

Bessel beams: diffraction in a new light

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Diffraction is a cornerstone of optical physics and has implications for the design of all optical systems. The paper discusses the so-called ‘non-diffracting’ light field, commonly known as the Bessel beam. Approximations to such beams can be experimentally realized using a range of different means. The theoretical foundation of these beams is described and then various experiments that make use of Bessel beams are discussed: these cover a wide range of fields including non-linear optics, where the intense central core of the Bessel beam has attracted interest; short pulse non-diffracting fields; atom optics, where the narrow non-diffracting features of the Bessel beam are able to act as atomic guides and atomic confinement devices and optical manipulation, where the reconstruction properties of the beam enable new effects to be observed that cannot be seen with Gaussian beams. The intensity profile of the Bessel beam may offer routes to investigating statistical physics as well as new techniques for the optical sorting of particles.

1. Introduction

Diffraction is a phenomenon intricately linked to the wave nature of light and occurs when a wave encounters an obstacle. In this process the wave may be altered in amplitude and/or phase and diffraction takes place. The parts of the wavefront that propagate beyond the obstacle interfere in some manner and yield a diffraction pattern. Diffraction is core to the propagation of Gaussian light beams. The output of a laser is deemed ‘pencil-like’ in nature and has a very low divergence yet it is subject to diffraction that causes the light to spread. Gaussian beam theory shows us how to manipulate the light field to yield tight focusing or a collimated beam depending upon the application of interest. The Rayleigh range Z_R is the typical parameter used for characterizing the spread of a Gaussian light field and denotes the distance over which a Gaussian beam increases its cross-sectional area by a factor of two:

$$Z_R = \frac{\pi w_0^2}{\lambda}, \quad (1)$$

where λ is the wavelength and w_0 is the beam waist size. The notion of overcoming diffraction is very evocative and indeed appealing from the viewpoint of numerous applications including atom optics and medical imaging.

This paper deals with the topic of Bessel light beams. Such beams appear to offer some immunity to diffraction and thus are potentially an attractive alternative to using Gaussian beams in a number of scenarios. The Bessel beam as a mathematical construct was first noted by Durnin [1]. Durnin looked at Whitaker’s solutions of the Helmholtz equation (2) and saw that particular solutions of the Bessel type were independent of the propagation direction. More importantly perhaps were the properties of this solution; in particular such a beam could have near diffraction limited features (the centre of the beam has been shown to have a

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minimum diameter of $\frac{3}{4}\lambda$ with λ being the beam wavelength [1]) which did not spread. Of course the reality is a little more down to earth. A Bessel beam gets its name from the description of such a beam using a Bessel function, and this leads to a predicted cross-sectional profile of a set of concentric rings. Mathematically the Bessel beam can contain an infinite number of rings, and so over an infinite area would carry infinite power. So the conclusion must be that we cannot make a Bessel beam. What Durnin and co-workers went on to show [3] was that one could make an approximation to a Bessel beam (a quasi-Bessel beam) experimentally which possesses the properties of the mathematical entity over a finite distance.

The concept of a ‘non-diffracting’ beam proved rather controversial at first, and perhaps still remains so, with a comment [4] published in *Physical Review Letters* relating to Durnin, Miceli Jr and Eberly’s original Bessel beam paper [3] suggesting that the Bessel beam was really just a line image and was nothing particularly new: it was just something akin to Poisson’s spot [5]. Durnin *et al.* explained [6] that there were many ways of generating line images and that they need not exhibit the properties of the Bessel beam. Still the idea remains anathema to some; four years after the original paper was published a second comment was published [7] suggesting that, depending on interpretation, the Bessel beam was not non-diffracting at all. This is a point that must be borne in mind: ‘what do we mean by a non-diffracting beam in the case of a Bessel beam?’, as Durnin *et al.* emphasized in their reply to the second comment [8], ‘We have confirmed that beams exist whose *central maxima* are remarkably resistant to the diffractive spreading commonly associated with all wave propagation’. That is, when we compare a Bessel beam to a Gaussian beam the comparison we are making is between the central core of the Bessel beam and a Gaussian beam of similar spot size and it is this central core that is propagation invariant.

In this paper we explore the idea of Bessel light beams and look at some of the applications that they have found in the last decade. After elucidating the basics of Bessel beam generation, applications in non-linear optics, atom optics, micromanipulation and studies in the pulsed regime will be reviewed.

2. Properties and generation of Bessel beams

An ideal Bessel beam can be described by

$$E(r, \phi, z) = A_0 \exp(ik_z z) J_n(k_r r) \exp(\pm i n \phi) \quad (2)$$

where J_n is an n th-order Bessel function, k_z and k_r are the longitudinal and radial wavevectors, with $k = \sqrt{k_z^2 + k_r^2} = 2\pi/\lambda$ (λ being the wavelength of the electromagnetic radiation making up the Bessel beam) and r , ϕ and z are the radial, azimuthal and longitudinal components respec-

tively. The intensity structure for a zeroth-order Bessel beam is shown in figure 1(a). Beams described by higher order Bessel functions ($n > 0$), the high-order Bessel Beams (HOBBs), have a phase singularity on the beam axis and hence have a non-diffracting dark, rather than bright, core [9]. The intensity profile for a first-order Bessel beam is shown in figure 1(b).

The result that garners the most attention is that this solution to the Helmholtz equation satisfies the equality, that for propagation in the z direction, the intensity, I , obeys:

$$I(x, y, z \geq 0) = I(x, y) \quad (3)$$

This means that there is no change in the cross-section as the beam propagates and thus the beam can be considered diffraction free, or propagation invariant. One way of thinking about the Bessel beam is to consider a set of plane waves propagating on a cone. Each propagating wave undergoes the same phase shift, $k_z \Delta z$ over a distance Δz . This decomposition of the Bessel beam into plane waves manifests itself in the angular spectrum (figure 2) of the beam, which is a ring in k -space. Thus the optical Fourier transform of a ring will result in a Bessel beam, and this is how Durnin *et al.* [3] first experimentally observed an approximation to a Bessel beam, as is detailed below.

The decomposition of the beam into plane waves also gives a means to characterize the Bessel beam, that of the opening angle of the cone defined by the waves traversing its surface. The angle:

$$\theta = \tan^{-1} \frac{k_r}{k_z} \quad (4)$$

can be used to define the central core spot size of the beam, which is given by (from the properties of the zeros of a Bessel function):

$$r_0 = \frac{2.405}{k_r} \quad (5)$$

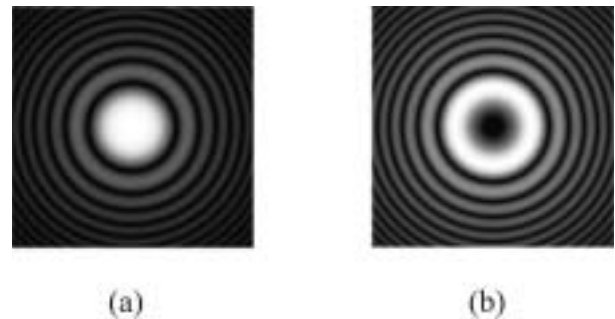


Figure 1. Bessel beam intensity profiles: (a) for a zeroth-order beam and (b) a first-order beam (J_1 beam). Both beams have the same k_r values.



Figure 2. Angular spectrum of a Bessel beam: (a) intensity profile of Bessel beam. With its accompanying angular spectrum – the Fourier transform of the Bessel function is a ring in k -space; (b) shows how the k -vectors of the Bessel beam propagate on a cone.

Since the Bessel beam can be considered as a set of waves propagating on a cone it also possess another interesting property, that of reconstruction [10, 11]. If we place an object in the centre of the beam, the waves that create the beam are able to move past the obstruction, casting a shadow into the beam, but ultimately reforming the intensity of the profile beyond the obstruction. The distance after which the beam is able to reform is given, from a simple geometric argument, by:

$$z_{\min} \approx \frac{ak}{2k_z} \quad (6)$$

Where a is the width of the obstruction measured from the beam centre. The reconstruction effect can be seen in figure 3.

It is useful to note that the Bessel beam has its energy (or power) evenly distributed between its rings [12, 13], so the more rings the beam has the lower the energy in the central core: important in many experimental situations (although a larger number of rings comes with an increased propagation distance). Durnin *et al.* [12] showed that this fact did not necessarily mean that a Bessel beam could not carry power as efficiently as a Gaussian beam, which one might naively expect. Indeed optimized Gaussian and Bessel beams are comparably efficient at transporting power. They also showed that the depth of field of a Bessel beam could be made far larger than that of a Gaussian but at the expense of power in the central core.

The fact that a Bessel beam can be considered as being made up from plane waves propagating on a cone also leads to the realisation that the nodes of the beam correspond to a π phase shift between adjacent rings, a fact demonstrated by Lin *et al.* [13].

The generation of a Bessel beam can be carried out in a number of ways. The Bessel beam can be thought of as the Fourier transform of a ring – and this was how Durnin *et al.* [3] (see also the tutorial account in [14]) initially observed a Bessel beam. A ring (an annular slit) was placed in the back focal plane of a lens to form the beam, using a setup similar to that shown in figure 4. The opening angle of this cone is given by:

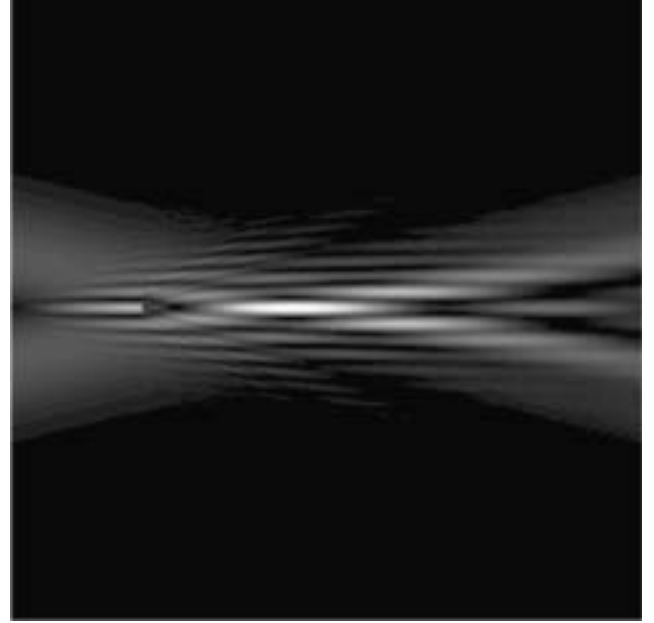


Figure 3. Bessel beam reconstruction. The figure shows a cross-section of the Bessel beam as it propagates from left to right. The axicon that creates the beam is present (within the mathematical model that generates the figure) at the hard left of the figure. The intensity profile of the Bessel beam is seen to reform even after encountering an obstruction. The obstruction is approximately one-fifth of the way along the figure from the left and covers the central core of the beam. This reconstruction is due to the conical wavefronts of the beam.

$$\tan \theta = \frac{d}{2f} \quad (7)$$

where d is the diameter of the ring and f the focal length of the imaging lens.

Geometrically we can estimate the propagation distance as:

$$z_{\max} = \frac{R}{\tan \theta} \quad (8)$$

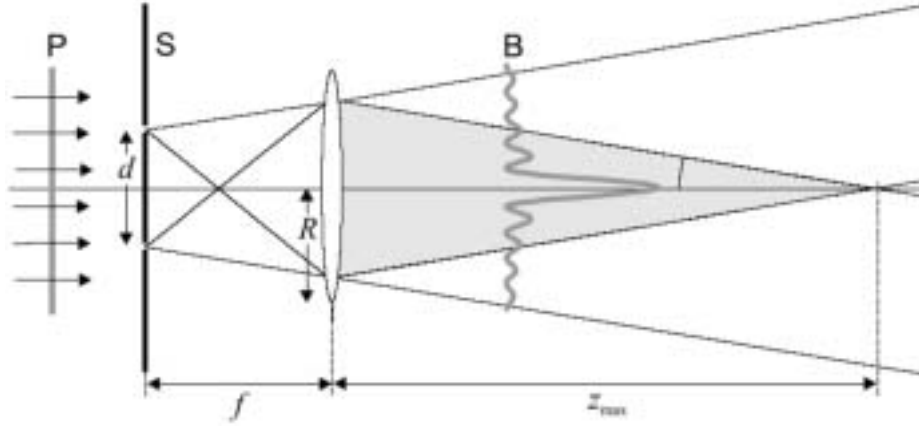


Figure 4. Creation of a Bessel beam by placing an annular aperture S with a diameter d , in the back focal plane of a converging lens of radius R . The Bessel beam is created from a plane wave P. The Bessel beam, denoted B in the figure has a propagation distance of z_{\max} .

(where R is the radius of the imaging lens) which, from (3), can be estimated as $(k/k_r)R$, when we consider the beam close to the optic axis. Durnin *et al.* [3] also found that the on-axis intensity of the beam oscillated rather widely before falling off rapidly at the end of the propagation distance. Further, while the initial experimental results demonstrated very clearly the properties of the Bessel beam, the method of generation is rather inefficient, as most of the incident power (from a Gaussian laser beam) is obstructed by the annular slit. For this method it was shown that the Bessel beam had both the smallest spot size and longest depth of focus, i.e. propagation distance, when compared to a Gaussian beam and an Airy disc created using the same generating lens [13] and that the peak intensity of the Bessel beam was the smallest of the three cases.

A Bessel beam may also be created using an axicon [15, 16], or conical lens element. This idea is shown in figure 5, and quite neatly illustrates the concept of a Bessel beam as being made up of a set of waves propagating on a cone. The opening angle of the cone is given by:

$$\theta = (n - 1)\gamma \quad (9)$$

where n is the refractive index of the axicon material and γ is the opening angle of the axicon. Therefore we can also say, that for the axicon generated beam:

$$z_{\max} = \frac{k}{k_r} w_0 \approx \frac{w_0}{\theta} \quad (10)$$

The use of an axicon is a far more efficient method of Beam generation than the annular slit as it utilises the whole, or at least most of, the incident Gaussian beam. Using an axicon removes the rapid on-axis intensity oscillation associated

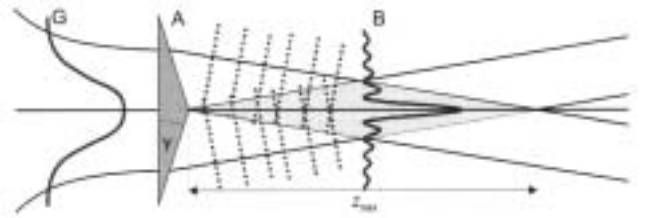


Figure 5. Creation of a Bessel beam making use of an axicon lens A. In this case the Bessel beam is formed using a Gaussian beam G. As in figure 4 the Bessel beam has a finite propagation distance given by z_{\max} .

with the annular slit method, giving a far smoother intensity variation, as is shown in figure 6. The alignment of the illuminating beam with the axicon is critical in producing a Bessel beam. Any oblique illumination results in an element of astigmatism being introduced and this leads, not to an on-axis spot with a set of concentric circles, but rather to a chequerboard type pattern. This has been studied by several authors [17–19].

Other efficient methods of creating Bessel beams exist, including those using holographic techniques, which imprint the appropriate phase of a Bessel beam onto an incoming Gaussian beam. These can be static, etched holograms [20]; or variable holograms made using computer controlled devices such as spatial light modulators [21, 22]. It is also possible to use a Fabry–Perot cavity in combination with an annular aperture [16, 23] which produces a more smoothly varying intensity profile than the annular slit with lens combination. Uehara and Kikuchi showed that it was possible to create a Bessel beam inside an Ar^+ laser cavity [24]. However in this approach the effective beam propagation distance is only slightly longer

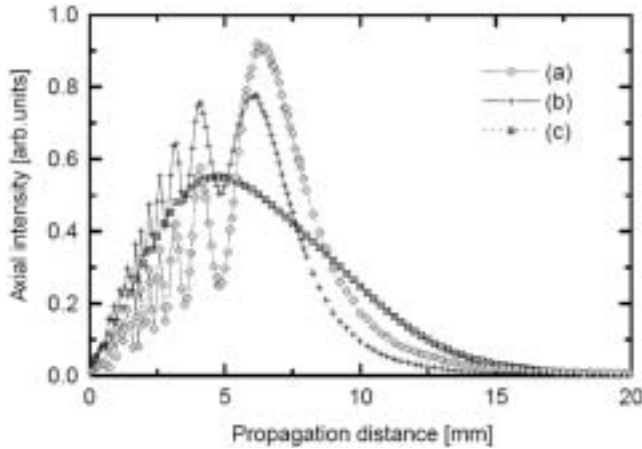


Figure 6. Intensity variation along the beam propagation direction for an experimental approximation of a Bessel beam created using (a) an axicon with a hard aperture illuminated with a plane wave; and (b) as in (a) but with Gaussian beam illumination; (c) makes use of a Gaussian beam illuminating an axicon with no hard aperture.

than a comparable Gaussian beam and so this technique is perhaps not that useful. More recently there have been other proposals to create Bessel beams inside cavities containing axicon elements [25–29], as well as the experimental demonstration of such a cavity, generating a zero-order beam, in a dye laser [30].

In discussing Bessel beams it would be remiss not to point out that there are other families of non-diffracting beams. The Bessel beam is a circularly symmetric light pattern, but just as we have elliptical Gaussian laser modes so we have an elliptical family of non-diffracting beams which are described by Mathieu functions and these are called Mathieu beams [31–33]. In Cartesian co-ordinates we have the plane wave solutions, while another member of the non-diffracting family, a parabolic solution, has recently been found by Bandres *et al.* [34].

2.1 High-order beams

We return briefly to the class of Bessel beams described by higher-order Bessel functions, i.e $n > 0$ in equation (1). These were first observed shortly after the demonstration of zero-order beams by Vasara *et al.* [20] using computer generated holograms; later this method was looked at in more detail by Paterson and Smith [35]. Such beams were subsequently generated using alternative methods such as using a spatial light modulator [22], an axicon illuminated with a Laguerre-Gaussian beam [36, 37] as well as propagating a zeroth-order beam through a biaxial crystal [38]. Jarutis *et al.* examined the case of focusing a Laguerre-

Gaussian beam with an axicon in detail and calculated the complex amplitude of such a beam to be:

$$c_1(r, \phi, z) = a_0 \sqrt{2\pi k_r \omega_0} \left(\frac{z}{z_{\max}} \right)^{l+\frac{1}{2}} \exp \left(-\frac{z^2}{z_{\max}^2} \right) \exp \left[i \left(l\phi - \frac{l\pi}{2} - \frac{\pi}{4} \right) \right] J_l(k_r r) \quad (9)$$

where l is the azimuthal mode index of the Laguerre-Gaussian beam illuminating the axicon, while r , ϕ and z denote cylindrical beam co-ordinates. The use of such beams has allowed the production of interesting non-diffracting patterns by making use of interfering Bessel beams [21, 39, 40]. They are also of interest because they carry orbital angular momentum [41] which is associated with the inclined wavefronts of such beams, and is discussed more in the section on micromanipulation below.

There is significant interest in beams which carry optical vortices [42–45]: these are intensity nulls associated with phase singularities in the beam. Both Orlov *et al.* [40] and Schwarz *et al.* [46] have investigated the propagation of Bessel beams carrying optical vortices. In particular Orlov *et al.* examined the case of superimposed Bessel beams carrying vortices and found that in the near field (here the near field indicates the Bessel beam itself) the vortex structure is richer than in the far field (a ring in the case of the Bessel beam, see figure 2). This was confirmed both experimentally and theoretically. Schwarz *et al.* [46] examined the propagation of Bessel beam with an embedded vortex placed away from the beam axis. This was achieved by passing a zero-order beam through a phase grating containing a phase dislocation. They show that the vortex, in contrast to what one sees with a Gaussian beam, actually moves toward and ends up in the beam centre as the beam propagates. Bouchal [47] has examined the case of an on-axis obstruction, in both intensity and phase, and has shown that the Bessel beam is able to reconstruct itself around such obstructions. This is work that has been experimentally examined by Garcés-Chávez *et al.* [48].

Recent work [49] has shown the demonstration of so-called ‘fractional’ Bessel beams, where the order n of the Bessel function is non-integer. Such beams can be easily generated with computer generated holograms and have a non-rotating ‘sprinkler’ type intensity profile, in which the normally solid inner ring of the integer Bessel beams has a hole in it (see figure 7).

2.2 Beam propagation

The ‘non-diffracting’ nature of the Bessel beam has fuelled much interest over the years and the properties of such beams have been widely investigated. For instance Chávez-Cerda [50] has interpreted the Bessel beam as being made of

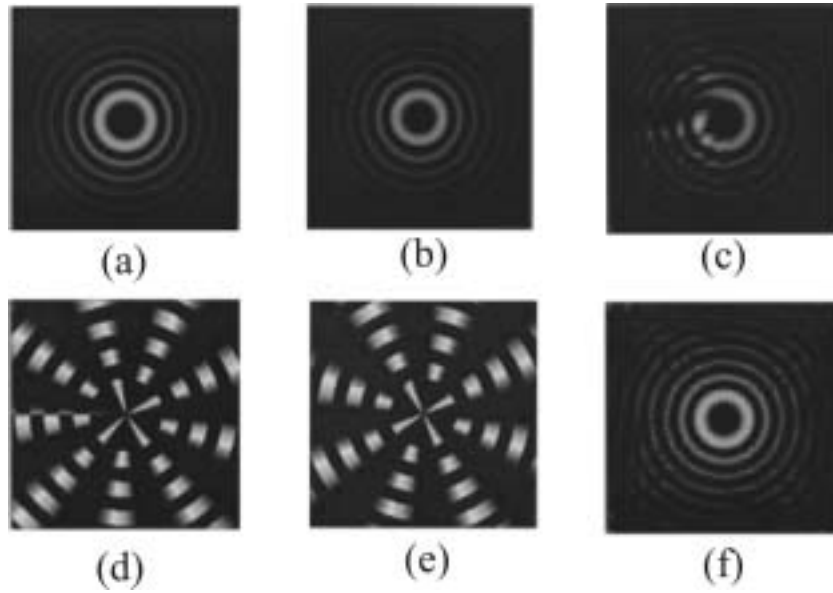


Figure 7. Simulations of fractional Bessel beams: (a) shows a fractional Bessel beam with $n = 4.5$ and (d) shows the phase of this beam; (b) shows an integer Bessel beam with $n = 4$ and (e) shows the phase of this beam; (c) shows the fractional Bessel beam in (a) after it has propagated 300 mm while (e) shows the integer Bessel beam in (b) after propagating a similar distance. (Printed with permission from [48].)

a superposition of travelling waves described by Hankel functions, which are Bessel functions of the third kind. In this basis the Bessel beams cannot have an infinite extent, while still describing the observed propagation properties of the beams. The notion of propagation invariance has been extended to more complicated beam intensity structures. Paterson and Smith [51] examined the case of a non-diffracting beam whose intensity pattern rotated with propagation while still maintaining the non-diffracting properties of the Bessel beam. These ‘helicon’ waves are superpositions of higher order Bessel functions.

Bouchal has examined the possibility of creating arbitrary intensity patterns that are diffraction free [52]. This idea could be realized by illuminating a Fourier filter, such as an annular disc placed in the back focal plane of a lens. A source then illuminates an amplitude mask placed in front of the filter. Making use of diffuse, incoherent light sources, in such a system leads to better replication of the original amplitude profile, as destructive interference effects are minimized. By appropriate choice of filter it may be possible to use this technique to carefully control the three-dimensional propagation of non-diffracting fields. The generation of somewhat simpler non-diffracting arrays using diffractive elements has also been studied by Lautanen *et al.* [53].

The superposition of Bessel beams has also been of interest. Jaroszewicz *et al.* [54] examined the possibility of making even narrower core Bessel type beams by interfering two zero-order beams with different radial and

longitudinal wavevectors together. The central core can be made almost arbitrarily small, although this is accompanied by a dramatic loss in intensity. This work was later extended to treat the interference of Bessel beams of differing order (e.g. J_0 and J_2 beams). Experimental work on interfering Bessel beams has also been undertaken [55] where beams with different radial wavevectors are interfered. The resulting beams show self-imaging effects, confirming earlier theoretical work [56].

3. Experiments

In the previous sections we have discussed the generation and some of the interesting properties of the Bessel beam. In the following sections we turn our attention to the applications that have been found for such beams, and indicate where these have been successful or not, while looking to future applications of the Bessel beam.

3.1 Optical manipulation

The optical manipulation of particles using Bessel beams was demonstrated by Arlt *et al.* [57] in 2001. Micro-manipulation using optical tweezers [58] is a well established technique which makes use of the optical gradient force to trap and confine microscopic particles at the focus of a laser beam. The Bessel beam has some advantages and disadvantages over this technique. The main disadvantage is that such beams do not have a specific focus and thus

cannot form a genuine three-dimensional optical trap. Particles can however be confined in two dimensions and pushed up (or down) against a sample coverslip. An example of a typical experimental setup is shown in figure 8. The long, thin central core of the Bessel beam makes it possible to trap long thin objects, such as rods and E-coli with relative ease. The ability to trap simultaneously both high and low refractive index particles in a Bessel beam is made possible by its ring structure [59–61]. Low index particles can be caught in the dark regions between the bright rings, while high index particles are caught in the rings themselves. High-order Bessel beams (HOBB) can also be used for micro-manipulation and can be used to study the orbital angular momentum of light beams [48, 59], including the first demonstration of the transfer of orbital angular momentum to a low-index particle, as shown by Garcés-Chávez *et al.* [59]. The first observation of the simultaneous transfer of spin and angular momentum to a particle which is not positioned on the beam axis was also observed using a Bessel beam [62] and the ring structure of the beam allowed the quantification of the spin rates of the particles as a function of radius. In this experiment the HOBB was circularly polarized, and this polarization state was altered by passing the light through a birefringent particle which rotates as the spin angular momentum of the light is transferred to the particle. The orbital angular momentum was transferred via scattering and is a result of the inclined wavefronts of the HOBB.

The reconstruction properties of the Bessel beam have also found use in optical micromanipulation. Standard

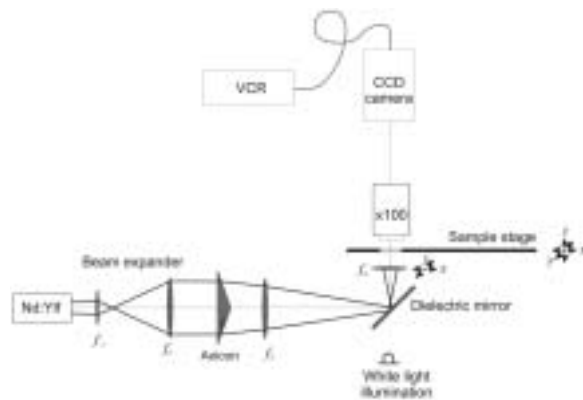


Figure 8. Experimental setup to create Bessel beam optical tweezers. The beam from an infrared laser is expanded using the beam expander f_1-f_2 before passing through the axicon to form a Bessel beam. This beam is then telescoped down to the appropriate size for optical tweezing using the second telescope f_3-f_4 . The beam is then sent through a microscope sample where the tweezing takes place. The microscope objective, white light illumination and CCD camera are used to image the sample.

optical tweezers are made from highly focused Gaussian beams and such is the divergence of the beam that the optical forces even a few microns away from the focal point may be insufficient to trap particles. Also the forces generated at the focus will draw any particle in, meaning that a standard optical trap can only trap particles a very small distance from the focus of the beam, and if multiple particles are trapped they will form stacks (stacks are formed due to the trapping forces a small distance away from the beam focus still being large enough to trap particles, so if a particle is trapped and another particle moves near the focus it is unlikely to push the trapped particle out of the trap but rather get drawn into the trap above the first particle). It is not possible to trap multiple particles simultaneously that are spatially separated by large (millimetre, say) distances. Reconstruction makes such manipulation possible. If a particle is trapped by a Bessel beam, the reconstruction of the beam around the object means that the beam reforms at a point beyond the object, and this beam can then be used to trap another particle. Thus a one-dimensional array of particles can be trapped and manipulated simultaneously [60]. As shown in figure 9 the particles can be separated by distances

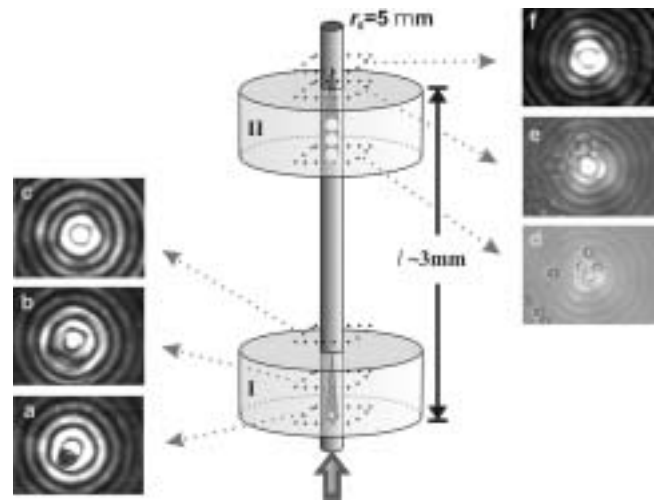


Figure 9. Reconstructing Bessel beam optical tweezers. Making use of the experimental setup in figure 8 the Bessel beam in this experiment passes through not one, but two sample chambers, separated by 3 mm. Due to the long propagation distance of the beam coupled with the reconstruction properties of Bessel beams, particles in both chambers can be manipulated simultaneously. (a–c) show a particle being tweezed in chamber I, with each figure showing different z -positions. Thus in (c) the beam has reformed despite tweezing a particle at the bottom of the cell in (a). (d–f) show the same beam tweezing a particle in chamber II, with (f) indicating that the beam has reformed again by the time it exits the second chamber.

approaching a centimetre and this is limited by the propagation distance of the beam (so long as the beam can keep reconstructing it can trap particles). The particles can be in different sample chambers but can still be manipulated simultaneously. The distance after which the beam reconstructs after encountering a particle is to some extent dependant on the refractive index properties (not just the size) of the particle and this may be an aid to the characterisation of particles and used for example in differentiating cancerous from non-cancerous cells.

While the Bessel beam does not provide a true three-dimensional trap, the confinement of the central core, coupled with an ever present source of radiation pressure, leads to the idea of controlled long distance optical guiding. This has been demonstrated for extended distances. In, for example, [63], particles are guided for a distance over 3 mm in a beam with a central core diameter of $5.6 \mu\text{m}$. This has to be compared to the $23 \mu\text{m}$ Rayleigh range of a Gaussian beam with comparable beam waist and demonstrates the ability of the Bessel beam to provide long range point to point optical guiding of particles.

Bessel beams are also finding application in the areas of statistical physics which examine how particles with inherent Brownian motion act in the presence of an external potential. The interest stems from the fact that activated escape from a metastable state underpins many phenomena right across the physical sciences and many of the theoretical predictions have yet to be experimentally studied. A particle in an optical trap is an excellent system with which to elucidate the underlying physics of this situation [64]. Such a particle undergoes Brownian motion within the trap [65] and can be thermally activated to escape from the trap region. The motion of a particle in a harmonic potential as well as ratchet-like potentials can be studied [66]. Activation (jumping) over a barrier is given by an exponential law of the form $T_K = t_R \exp(\Delta U/kT)$ where T is the temperature, ΔU the difference in optical potential experience by the object and t_R is a characteristic time for the process. Studies of the escape and synchronisation of a particle between two adjacent optical tweezers traps has been observed [67].

In recent work, it has been recognized that the Bessel beam offers an important avenue for studying such activation in a 2D circularly symmetric optical potential [63]. The geometry of the Bessel beam is advantageous in some respects for this work. The fall in amplitude of the light beam has a reciprocal relationship to the radius with the central ‘non-diffracting’ core providing the most intense region of the beam. The large number of rings of the beam can thus be employed to assist in the amalgamation process and we can make use of this thermal activation to load particles from a large area ($2.5 \times 10^{-9} \text{m}^2$) into the beam centre. The first study of this type of loading used an optical guiding geometry where particles were seen to

accumulate within beam centre when a tilt was applied to the beam [68]. This tilt created a washboard-like potential (see figure 10). A washboard potential is again a rather ubiquitous and seminal form of landscape across the physical sciences. In this instance it was possible to create directed transport of particles towards the beam centre. This in turn meant that the ‘non-diffracting’ central maximum of the beam was loaded with particles collected from a 50–100 micron radius. In contrast to previous guiding using light beams this offers obvious advantages: the outer rings act to direct particles towards the beam centre: once loaded there they were constrained by the gradient force to a narrow region at beam centre and pushed along the beam propagation direction by radiation pressure.

Researchers have also recently started to look at the idea of this hopping in a vertical geometry where the actual transit of microscopic objects between rings of an untilted Bessel beam may be observed in real time [69]. It is important to stress that this is in the absence of any flow. Figure 11 shows this for 2.3 micron spheres with time denoted for each frame. Interestingly when we place

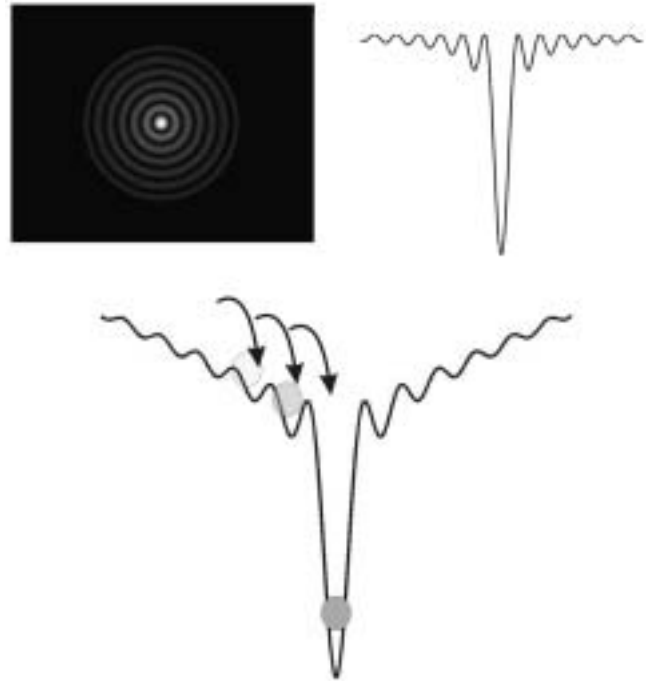


Figure 10. A tilted ‘washboard’ potential. The upper left figure shows the Bessel beam cross-section intensity profile with a line profile of the potential created by such a beam shown in the upper right. Note that all the Bessel function zeros go to zero. In the tilted case at the bottom of the figure a bias has been added that lifts the zeros. This enables particles trapped in the beam to move towards the centre of the beam more rapidly than in the untilted case.

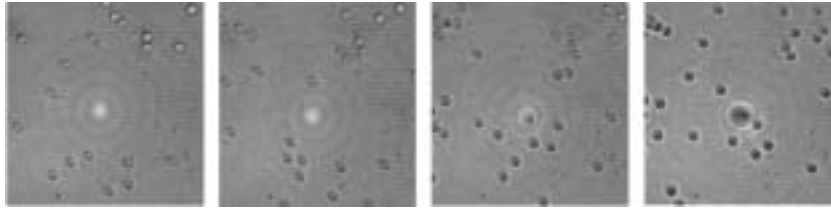


Figure 11. The amalgamation of 2.3 micron spheres in the centre of a Bessel beam due to the Brownian motion of the particles interacting with the optical potential of the beam. The frames are taken every 60 s.

particles of different sizes on the pattern they respond in a different manner. For example 5 micron spheres and 2.3 micron spheres behave differently. The smaller spheres may get ‘locked’ into the pattern and only slowly migrate towards beam centre whereas for this particular Bessel beam size the larger 5 micron spheres are too large to get locked into any ring but rather respond to the overlying $1/r$ intensity gradient and migrate more rapidly towards beam centre. This is of interest for separation and sorting of microscopic objects in the absence of any flow.

3.2 Atom optics

The mechanical effects of light interactions with matter have led to a number of truly landmark results in the last decade. Laser cooling and trapping have come to the forefront of atomic physics and emerged as a powerful technique whose applicability is ever increasing. The increase in phase space density offered by laser cooling is the first step towards compression and cooling of atomic gases to achieve Bose–Einstein condensation [70]. Nowadays upwards of one hundred laboratories worldwide have laser cooling apparatus in place and there are upwards of forty BEC experiments in progress at the time of writing. Ultra-cold atomic gases have truly opened new vistas of both fundamental and applied science ranging from new studies of matter wave properties to ultra-stable atomic clocks. In tandem with this progress the field of atom optics has developed with the key goal to manipulate and control atoms in the manner that conventional optics does for laser light. Thus intense activity has centred upon the creation of ‘atom optics components’: mirrors, beamsplitters and guides for example. This can be achieved with optical or magnetic forces but optical forces have become rather attractive in recent years due in part to the recognition that they are state independent and thus more widely applicable [71]. Light can exert an optical dipole or gradient force on atoms that depends upon the detuning of the light frequency with respect to the appropriate resonant transition. Essentially this gradient force is analogous to the gradient force used in optical tweezers for microscopic particles. If the light is detuned above resonance the

atoms are repelled from the light intensity and thus hollow regions fully enclosed in two or three dimensions can be used for atom confinement. In the red-detuned (below resonance case) we have the atoms drawn towards the light intensity (just as in standard optical tweezers). Both forms of detuned light can result in both traps (dipole traps) or guides for atoms. This latter property can be used to transport atoms over large distances and the gradient force counteracts any tendency for the atomic ensemble to spread or disperse in space. Broadly speaking red detuned guides require higher power as they are usually operated far from resonance to obviate heating from the guide beam. Blue detuned guides require less power as they can be used very close to resonance as the atoms primarily reside well away from the beam and penetrate and interact with the light field relatively infrequently.

An area where Bessel beams have created significant interest in atom optics is a non-diffracting guide (see figure 12). The idea here is to confine the atoms within the central minimum of a high order Bessel light beam. The atoms may be channelled into the Bessel beam guide from a hollow Laguerre-Gaussian light mode. The important point is that the non-diffracting nature of the central region allows the atoms to be transported an extended distance without transverse spreading as would be the case for a normal hollow light beam. Figure 12 shows a potential geometry for coupling into a hollow Bessel beam. It is important to note that researchers have been able to transport atoms into hollow fibres that have dimensions of a few to tens of microns. This is useful for atom deposition and atom fibre interferometry. The Bessel beam offers an all-optical realization of a fibre guide. Also the central region of the Bessel beam can be of the order of the wavelength. Thus if we are able to couple a Bose–Einstein condensate into such a guide one may be able to allow the condensate to propagate in only a restricted number of modes that can eventually result in the observation of atomic speckle patterns.

Bessel beams can also be used for atomic dipole traps. Optical dipole traps have seen an immense resurgence of interest and application in the last few years. A translated optical dipole trap has been used to transport a BEC 30 cm

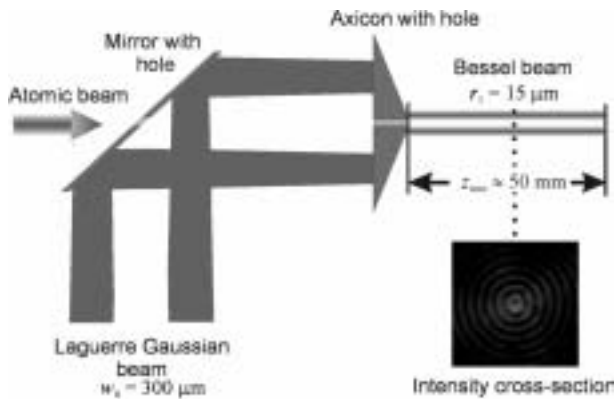


Figure 12. Bessel beams as atomic guides – a Laguerre-Gaussian beam is reflected off a mirror with a hole in which allows an atom beam to enter the beam. The optical and atomic beams pass through an axicon with an on-axis hole and the atoms are subsequently guided in the narrow Bessel beam guide.

[72, 73], a crossed beam dipole trap with variable potential depths can be used to instigate evaporative cooling directly [74] to BEC and such traps have been used recently to help achieve BEC in caesium [75]. The Bessel beam offers an interesting prospect for an atomic dipole trap. For a Gaussian beam we are restricted in that the Rayleigh range of the beam is proportional to the square of the beam waist, which thus links these two parameters. For an axicon-generated Bessel beam, however, we have independent control of the distance and size of the central maximum. In this example the opening angle of the axicon dictates the size directly of the central maximum of the beam whereas the incident beam diameter is related to the propagation distance of the Bessel beam. This allows us to create an optical dipole trap with a very large aspect ratio [76]. Low temperature Bose systems in 1D are currently the subject of intense interest as they may exhibit phenomena not seen in 2D or 3D. Notably if we consider the interparticle repulsive interaction in this region and then look at the typical parameters for the correlation length of the system versus mean interparticle separation we can violate the normal case of a weakly interacting system. The quantum gas then exhibits Fermi like properties [77] as wavefunction decreases at short interparticle distances. This is termed a Tonks gas of impenetrable bosons [78] and is a key current experimental goal in the field, which has only recently been realized [79].

Work by Okamoto has also considered using high-order Bessel beams for the focussing of cold atoms [80]. Here the use of a blue detuned first-order Bessel beam is proposed to write a guided beam of atoms, initially generated using a zeroth-order Bessel beam, down to spots of the order of 100 nm over distances of several millimetres.

3.3 Non-linear optics

The idea of using a Bessel beam to enhance non-linear optical processes is an attractive one. In conventional Gaussian optics, perhaps the simplest non-linear process is that of single pass second harmonic generation (SHG). The first experimental work on this topic was carried out by Wulle and Herminghaus [81]. The phase-matching efficiency (a measure of the efficiency of the non-linear conversion process) was found to be dependent on the longitudinal wavevector of the Bessel beam (as one would expect). However this could be tuned by appropriately focusing the beam and thus, it was argued, that such beams can be considered as being light beams with a tuneable wavelength in such experiments. Wulle and Herminghaus suggested that Bessel beams could be used in conditions where traditionally it was difficult to phase match, for instance with new non-linear materials where normal temperature and angle phase matching conditions were not suitable.

An ideal plane wave can be frequency doubled, in the absence of depletion, with an efficiency that is proportional to the square of the beam intensity and to the length of the non-linear crystal. Thus by focusing down a Gaussian beam we can increase the efficiency; however, if we focus too tightly the beam divergence becomes very rapid and this leads to a decrease in the conversion efficiency. The solution would seem to lie in a beam which does not diverge, and it was this topic that Shinozaki *et al.* [82] turned their attention to in 1997. They compared the conversion efficiency of Gaussian and Bessel beams in bulk crystals with similar interaction lengths inside the crystal. They calculated that the Bessel beam was up to 48% more efficient than the Gaussian beam due to the increased intensity of the beam along the optical axis when compared to the Gaussian. Unfortunately Shinozaki *et al.* had used a somewhat simplified model for their approximation to a Bessel beam, which did not take into account the intensity variation along the beam axis of an experimentally realized beam or the full cross-section of the beam. Arlt *et al.* [83] addressed these points in their paper, carrying out an experimental comparison between Bessel and Gaussian beam second harmonic generation. Boyd and Kleinman [84] had examined the optimal Gaussian beam parameters for second harmonic generation and Arlt *et al.* found that the conversion efficiency was in fact less for the Bessel beam than for a Boyd–Kleinman focused Gaussian beam. Indeed they found that a Bessel beam can never exceed the conversion efficiency of a Boyd–Kleinman focused Gaussian of similar power, due to the fact that the power of the Bessel beam is distributed equally among its rings. Experimentally they made use of a holographically generated Bessel beam passing through an LBO crystal, and the experimental results were in good agreement with

their model. This work was confirmed by the later theoretical study of Magni [85] in which he showed that the optimal beam for second harmonic generation was in fact a beam made of a combination of Laguerre-Gaussian beams with a cross-section only marginally different than that of a Gaussian and this would only produce a 2% increase in conversion efficiency over an optimised Gaussian beam.

Further second harmonic experiments were carried out by Piskarskas *et al.* [86] in which non-collinear phase matching was demonstrated in PPKTP (periodically poled KTiOPO₄) and by Jarutis *et al.* [87] in which the SHG of high-order Bessel beams was investigated.

The Bessel beam was also found wanting when used in pumping an optical parametric oscillator (OPO), a device which converts the pump field into lower energy signal and idler fields. An experiment was carried out by Gadonas *et al.* [88] that considered optical parametric generation which created two conical beams with differing polarisations. The use of OPOs pumped by Bessel beams was first considered, theoretically, by Belyi *et al.* [89]. The authors used an ideal Bessel beam in their models and showed that in this case the efficiency of the parametric generation was increased by using a Bessel beam instead of using a Gaussian beam. The first parametric oscillator device pumped using a Bessel beam was produced by Piskarskas *et al.* [90]. This was a device based on KTP and produced a characteristic output beam due to the non-collinear phase matching consisting of a ring beam surrounding a central spot. Binks and King [91] demonstrated the first periodically poled material based OPO pumped using a Bessel beam using PPLN (periodically poled lithium niobate) which produced a Gaussian signal beam. Binks and King went on to consider such a device in more detail [92], and showed that there was no practical advantage to pumping an OPO with a Bessel beam rather than a Gaussian beam. The Gaussian beam pumped device showed lower threshold characteristics and a better slope efficiency than the Bessel beam pumped device, while both had a similar threshold fluence.

Other wave-mixing effects have been investigated using Bessel beams, for instance the enhancement of photo-refractive two-wave mixing [93], the creation of Bessel beams within the coherent anti-Stokes Raman scattering process [29], stimulated Raman scattering [94, 95], higher order stimulated Brillouin scattering (SBS) and third harmonic generation [96–101].

3.4 Pulsed Bessel beams

Following the seminal work of Durnin, Micelli and Eberly, [3] the work on Bessel light beams concentrated on monochromatic continuous wave realisations of such light fields. This is what we have concentrated upon in the discussion in this article. However laser light can also

propagate in a pulsed mode where in addition to spatial characteristics we have to consider the temporal behaviour of the light. Indeed mathematically there have been several solutions that are spatio-temporal invariant and offer interesting applications. Analogously to the monochromatic case, these are immune to spreading and retain their localisation in both space and time as they propagate. Applications of such a beam include optical communication and advanced imaging. In this section we briefly explore these light fields. It is to be noted that when we go to such broadband light considerations several publications have dealt with pseudo non-spreading functions. This includes focus wave modes, Bessel–Gauss pulses, electromagnetic bullets, slingshot pulses, and X waves, among others. The key point is all these solutions maintain their spatio-temporal characteristics as the pulse propagates in free space.

Obviously a key issue is the experimental realization of such beams and the monitoring of their evolution. Once such variant of the solution, the X pulse has attracted particular attention [102]. Its name derives essentially from the form of the spatio-temporal pulse when a snapshot is taken in space and time a pulse is obtained that resembles the letter X rotated about its horizontal axis (see figure 13). X pulses are known from acoustic [103, 104] and terahertz radiation regions. Such X waves are termed thus due to the X shaped intensity distribution over time and radius that they represent and are the result of interference of few-cycle wavepackets from ultrashort pulse light sources. The

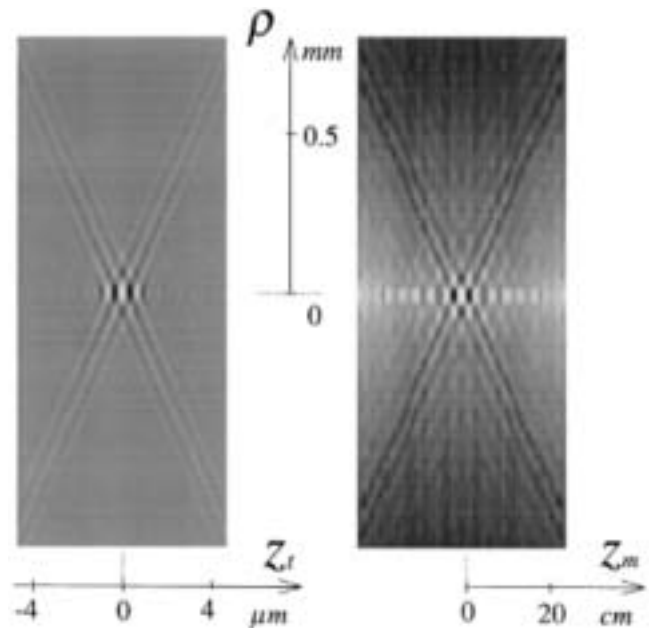


Figure 13. Bessel X-wave – left panel simulated result; right panel experimental result (printed with permission from [102]).

transmission of such beams in dispersive media has also been considered [105].

In the acoustic region this beam has been of interest as a candidate for medical ultrasonic imaging. In the optical domain a restriction on generating such beams is primarily the extremely broad spectral content. The reduction in the temporal spread (in addition to the spatial spread) is very interesting in the short pulse region as it offers a scheme to overcome group velocity dispersion. Work has characterized optical X waves in free space [102]. To record the pulse a gate could be used but is not feasible at these ultrashort timescales. Instead a cross-correlation technique was employed using a reference wave. A key point about such ultrashort Bessel beams (BB) was utilized in this scheme: the conical wavevectors mean the beam exhibits superluminal propagation meaning it propagates at a different speed than a reference wave. The superluminal BB characteristics were recorded by Alexeev and co-workers in laser plasma [106]. Optical guiding of intense laser pulses in plasmas has generated attention due to key applications in X-ray generation and laser driven electron generation. Indeed the superluminal measurements were performed with ultrashort 70 fs pulses and three diagnostics used to verify the rapid plasma generation by the pulse. Notably the superluminal propagation of the group velocity has generated some debate but is not a violation of special relativity: by considering the energy flow of the beam (Poynting vector) and energy flow speed which is actually less than c . By consideration of the manner of formation of the Bessel beam (the interference of conical wavefronts) one finds that the constant phase intersection points will move faster than c but once again we re-iterate that the energy flow of the beam is sub-luminal.

The space time coupling in a Bessel beam has also been studied [107]. Here a variation in the spatial frequency spectrum and contrast in the intensity pattern as a function of pulse duration was recorded. At very short pulse durations (~ 10 fs or less) a reduction in fringe contrast is expected due to the enhanced spectral content of the illuminating pulse. Grunwald and co-workers looked at micro-axicon arrays to generate femtosecond Bessel light beams and used pulses of duration 10–30 fs for their studies. The micro-axicon arrays were fabricated by vapour deposition of dielectric layers penetrating through shadow masks and using a scheme of planetary rotation of the substrate. Illuminating this array from an amplified Ti-sapphire laser system yielded multiple Bessel beams. Moving from the CW regime to pulses as short as 12.5 fs, the contrast and absolute intensity of the beam profile was altered. The reduction of pulse width was directly correlated to the increase in spectral bandwidth of the beam. Superposing several spectral components with a range in excess of 100 nm gave a spectral off-axis modulation which in turn resulted in a reduction in

intensity and contrast of the outer fringes of the Bessel beam profile. Short pulse durations lead to time dependent geometry of the interference region. A reduced contribution of the interfering partial waves from adjacent (neighbouring) axicons was observed and a reduction in the axial extent of the Bessel zone.

Ultrashort Bessel beam have been studied in the context of laser plasmas. The coupling of intense laser pulses to plasmas is key to inertial confinement fusion, high harmonic generation, X-ray sources from laser plasmas and more elaborate laser based acceleration methods. These applications require laser pulses to propagate at high intensity above the ionisation threshold of the atoms. Within this self focusing processes are of relevance as here the light field can self-trap and propagate at high intensity over distances far in excess of the Rayleigh range. Bessel beams have been observed to exhibit resonant self-trapping correlated with an enhanced absorption [108]. The self-trapping of the Bessel beam in a self-generated plasma was recorded by integrating CCD camera images at the exit plane of the plasma channel.

A tubular plasma fibre has been generated by optical breakdown of a target using a Bessel beam of the fifth order generated by using a phase plate and axicon in tandem [109]. The insertion of the hollow Bessel beam in this instance caused the breakdown of the gas and the generation of the tubular plasma with a maximum electron density located radially away from the optical axis. The index of refraction of such a plasma decreases with radius implying we can utilize this as a waveguide for high intensity laser pulses. In the experiment, electron density was rapidly recorded in the high intensity regions of the beam well before any ion mass motion could have taken place. This results in plasma waveguides that have smaller effective core diameters than those produced through shock expansion. This is a key point in this study as obviating the need for hydrodynamic evolution to establish deep electron density profiles means that intense laser pulses of very small waist sizes can propagate.

4. Conclusions and outlook

The idea of non-diffracting fields is an intriguing one and has attracted a large amount of work in the seventeen years or so since Durnin *et al.*'s original papers [1, 3]. Of course, as we have seen the Bessel beam has not been a universal success; in many areas the perceived advantages of propagation invariant beams turn out to be not quite as good as imagined when the experimentally realizable versions of such beams are applied to the problem at hand. The problem with the power being shared between the rings of the Bessel beam has also caused experimental difficulties and although this can usually be circumvented simply by increasing the overall power of the whole beam, this may

not always be possible and may lead to other unwanted problems. The Bessel beam has had its successes, however. As we have seen the Bessel beam has extended the optical ‘toolbox’ in micromanipulation providing many techniques that are not possible using conventional Gaussian beams. In atom optics the theory seems sound, although the experiments have not yet been carried out, but unlike in other areas the practicalities of using quasi-Bessel beams seem good. There has been much detailed work on the properties of Bessel beams themselves and this work has provided a sound footing for the current crop of experimental work as well as that which is to come. As a result of the study of Bessel beams an improved knowledge of beam propagation and classes and properties of light beams has emerged. Perhaps the most intriguing areas of study are those looking at the motion of microscopic particles in optical potentials as well in the number of topics that involve the use of pulsed Bessel beams. There is plenty of work still to be done and the future looks healthy for the still controversial idea of a beam that does not diffract.

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