \underbrace{Q^{T^{-1}}) Q^T}_{\mathbb{I}} V^{-1} N = P$$

$$(Q^{-1} \underbrace{V) V^{-1}}_{\mathbb{I}} N = P$$

$$Q^{-1} N = P$$

Why we want to do this

- Prove neutrino oscillations and KamLAND's conclusions empirically
- Gain knowledge about how neutrinos behave, which could lead to a better understanding of dark matter
- Gain knowledge about neutrinos to be able to control nuclear reactors efficiently by monitoring neutrinos that leave

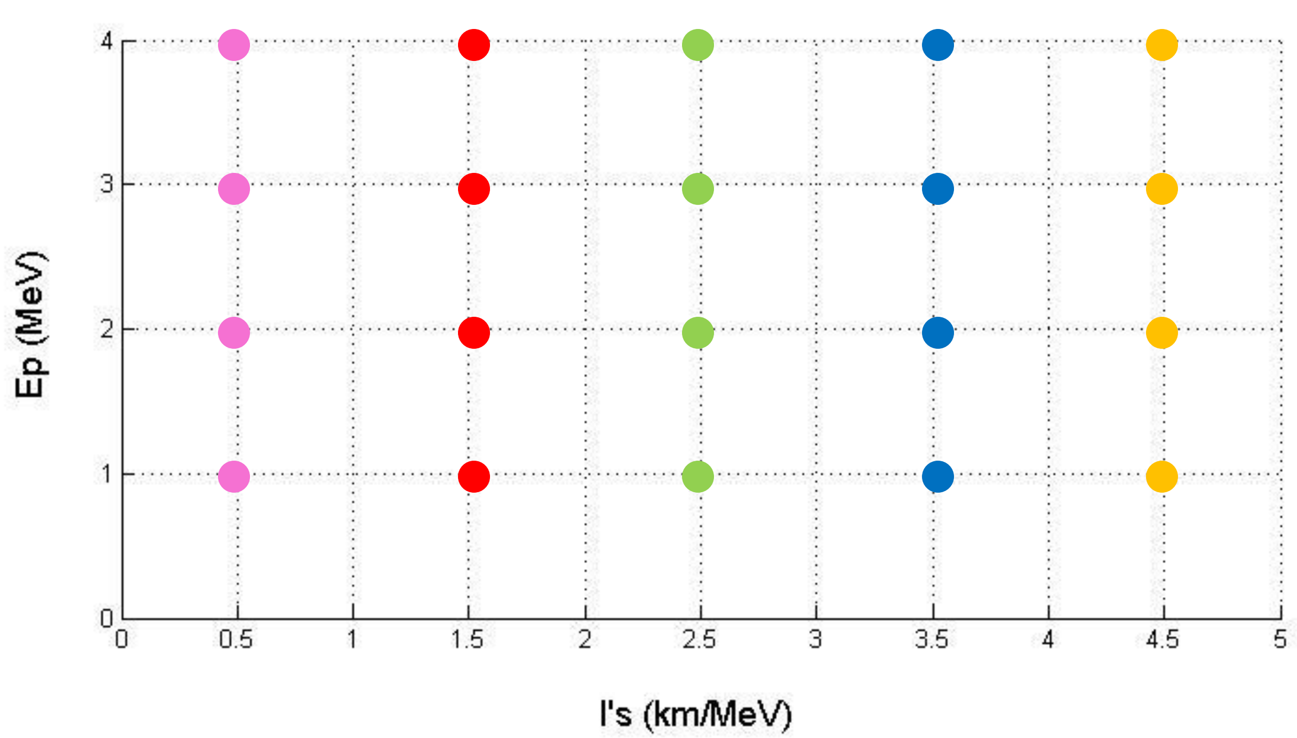
Forming the Q matrix

For ex:

$$\begin{pmatrix} N(E_{p1}) \\ N(E_{p2}) \\ N(E_{p3}) \\ N(E_{p4}) \end{pmatrix} = \begin{pmatrix} Q(E_{p1},l_1) & Q(E_{p1},l_2) & Q(E_{p1},l_3) \\ Q(E_{p2},l_1) & Q(E_{p2},l_2) & Q(E_{p2},l_3) \\ Q(E_{p3},l_1) & Q(E_{p3},l_2) & Q(E_{p3},l_3) \\ Q(E_{p4},l_1) & Q(E_{p4},l_2) & Q(E_{p4},l_3) \end{pmatrix} \begin{pmatrix} P(l_1) \\ P(l_2) \\ P(l_3) \end{pmatrix}$$

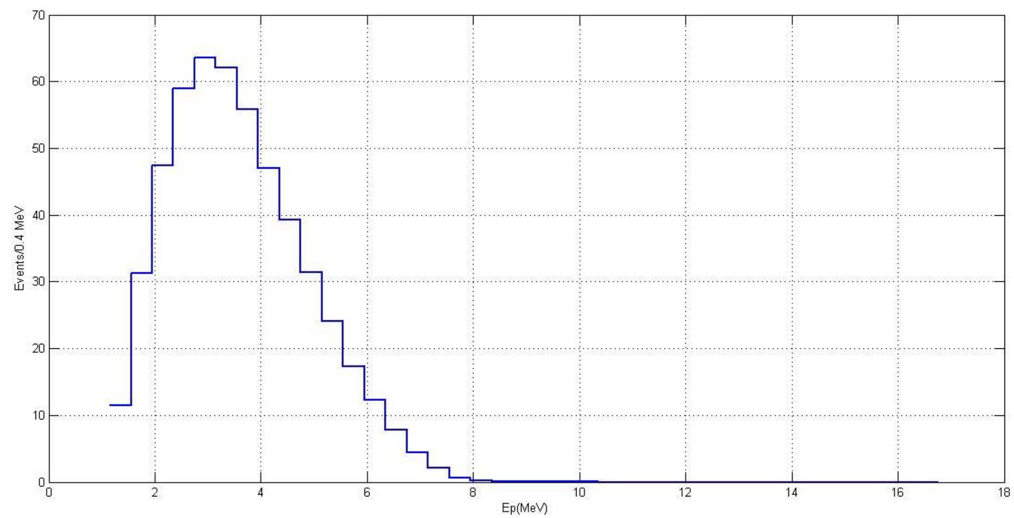
- Test for as many l's as possible, binning them
- If the E_ν lies between 1.8 MeV-10 MeV, then plug the values into the Q equation
- If the E_ν lies outside of that range, it does not contribute to the detector, so we input zero for that matrix element
- Obtain a different Q matrix for each reactor
- Superpose all the Q matrices

Forming the Q matrix

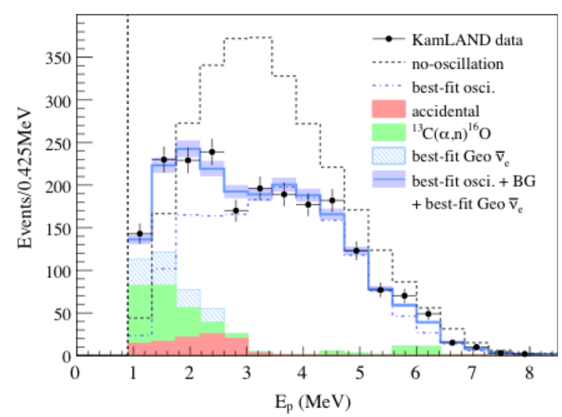


Our ‘no oscillations’ graph

our ‘no oscillations’ graph (without taking into account certain small factors)



KamLAND’s ‘no oscillations’ graph



Setting up the test

$$N_0 = Q_0 P_0$$

where, $P_0 =$ appearance probability if there were no oscillations

$$\underbrace{(\boxed{Q_0'^T V^{-1} Q_0'})^{-1}}_{C} \underbrace{Q_0'^T V^{-1} N}_{Y} = P$$

where, $V^{-1} = \begin{pmatrix} \frac{1}{N'_{0_1}} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{N'_{0_2}} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{N'_{0_3}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & & \frac{1}{N'_{0_{17}}} \end{pmatrix}$

Setting up the test

Since,

$$C = C^T$$

$$C = RDR^T$$

$$C^{-1} = R^{T^{-1}} D^{-1} R^{-1}$$

$$\tilde{C}^{-1} = R^{T^{-1}} \tilde{D}^{-1} R^{-1}$$

Where R is a matrix containing the orthonormal eigenvectors for each eigenvalue, and D is a diagonal matrix containing all the eigenvalues of C

Has the smallest eigenvalue element equal to zero

$$N_{1true} = Q_0 P_1$$

$$N'_{1observed} = N'_{1true} + \eta_{noise}$$

$$\boxed{\tilde{C}^{-1} Y N'_{1observed} \approx P_1}$$

The Binning

Why bin the Ep's?

- The greater the counts per bin, the smaller the relative error
- As a result, approaches a Gaussian

Why bin the I's?

- More functions than unknowns
- A higher sum in each I column will provide for a smaller error

Why find the eigenvalues of C?

- If product of eigenvalues is big, error is small when inverting C
- If difference is big, magnifies error

Why test it this way

- Accounting for bias by using N_0 prime to calculate V inverse instead of N_1 prime:

  - This method gives each element in N_1 prime their corresponding importance according to how many number of counts they each contribute and, therefore, how much data they contain

For example:

$$V^{-1}N_0' = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

while

$$V^{-1}N_1' = \begin{pmatrix} 0.97 \\ 1.00 \\ 0.10 \\ 1.32 \\ 0.61 \end{pmatrix}$$

Why omit the smallest eigenvalue?

$$P_1 \approx \tilde{C}^{-1} Y N_1^{'}$$

Contains background noise

$$P_1 \approx \tilde{C}^{-1} Y (N_{1true}^{'} + \eta_{noise})$$

Contains inverse eigenvalues

$$P_1 \approx \tilde{C}^{-1} Y N_{1true}^{'} + \underbrace{\tilde{C}^{-1} Y \eta_{noise}}$$

The smaller the eigenvalue, the more noise error it contributes

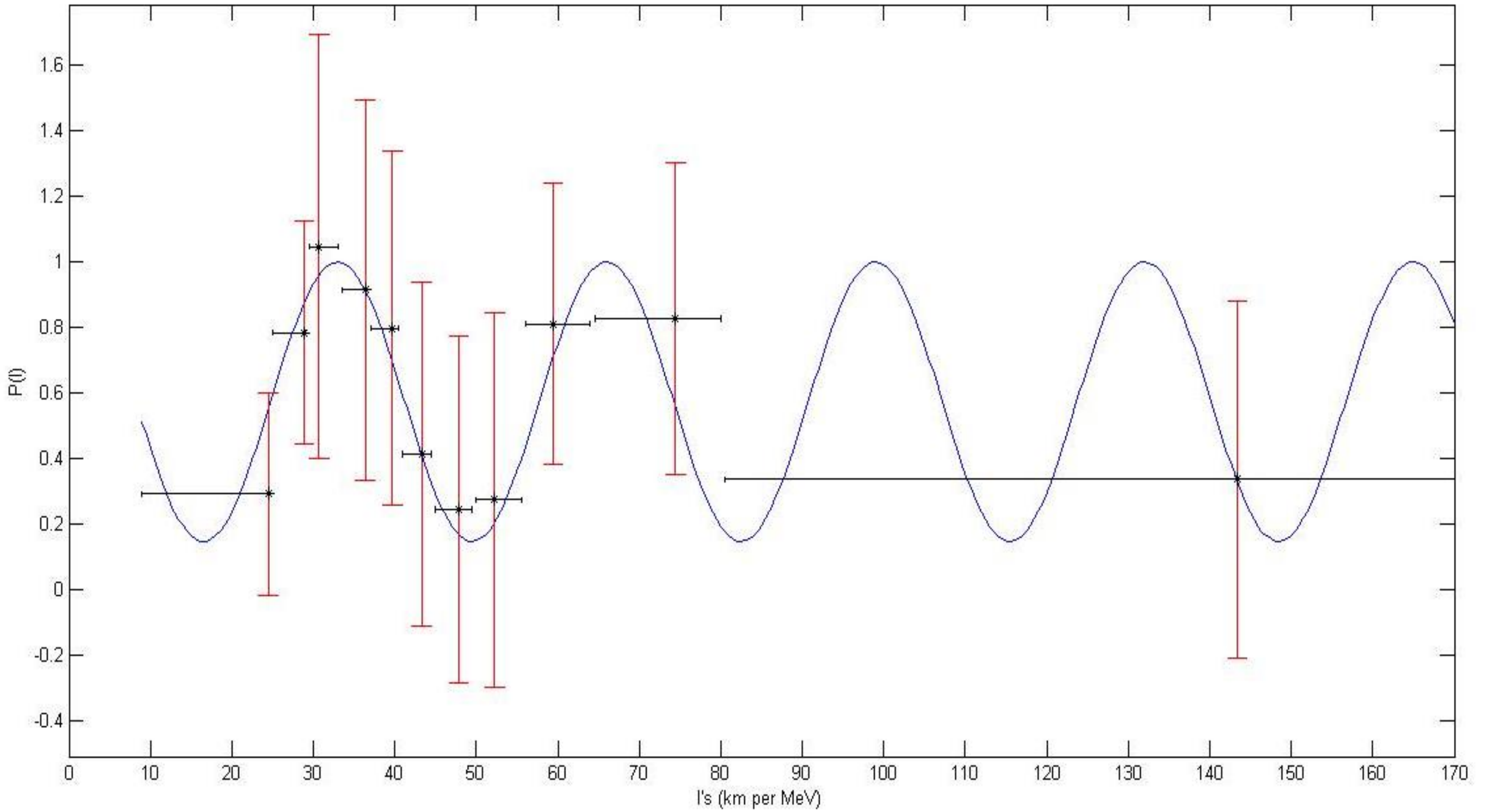
Error

Biggest error contributors:

- Background noise in the data N_1 from the experiment
- Approximation of I values due to the I binning in Q_0

Producing reasonable error bars for our test of specific P(I)s :

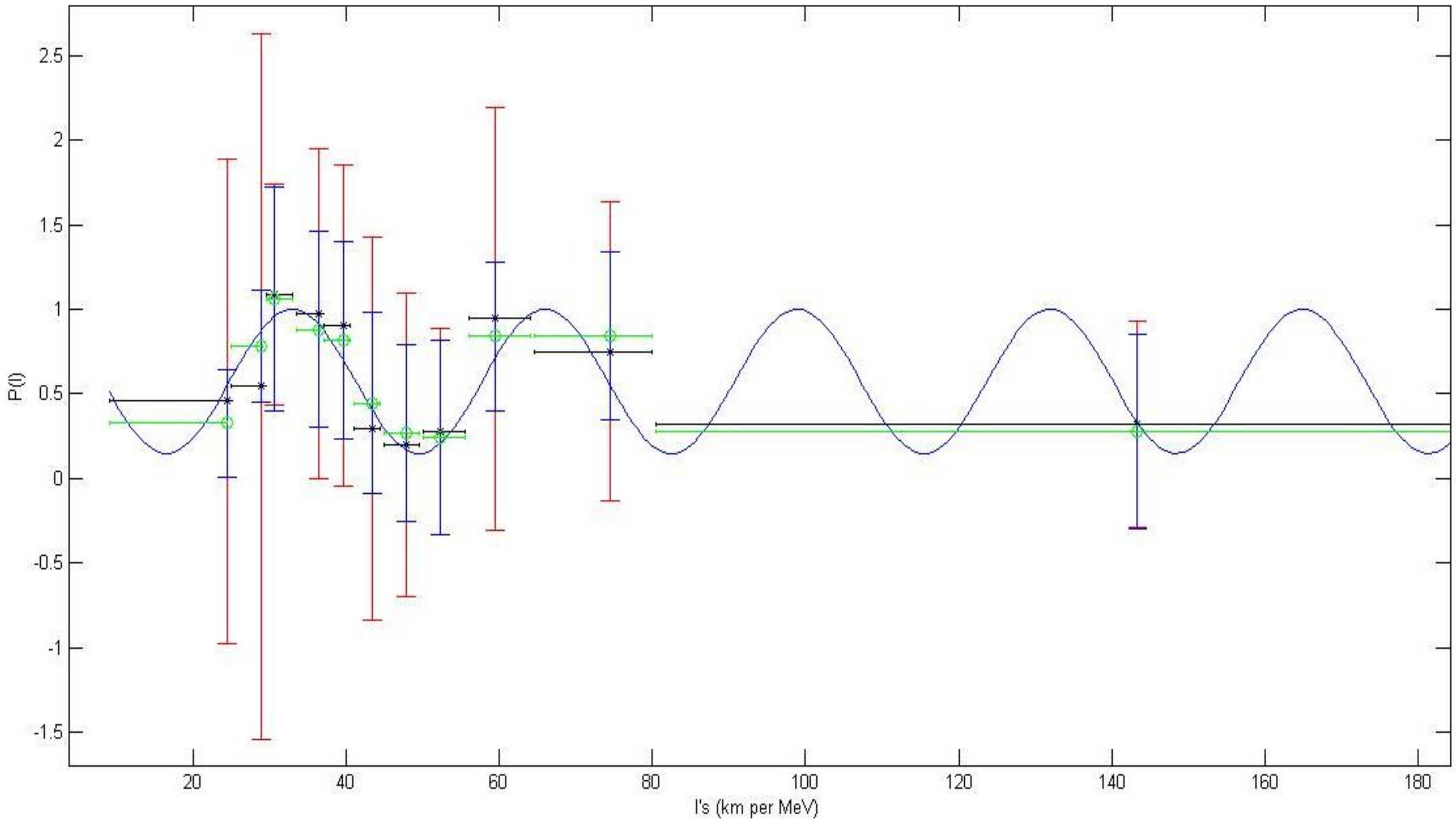
- Create N'_{1true} with a specific P(I)
- Add randomized background noise to N'_{1true}
- Create 1000 different P(I)s, each using a different randomized $N'_{1observed}$
- Find the average P(I) and its standard deviation to obtain different error bars for each P(I) entry



Omitting the smallest eigenvalue

16 Ep bins,11 l bins

Smallest error bars so far, but not as small as were expected

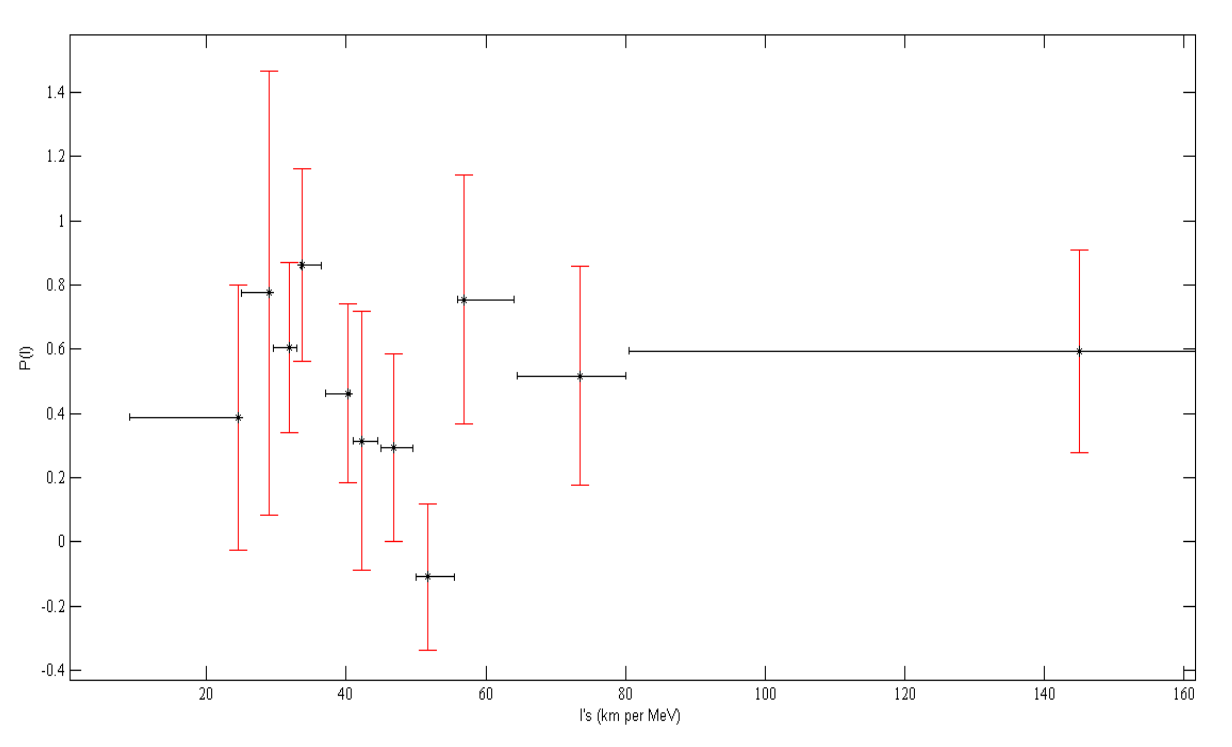


Omitting vs not omitting smallest eigenvalue

16 Ep bins,11 l bins

Smaller error bars

Testing KamLAND's N



$$P_{1_{estimate}} = \begin{pmatrix} 0.39 \\ 0.78 \\ 0.60 \\ 0.86 \\ 0.46 \\ 0.31 \\ 0.29 \\ -0.11 \\ 0.75 \\ 0.52 \\ 0.59 \end{pmatrix}$$

Comparing Chi Squares

Chi square of the N between our estimate and the N observed:

6.68

Chi square of the P between our estimate and the closest straight line of 0.44 without taking into account covariance:

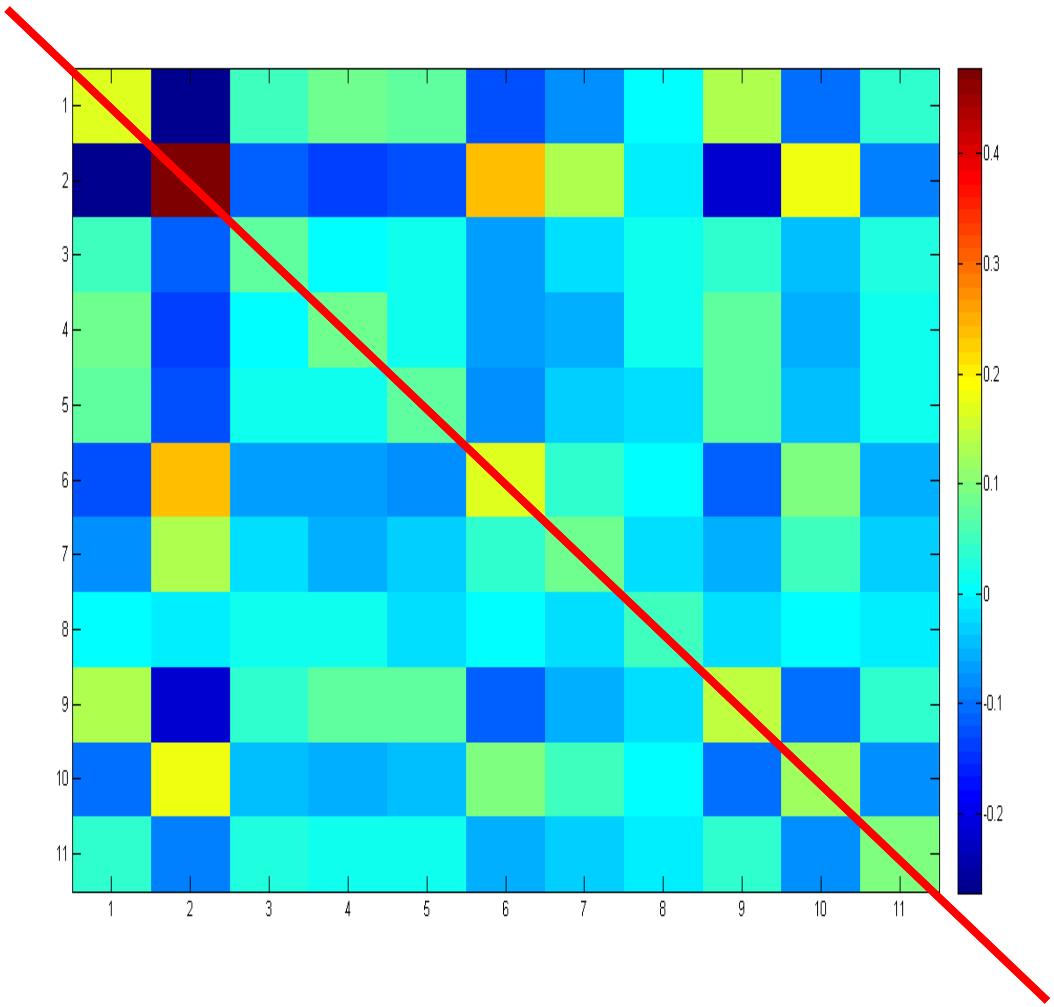
9.65

The above chi square with covariance:

67.95

$$N_1' = \begin{pmatrix} 50.1 \\ 88.2 \\ 106.8 \\ 73.7 \\ 79.3 \\ 92.5 \\ 66.6 \\ 65.8 \\ 83.0 \\ 97.0 \\ 137.3 \\ 119.4 \\ 126.2 \\ 169.0 \\ 112.2 \\ 104.9 \\ 102.6 \end{pmatrix} \qquad N'_{1_{estimate}} = \begin{pmatrix} 53.1 \\ 85.5 \\ 102.5 \\ 81.4 \\ 83.3 \\ 84.1 \\ 65.4 \\ 69.8 \\ 87.8 \\ 89.0 \\ 129.1 \\ 136.0 \\ 126.5 \\ 157.0 \\ 115.5 \\ 109.9 \\ 99.2 \end{pmatrix}$$

The Covariance Matrix



Conclusions

- Obtained appearance probabilities for 11 values of L/Eν without assuming an average L
- Appearance probability cannot be constant
- Predicted N matched KamLAND's observed N

Acknowledgements

- Dr. Horton-Smith & Dr. Weaver
- Dr. Corwin
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