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The Axicon: A New Type of Optical Element

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A search for a universal-focus lens has led to a new class of optical elements. These are called axicons. There are many different kinds of axicons but probably the most important one is a glass cone. It may be either transmitting or reflecting. Axicons form a continuous straight line of images from small sources.

One application is in a telescope. The usual spherical objective is replaced by a cone. This axicon telescope is in focus for targets from a foot or so to infinity without the necessity of moving any parts. It can be used to view simultaneously two or more small sources placed along the line of sight.

If a source of light is suitably added to the telescope it becomes an autocollimator. Like ordinary autocollimators it can be used to determine the perpendicularity of a mirror. In addition, it can simultaneously act as a telescope for a point target which may be an illuminated pinhole in the mirror.

The axicon autocollimator is also a projector which projects a straight line of images into space.

INTRODUCTION

THE word axicon has been coined to cover a type of optics. All axicons are figures of revolution. An axicon has the property that a point source on its axis of revolution is imaged to a range of points along its axis. Axicons do not, therefore, have a definite focal length. The name axicon means axis image. Axicons form images only of small bright sources like lamp filaments or brightly illuminated pinholes.

EXPERIMENTAL

The first attempt to construct an axicon was simply to make a narrow circular opening in an opaque disk and use the interference pattern produced by it, see Fig.

1. This idea is, of course, very old but recently it has been publicized by van Heel.* The image was, however, very dim.

It was realized that diffraction theory was necessary to explain the bending of the light towards the axis at the circular opening. This appeared to be a very inefficient way of getting light to the axis so the next move was to eliminate the necessity for diffraction at this place by using a circular groove to refract the light towards the axis. This groove would be a circular cylindrical lens or a toric lens, Fig. 2. When it was tried out a very much brighter image was produced but no diffraction pattern was apparent in the image.

The next model had several concentric grooves; this also gave an image of even greater brightness but still no indication of diffraction rings. The next sample was, indeed, a radical one because a disk of glass was mounted on the face plate of a lathe and rubbed with fine sandpaper to make a multitude of concentric circular scratches. This, too, gave a very good image of the point source and, of course, no indication of any diffraction pattern around the image.

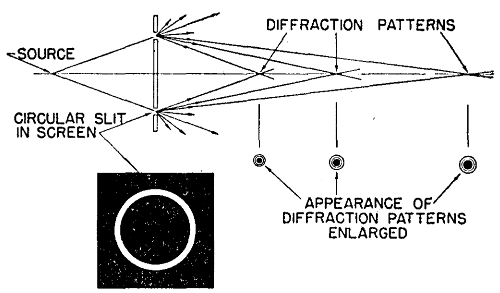

FIG. 1. Ray diagram showing formation of images of a point source by diffraction through a narrow circular opening.

AXICON THEORY

About this time a theory of the operation of these various forms of circular optics began to be formed. It is based on geometrical optics only. Suppose we consider a very short length of a refracting ring or of a scratch. It will resemble a cylindrical lens or a prism of some sort. It is simpler to think of it as a cylinder or groove. The incident light will arrive normal to the direction of the cylinder because it comes from a point source on the axis. The important factor is that the refracted light will also be normal to the direction of the cylinder (first law of refraction). This means that a fan of light will leave the little element, and the plane of the fan of light will pass through the axis. Therefore, this little element or cylinder will send some light to a continuous range of image points along the axis. The important fact to realize is that every other little element of every circular groove sheds some light along the axis. Conversely, if a point in the image space is to receive light from all parts of all grooves, it *must be on the axis*. If the reference point is off the axis it will receive light

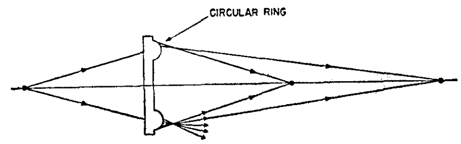

FIG. 2. Ray diagram showing formation of images of a point source by refraction through a circular ring or toric lens.

only from elements of grooves in the plane containing the axis and the reference point. This means that very much less light falls on points off the axis. A familiar example of the off-axis condition is the radial streak of light seen in an automobile windshield produced by the minute circular scratches made by the windshield wiper.

Let us now proceed to more details of the main theory. Suppose the source is a small circular disk, and suppose we catch the image on a screen, Fig. 3. In order to isolate all but one element of a groove let the axicon be covered with an opaque screen having a pinhole over one element. The fan of light from this element will be caught on the screen as a strip of light. The length of this strip will be determined by the maximum deviations produced by the groove. The width of the strip will be determined by the diameter of the source and by the magnification produced by the pinhole acting as a lens in one direction.

A view of the screen with the strip of light is illustrated in Fig. 4, upper left. If we now take more pinholes around the circular scratch or groove, we get a pattern on the screen like Fig. 4, upper right. Each pinhole produces a strip but all of the strips cross each other at the center. If still more pinholes are used, the pattern would look like Fig. 4, lower left, and finally if all of the grooves are uncovered we will get the axicon image with merely a continuous flare around it, Fig. 4, lower right.

The first three illustrations above are negatives made from crossed strips of neutral density film. The fourth one is a real photograph of an axicon image.

EXAMPLES OF AXICONS

We have seen that the basic fact about an axicon is that it must be a figure of revolution such as to direct light from a point source on its axis of revolution to a range of points on its axis. With this in mind let us look at some other possible forms of axicons.

One that is closely related to the circular groove or

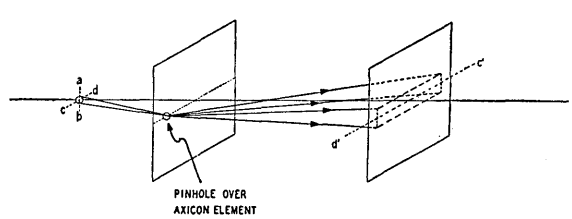

FIG. 3. Ray diagram showing how one element of an axicon produces a strip of light on a screen.

* A. C. S. van Heel, J. Opt. Soc. Am. 40, 809 (1950).

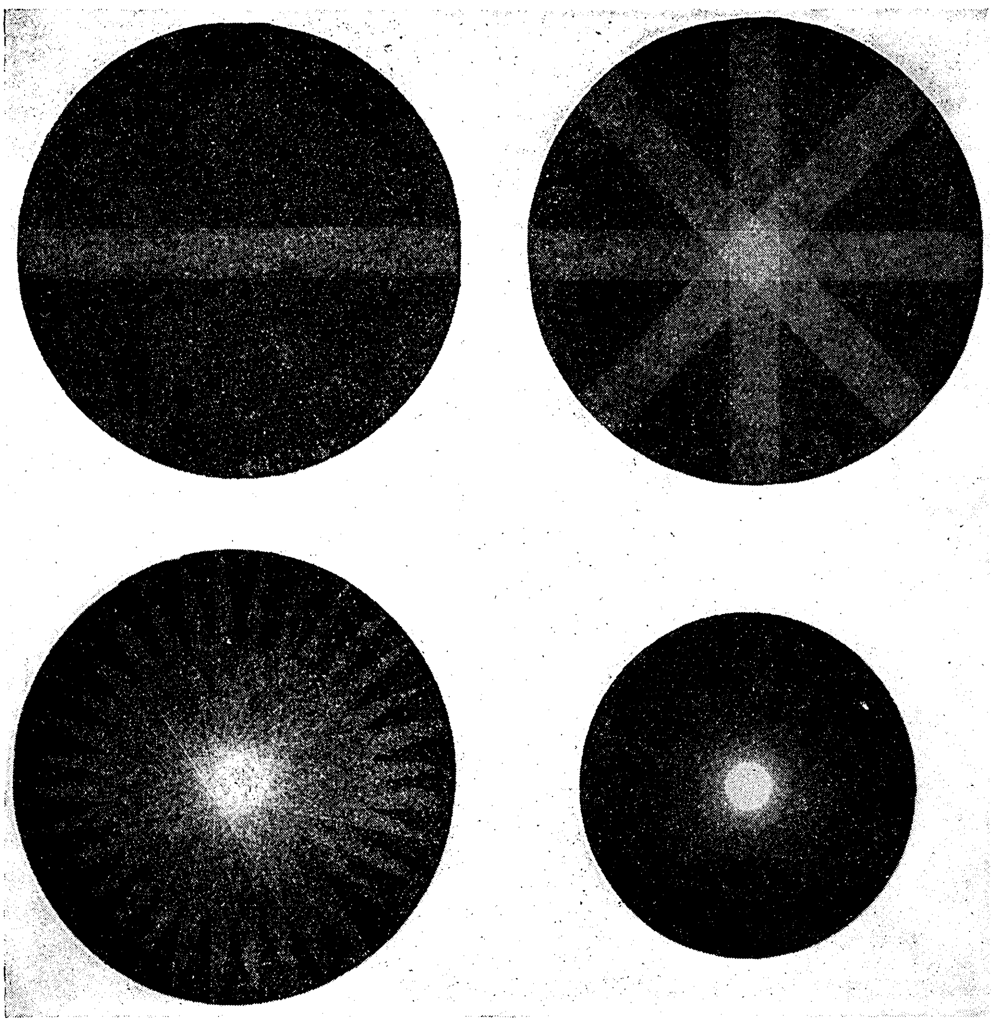

FIG. 4. Upper left—photograph of a screen showing strip of light from one element of an axicon. Upper right—a similar photograph showing crossed strips from four elements. Lower left—showing crossed strips from many elements. Lower right—a photograph of an axicon image using complete axicon.

scratch would be a toric lens, either positive or negative as in Fig. 5 and Fig. 6.

A cone, as in Fig. 7, is another axicon. It may be considered as a toric lens with infinite radius.

There are a great many forms of reflecting axicons. Most of the examples mentioned above may be changed to reflecting axicons by making one of the surfaces reflecting.

Other reflecting types are shown in Fig. 8. The cylinder is interesting because the magnification is always unity. The hollow sphere is interesting because it is completely symmetrical and therefore will form images from any angle.

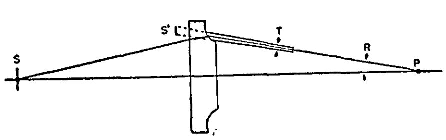

FIG. 5. Ray diagram showing how a strong toric lens forms a small virtual ring image S', and a real axicon image at P of a source S.

Most of these types have been tried out experimentally and found to work according to theory.

ILLUMINANCE

Let us come back to the single element as illustrated in Fig. 3. The illuminance across the narrow dimension of the strip of light produced by it is not uniform. The

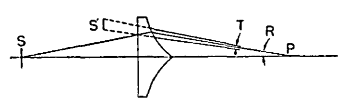

FIG. 6. Ray diagram showing how a weak toric lens forms a larger virtual ring image S', and a real image at P of source S.

center of the strip $c'd'$ receives light from points all across the diameter cd of the source. The edge receives light only from small areas a and b at the edges of the source. The distribution, therefore, is proportional to the length of the chord across a circle or to $(r^2-s^2)^{\frac{1}{2}}$, where r is the half-width of the strip and s the distance out from the center of the strip. On the basis of this data it should be possible to calculate the distribution of illuminance over the screen when all elements are added up.

We have just seen how the illuminance varies over the screen. The next problem is to find out what factors govern the absolute illuminance. We must apply the fundamental law of illuminance, namely, that the illuminance at a point on a screen is the product of the luminance of the source times the solid angle subtended at the point by the effective aperture stop of the imaging system.

To simplify this problem let us consider an axicon in the form of a circular toric lens either positive or negative. Figure 5 and Fig. 6 illustrate negative toric lenses of this type. The source S is a small bright circular disk. At S' there is formed a virtual image of the source S. This image, if viewed from a point on the axis such as from P, will appear as a circle of light hereinafter referred to as the *ring image*. Let its angular diameter as seen from P be $2R$ and its angular thickness be T. The solid angle subtended at the point P is then $2\pi RT$ and the illuminance will be proportional to $(2\pi RT)B$ where B is the luminance of the source. The value of R is dependent on the size and shape of the axicon and on the distance to the point P. The angle

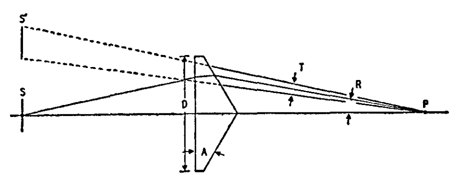

FIG. 7. Ray diagram showing how a cone forms a virtual ring image S' in the plane of the source S, and an axicon image at P.

T is determined by the thickness of the image S', and again by the distance to P. The illuminance in the case illustrated then varies inversely as the square of the distance from the ring image to the observation point P.

Let us examine what can be done to get brighter axicon images. We have seen that the illuminance is proportional to $2\pi RTB$.

T, the angular thickness of the ring image, may be increased by increasing the magnification from source S to its virtual image S'. Figure 6, as compared to Fig. 5, shows how a torus of larger radius will accomplish this. Figure 7 shows how the toric radius may be increased to infinity to give a cone and so increase the value of T still farther. Figure 9(A) or 9(B) shows how still greater increases in T may be obtained by effectively going to positive radii for the torus.

It might be pointed out that the range of focal lengths is in general reduced as T the angular size of the ring image, as viewed from P, is increased.

The other factor governing the value of T is the size of the source itself. Let us consider two special cases. In the first let S be so small that its image is always a diffraction ring pattern or a blur circle. Then as S is increased in size the illuminance in the pattern will

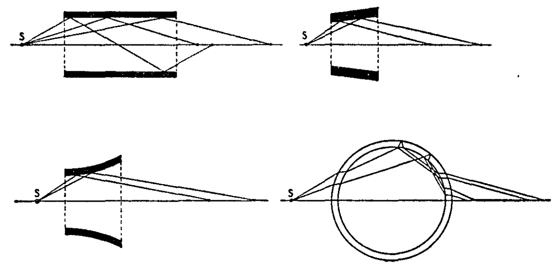

FIG. 8. Various forms of reflecting axicons. Upper left—a hollow reflecting cylinder. Upper right—a hollow reflecting cone. Lower left—a hollow flared reflector. Lower right—a hollow sphere of refracting material.

increase. In spherical lenses the increase would be proportional to the square of the diameter of the source. In axicon optics, however, another dimension has been added and the increase in illuminance should be proportional to the cube of the source size. No experimental confirmation of this relatiohship has as yet been made.

In the second case let the source be of such a size that the image is always larger than the diffraction pattern. Figure 4 illustrates such a case. In spherical optics the illuminance in the image would, of course, be independent of the source size. In axicons however the illuminance will be proportional to the *diameter of the source*. This is so because in one dimension the source is the aperture stop. Experiments show this to be true.

CONE

The simple cone Fig. 7 for many applications may be the most useful form of axicon. It is somewhat simpler than other forms to make and its range and illuminance are practical.

Cones have some interesting properties. For example, a cone may be thought of as a lens having so much coma that the useful field is very small indeed. It has a focal length from zero up to a maximum. It can be said that it has over-corrected spherical aberration equal to its focal length. It has no chromatic aberration, except for some at the limits of its range, because each color of light finds its own path through the cone to the image.

If the source is at infinity the illuminance in the axicon image of a cone remains constant for all positions of the image. This is so because the ring image is

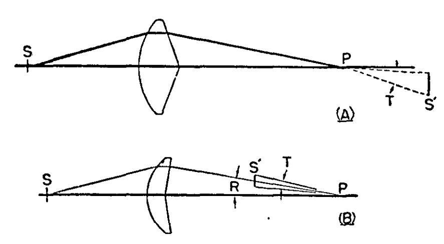

FIG. 9. (A) Ray diagram of a spherical lens plus positive cone forming a real ring image S' beyond an image point P. (B) Ray diagram of a spherical lens plus negative cone forming a real ring image S' in front of an image point P.

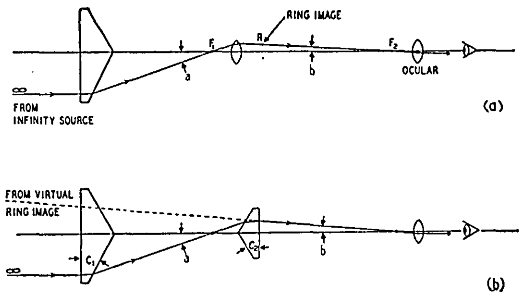

FIG. 10. (a) Ray diagram of combination of cone and spherical lens forming an axicon image at an ocular. (b) Ray diagram showing replacement of lens with another cone to form axicon image at ocular.

always at infinity for all zones of the cone and always subtends the same solid angle. Although the illuminance remains the same the image size is smaller close to the cone. This means that there is less total energy in the image close to the cone than in an image distant from the cone. This gives rise to the rather unusual phenomenon that the visibility *increases* with increase in distance from the cone. This is easy to observe, for example, in the image of the sun formed by a cone.

INTERFERENCE PHENOMENA WITH A CONE

As soon as axicons with single grooves or toric lenses were considered, it was realized that if they could be made perfectly enough, there should be interference patterns. Only with special care was it possible to realize this perfection. The first axicon to show a diffraction pattern was a cone of about 3 inches in diameter and 6-feet maximum focus. When tested on a lens bench a beautiful set of diffraction circles were observed around the image. It was noted that the diameters of the rings remained constant for all positions of the observing microscope. This is to be expected because the angle of the rays to the axis is the same for all image distances. As pointed out before, the visibility close to the cone became very low so the diffraction pattern was more easily seen farther from the cone.

Probably the most striking demonstration of diffraction rings was with a cone about 6 inches in diameter and maximum focal length about 50 feet. A very bright source about 0.003 inch in diameter (Zirconium Arc 2-watt lamp) was placed 100 feet along the axis of the cone on one side. Along the axis on the other side the diffraction pattern could be seen with an ordinary magnifying glass for as far as 100 feet. A crosshair could be located on the image with an accuracy of about 0.001 inch or 25 microns.

USE OF CONE AND LENS

Let us consider some of the properties of combination of axicons and spherical optics. Probably the simplest case is adding a magnifying lens or ocular to view the aerial image. This, of course, is a simple telescope.

Figure 10(a) shows how another lens such as a microscope objective may be added to the cone objective to give greater magnification to the image in front of the ocular. This change, however, adversely affects the illuminance at the ocular because not only is the angular diameter of the ring image reduced from a to b in the figure but the angular thickness is also reduced. The reduction in illuminance is therefore proportional to the square of the magnification of the microscope objective. The exit pupil will be a ring of light.

USE OF TWO CONES

Now suppose we replace the microscope objective with a cone of such an angle as to match the power of the microscope objective, Fig. 10(b). The angular diameter of the ring image is reduced from a to b as before but the thickness of the ring image is not reduced by the second cone. This, then, gives a reduction in illuminance proportional only to the first power of the magnification of the second cone. This means then that if we replace a microscope objective giving, for example, ten times magnification with a cone to give the same magnification, we will have an image ten times as bright. Qualitative experiments show this to be true.

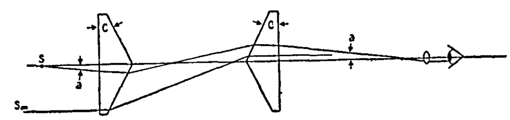

FIG. 11. Diagram showing two cones of equal angle forming 1—1 image of source.

USE OF TWO EQUAL CONES

Figure 11 shows the use of two similar cones. The magnification produced by the pair is always unity for all object distances and for all viewing distances. This is apparent from the fact that both cones produce the same deviations and if the ray in question crosses the axis between the cones, the two deviations are opposite, so that the incoming and outgoing rays are always parallel to each other and, from the sine law, the magnification is unity. Similar triangles in the figure will also show that the magnification produced by the first cone is always reciprocal to the magnification produced by the second cone and so the product is unity.

A pair of similar *spherical* lenses placed two focal lengths apart will always form an image at unit magnification but the position of the image is always at a distance of four focal lengths from the object. In the case of a pair of similar axicons, however, not only is the image always the same size, but it may be observed over a long range of distances for each position of the source.

When two cones of equal slopes were tried, the image did remain the same size as the object over a considerable range. However, when the source was at a great distance, the second cone was being used so close to its tip that the image deteriorated. This occurred in one experiment at about 40 feet distance. The field of view was about 2 mm wide for all object distances.

REFLECTING CONE

One rather interesting variation of the cone is obtained if we aluminize one face of it and use it in reflection. Either the plane surface or the cone surface may be the reflecting one. The light will in either case pass twice through the transparent material of the cone.

If the source and observation points are made coincident by a beamsplitter or pinhole mirror as in Fig. 12 the images will always be at unit magnification for any distance from the cone out to its maximum range. Moreover, the size of the diffraction pattern will always be constant. The illuminance will be inversely proportional to the first power of the distance, not to the usual second power. This is because the angular diameter of the ring image remains constant, and only the angular thickness is reduced as the distance increases.

Experiments have demonstrated that images formed by a reflecting cone of $3\frac{1}{2}$ inches in diameter and 6-feet maximum range may be located with a standard deviation of 1 to 2 wavelengths of light over its complete range.

TELESCOPES

Brief mention has been made of the axicon telescope where the usual spherical objective has been replaced by a cone. The outstanding property of the axicon telescope is that no focusing adjustment is needed and therefore no possibility exists of lack of straightness of line of sight caused by errors in the focusing mechanism.

This brings up the question—what determines the straightness of line of sight with an axicon? Different distances are focused by different zones of the axicon. If the line of sight is to be straight, each zone must be centered about the same axis as all the other zones. The method of making axicons appears to insure that all zones will have the same axis. This would appear to be particularly true in view of the fact that the accuracy may be such as to produce diffraction patterns over the complete range of the axicon.

An interesting property of axicon telescopes is that we can use them to see two or more point sources along the line of sight simultaneously, even when they are at different distances. All will, of course, be in focus. It may be interesting to note that the farther sources are then seen through the space *surrounding* the nearer ones. Thus the nearer sources do not get in the way of the light coming from the farther ones.

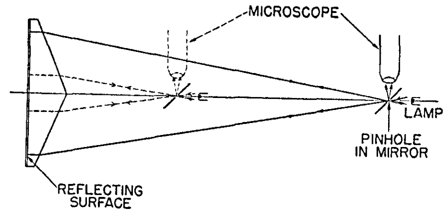

FIG. 12. Ray diagram of reflecting cone to show how an axicon image is returned to the source wherever the source is located along the axis of the cone out to a maximum distance.

PROJECTORS

An axicon line projector may be made by placing a small source of light just within the maximum focal length of an axicon objective. The system will project a continuous straight line of image out to infinity. If a single cone is used the image is very small close to the cone and gets larger and larger at greater distances from the cone. If two equal cones are used the projected image will remain constant in size with distance.

The use of equal cones, however, limits the range. If, however, the cone nearer the source has a somewhat larger slope than the objective one, the projected beam may go out to great distances with only a small increase in image size.

AUTOCOLLIMATOR

The projector and telescope may be combined into an autocollimator. One way to accomplish this is to use a beamsplitter to make the projected beam coincident with the line of sight of the telescope. Another way is simply to put a small source on the axis in front of the telescope axicon objective.

The axicon autocollimator, like ordinary autocollimators, can be used to determine if a mirror surface is perpendicular to the line of sight. The axicon instrument can, in addition, at the same time determine the lateral position of an illuminated pinhole in the mirror or of any other point source along the axis.

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