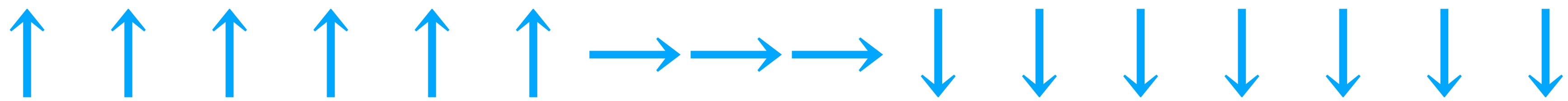
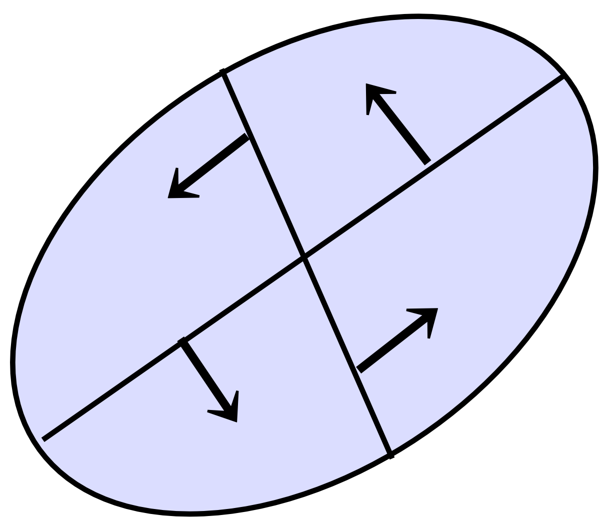
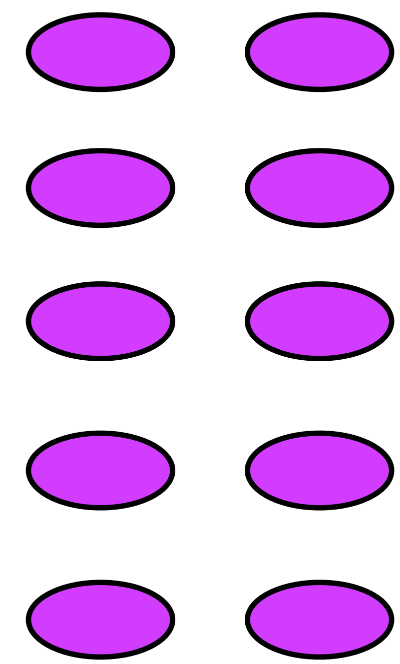


Effective Potentials of Vortices in Cylindrical Magnetic Dots





First Magnetic North Conference
University of Western Ontario
June 5-7, 2010

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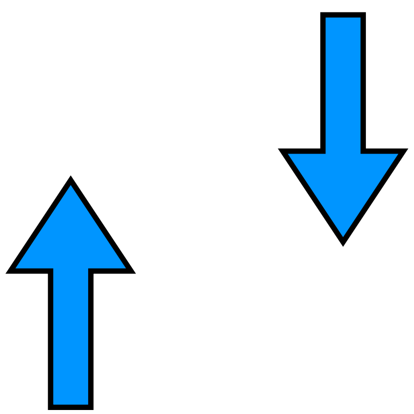
Overview: Consider some ideas about the
"Effective potential for a magnetic vortex in a nanodot"

- Magnetics & energetics for cylindrical dots.
- Vortices as particles, charges, switching prospects, etc.
- Finding the potential by using a Lagrange constraint.
- The numerical solution method.
- Results: Stability of a vortex in a nanodot,
  Using defects to pin and modify vortices.



Magnetic dots

- Approx. 50 nm - 5 um, magnetic elements & arrays, soft magnetic materials, grown with epitaxial & lithographic techniques.

- Can be islands on a non-magnetic substrate.   Form arrays of interacting particles.

- Will have new physics effects due to small length scales (modified spin wave modes, spin wave - vortex interactions, surface effects, special sensitivity as detectors).

Magnetic Dots: Applications, Advantages

- memory elements & signal processing
- nonvolatile storage (magnetic ram)
- use in giant magnetoresistance (GMR) sensors
- integration into spintronics devices (spin flipping etc., via spin-polarized current.)
- stable vortex state with low stray field.

Magnetic Vortex Core Observation in Circular Dots of Permalloy

T. Shinjo,[1*] T. Okuno,[1] R. Hassdorf,[1†] K. Shigeto,[1] T. Ono[2]

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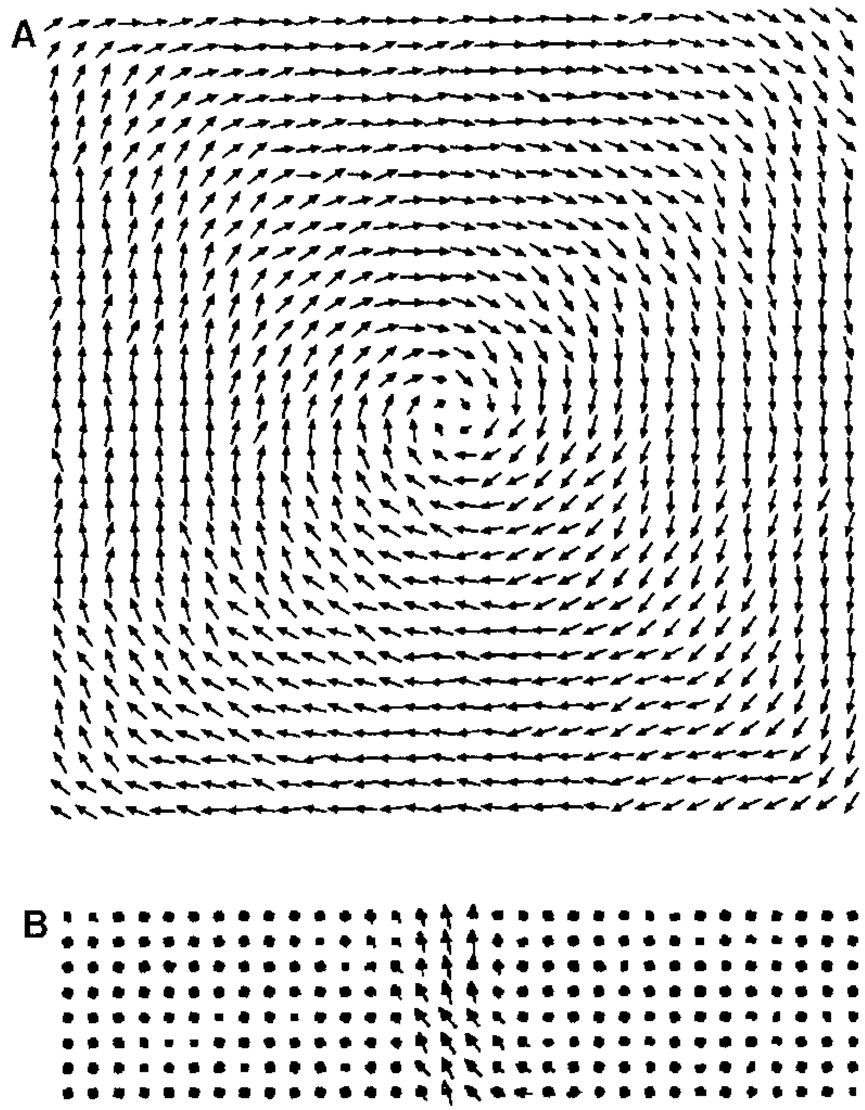
vortex core detection.

Fig. 1. Monte Carlo simulation for a ferromagnetic Heisenberg spin structure comprising 32 × 32 × 8 spins [courtesy of Ohshima *et al.* (*2*)]. (**A**) Top surface layer. (**B**) Cross-section view through the center. Beside the center, the spins are oriented almost perpendicular to the drawing plane, jutting out of the plane to the right and into the plane to the left, respectively. These figures represent snapshots of the fluctuating spin structure and are therefore not symmetric with respect to the center. The structure should become symmetric by time averaging.



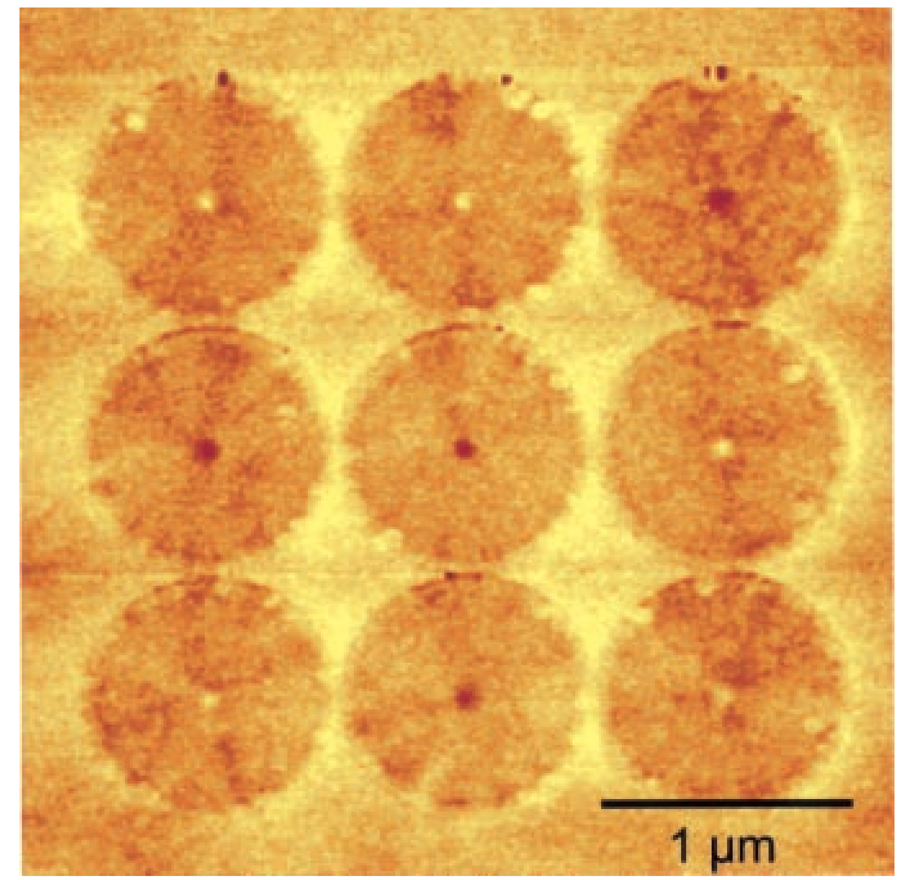


Fig. 2. MFM image of an array of permalloy dots 1 μm in diameter and 50 nm thick.

Can see up/down Sz vortex polarization!

Vortex control & switching?

How to control the position, circulation, and polarity of a magnetic vortex in a nanomagnet?

--voids or holes?
--applied fields, currents?
--optical impulses?

Theory: computer simulations of spin energetics & dynamics to study vortex motion and spin reversal.

053902-2 Vavassori *et al.*
JOURNAL OF APPLIED PHYSICS **99**, 053902 (2006)

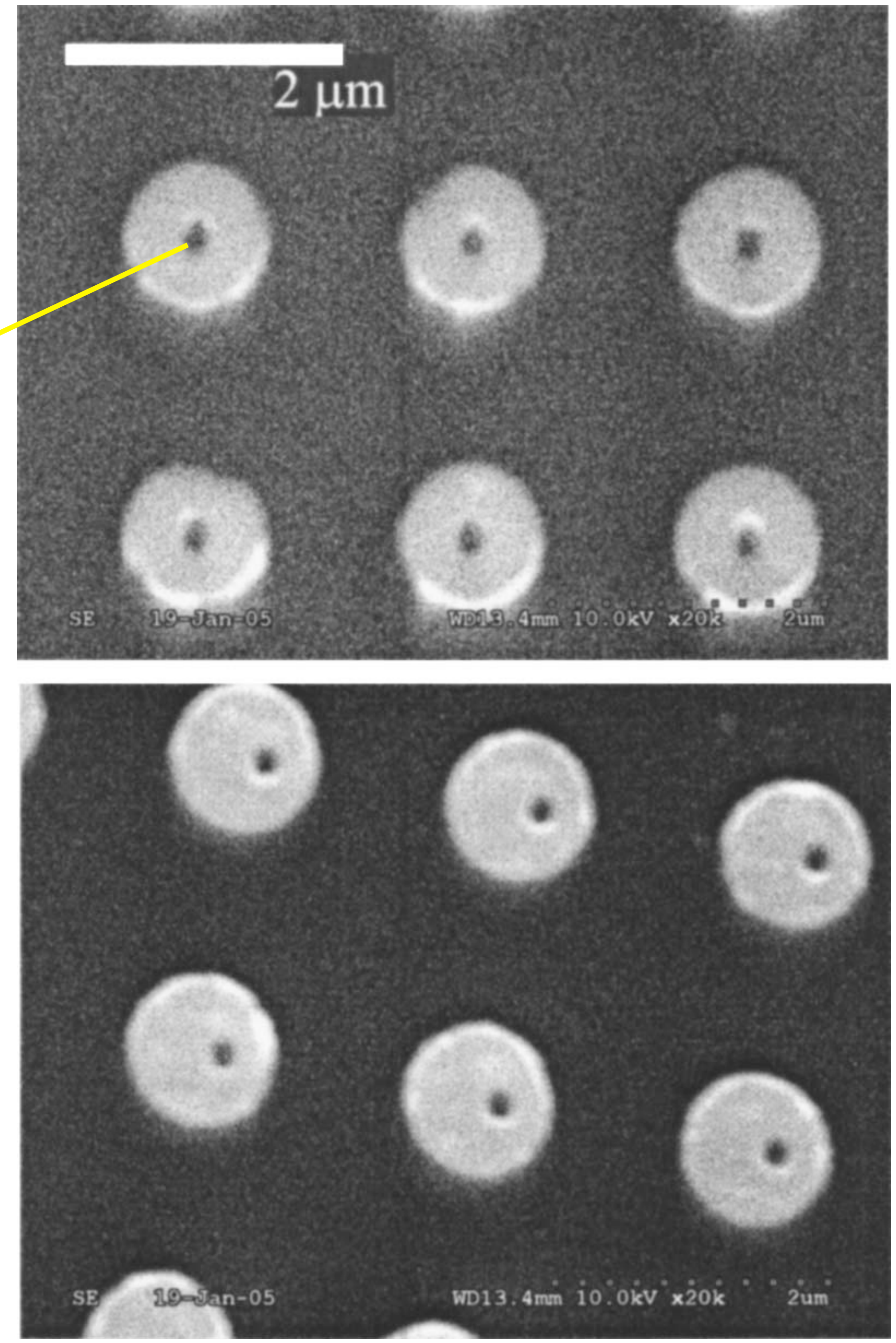
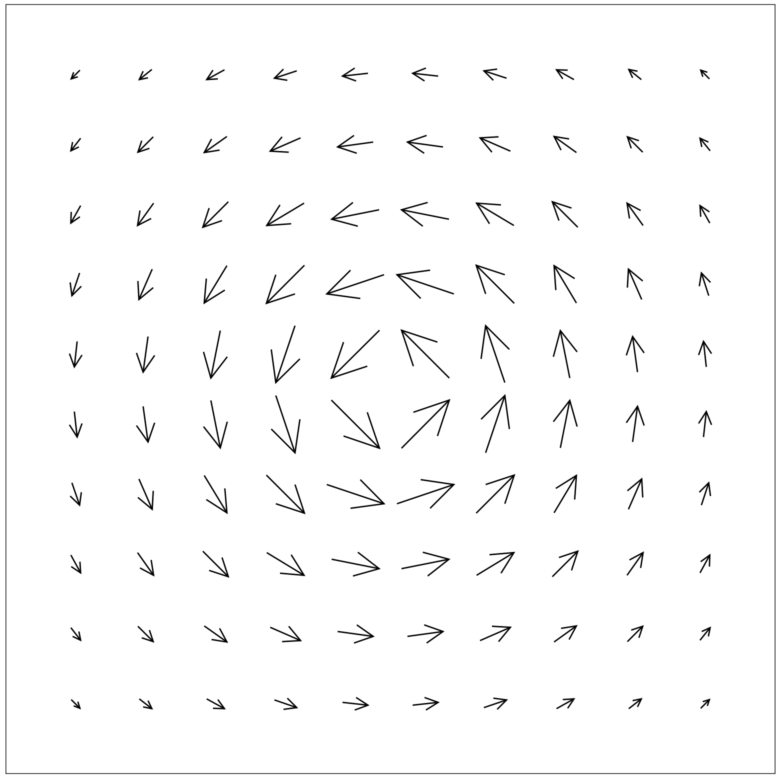


FIG. 1. Scanning electron images of a portion of the two patterns: symmetric rings (upper panel) and asymmetric rings (lower panel).

Vortex particle-like properties



"vorticity charge"

$$q = \frac{1}{2\pi} \oint \vec{\nabla}\phi \cdot d\vec{r} = 0, \pm 1$$

circulation or curling

$-1 \leq C \leq +1$

$$C = \frac{1}{N} \sum_i \hat{\sigma}_i \cdot \hat{\phi}_i \qquad \hat{\sigma}_i = \vec{\mu}_i / \mu.$$

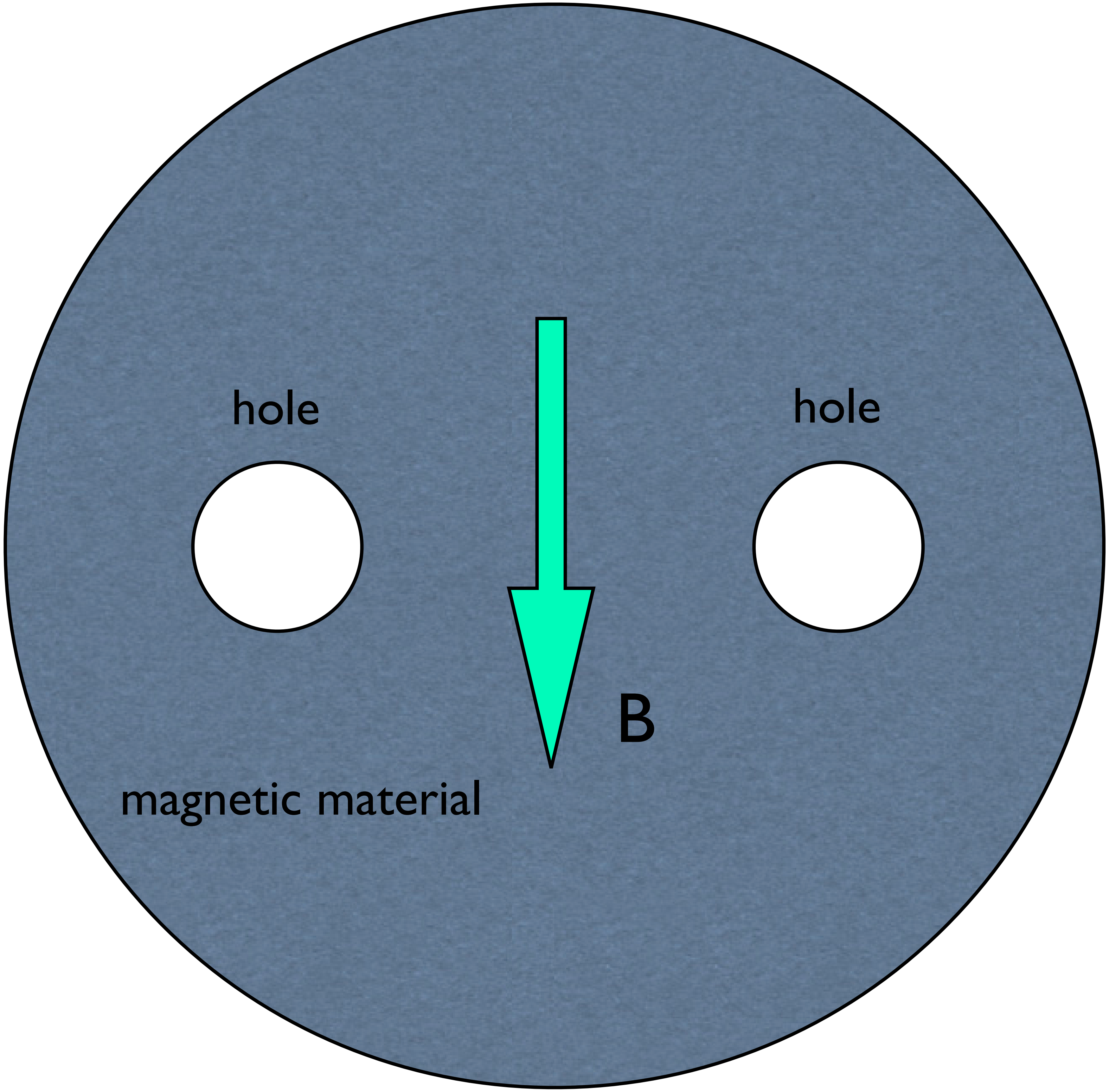
polarization

$p = m_z = \pm 1$ at vortex core

"topological charge"

$Q = 2\pi pq$ = solid angle mapped out by all spins

possible bistable nanomagnetic switches



Can an applied magnetic field control whether a vortex surrounds the left hole or the right hole?

Approaches:

Energy minimization for the metastable states.

Monte Carlo or Langevin dynamics simulation for including thermal fluctuations and seeing the switching process.

Microscopic Theory. Model for interacting atomic dipoles.

Hamiltonian: $H = H_{ex} + H_{dd} + H_B$

$$\mu_{\rm atom} = g\mu_B S$$

exchange: $$H_{\rm ex} = -J \sum_{(i,j)} \vec{S}_i \cdot \vec{S}_j$$

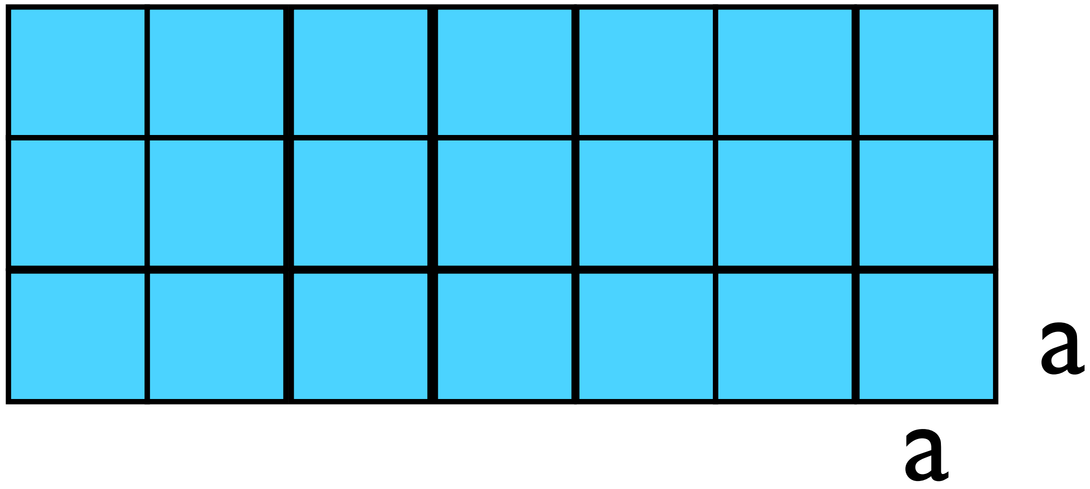
dipole-dipole: $$H_{\rm dd} = -\left(\frac{\mu_0}{4\pi}\right) \sum_{i>j} \frac{[3(\vec{\mu}_i \cdot \hat{r}_{ij})(\vec{\mu}_j \cdot \hat{r}_{ij}) - \vec{\mu}_i \cdot \vec{\mu}_j]}{r_{ij}^3},$$

applied field: $$H_{\rm B} = -\sum_i \vec{B} \cdot \vec{\mu}_i$$

Relative strengths: JS^2, $D = \left(\frac{\mu_0}{4\pi}\right) \frac{\mu_{\rm atom}^2}{a_0^3}$, $B\mu_{\rm atom}$

largest | smaller, but more terms | usually, pretty small

Micromagnetics. a more practical Model for interacting cells.



each cell has a unit dipole: $\hat{m} = \vec{M}/M_S.$

- Model for cylindrical nanodot, radius R, height L.
- partition the sample into cells of size a x a x L.
- assume magnetization is saturated (M_S) in each cell, but the directions vary from cell to cell.
- the cells interact as dipoles, with exchange & demagnetization fields.

Hamiltonian: $\mathcal{H} = \mathcal{H}_{\text{ex}} + \mathcal{H}_{\text{demag}} + \mathcal{H}_{\text{B}}$

exchange: $$\mathcal{H}_{\text{ex}} = A \int dV \; \nabla \hat{m} \cdot \nabla \hat{m},$$

demagnetization: $$\mathcal{H}_{\text{dd}} = \mathcal{H}_{\text{demag}} = -\frac{1}{2}\mu_0 \int \; dV \;\; \vec{H}_M \cdot \vec{M}$$

applied field: $$\mathcal{H}_B = -\mu_0 \int \; dV \;\; \vec{H}_{\text{ext}} \cdot \vec{M}$$

Plan Here: Use some kind of energy minimization to seek out stable equilibrium configurations.

Difficulties: Finding demagnetization field H_M and constraining the vortex location

Scale energies by the exchange between cells: $$J_{\text{cell}} = \frac{2Av_{\text{cell}}}{a^2} = 2AL.$$

"magnetic exchange length" $$\lambda_{\text{ex}} = \sqrt{\frac{2A}{\mu_0 M_S^2}}$$

demag. field: $\vec{H}_M = M_S \tilde{H}_M$

Hamiltonian on the grid of cells:

$$\mathcal{H}_{\text{mm}} = -J_{\text{cell}} \left\{ \sum_{(i,j)} \hat{m}_i \cdot \hat{m}_j + \left(\frac{a}{\lambda_{\text{ex}}}\right)^2 \sum_i \left(\tilde{H}_{\text{ext}} + \frac{1}{2}\tilde{H}_M\right) \cdot \hat{m}_i \right\}$$

Need $\left(\frac{a}{\lambda_{\text{ex}}}\right)^2$ less than 1 for reliable solutions.

(cells smaller than exchange length)

Finding the demagnetization field via Green/FFT approach.

➡ The magnetostatics problem has no free currents:

$$-\tilde{\nabla}^2\tilde{\Phi} = \tilde{\rho} \qquad \tilde{\rho} \equiv -\tilde{\nabla}\cdot\hat{m} \qquad \tilde{H}_M = -\tilde{\nabla}\tilde{\Phi}$$

use Green's function solution:

$$\tilde{\Phi}(\vec{r}) = \int d^3r'\ G(\vec{r},\vec{r}\,')\,\tilde{\rho}(\vec{r}\,') \qquad G(\vec{r},\vec{r}\,') = \frac{1}{4\pi|\vec{r}-\vec{r}\,'|}$$

specialize to a thin cylinder (2D) geometry: $\tilde{r} \equiv (x,y)$

$$\tilde{H}_z(\tilde{r}) = \int d^2\tilde{r}\,'\ G_z(\tilde{r}-\tilde{r}\,')\ m_z(\tilde{r}\,')$$

$$\tilde{H}_{xy}(\tilde{r}) = \int d^2\tilde{r}\,'\ \vec{G}_{xy}(\tilde{r}-\tilde{r}\,')\ \tilde{\rho}(\tilde{r}\,')$$

$$G_z(\tilde{r}) = \frac{1}{2\pi L}\left[\frac{1}{\sqrt{\tilde{r}^2+L^2}} - \frac{1}{|\tilde{r}|}\right]$$

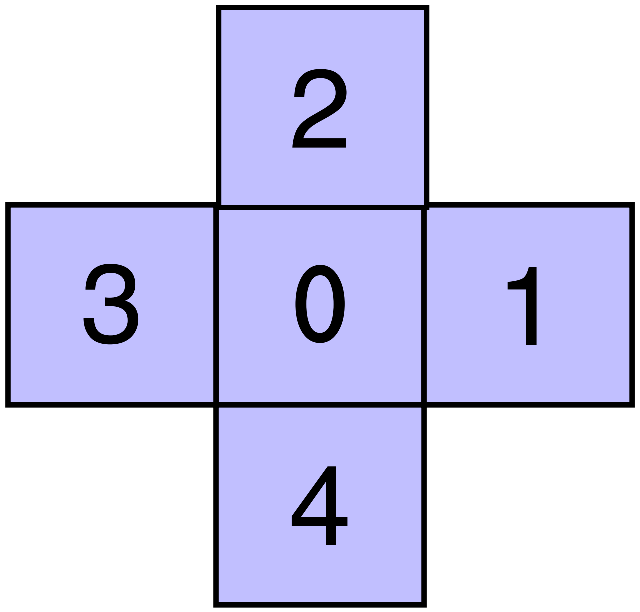
$$\vec{G}_{xy}(\tilde{r}) = \frac{1}{2\pi L}\left[\sqrt{1+\left(\frac{L}{\tilde{r}}\right)^2} - 1\right]\hat{e}_{\tilde{r}}$$

Some details.

The magnetic charge densities depend on the present magnetic configuration, such as:

$$\tilde{\rho}_0^{\text{vol}} = \frac{q_M^{\text{vol}}}{La^2} = -\frac{1}{2a}\left[m_1^x - m_3^x + m_2^y - m_4^y\right]$$

$$\tilde{\rho}_0^{\text{sur}} = \frac{q_M^{\text{sur}}}{La^2} = \sum_{\text{cell edges}} \frac{1}{2a}\hat{m}_0 \cdot \hat{n}_{\text{edge}}$$



Integrals are evaluated using fast fourier transforms.

Use zero padding to avoid the wrap-around problem.

The solution for demagnetization field is that for a disk isolated from others.

How to minimize the energy for a vortex in a desired location? → Use Lagrange undetermined multipliers technique.

Energy functional:

$$\Lambda[\vec{m}_i] = E[\vec{m}_i] + \sum_i \alpha_i(\vec{m}_i^2 - m^2) - \vec{\lambda} \cdot \sum_n^{\text{core}} \vec{m}_n$$

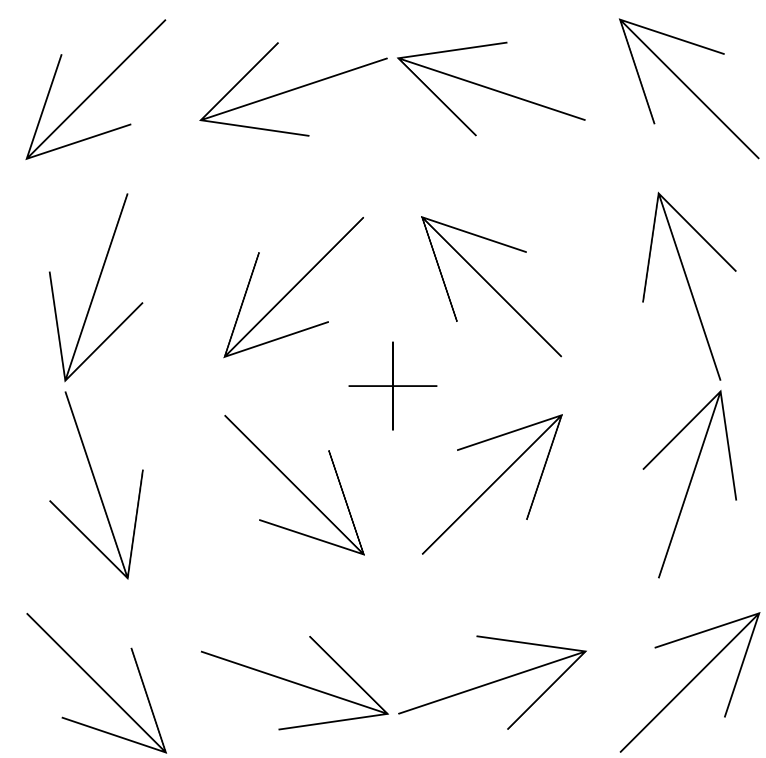
hamiltonian — length constraints — vortex position constraint

$$\frac{\partial \Lambda}{\partial m_n^x} = \frac{\partial E}{\partial m_n^x} + 2\alpha_n m_n^x - \lambda_x = 0$$

$$-F_n^x + 2\alpha_n m_n^x - \lambda_x = 0$$

in core:

$$m_n^x = \frac{1}{2\alpha_n}(F_n^x + \lambda_x)$$



But need to get α and λ by applying the constraints.

Iterations . . .

$$\vec{m}_n^2 = \frac{1}{4\alpha_n^2}\left[(F_n^x+\lambda_x)^2+(F_n^y+\lambda_y)^2+(F_n^z)^2\right] = m^2$$

(length constraints)

A.
$$\frac{1}{\alpha_n} = \frac{2m}{\sqrt{(F_n^x+\lambda_x)^2+(F_n^y+\lambda_y)^2+(F_n^z)^2}}$$

B.
$$\sum_{\text{core}} m_n^x = \sum_{\text{core}} \frac{1}{2\alpha_n}(F_n^x+\lambda_x) = 0 \longrightarrow \lambda_x = -\frac{\sum_{\text{core}} F_n^x/\alpha_n}{\sum_{\text{core}} 1/\alpha_n}$$

(vortex position constraint)

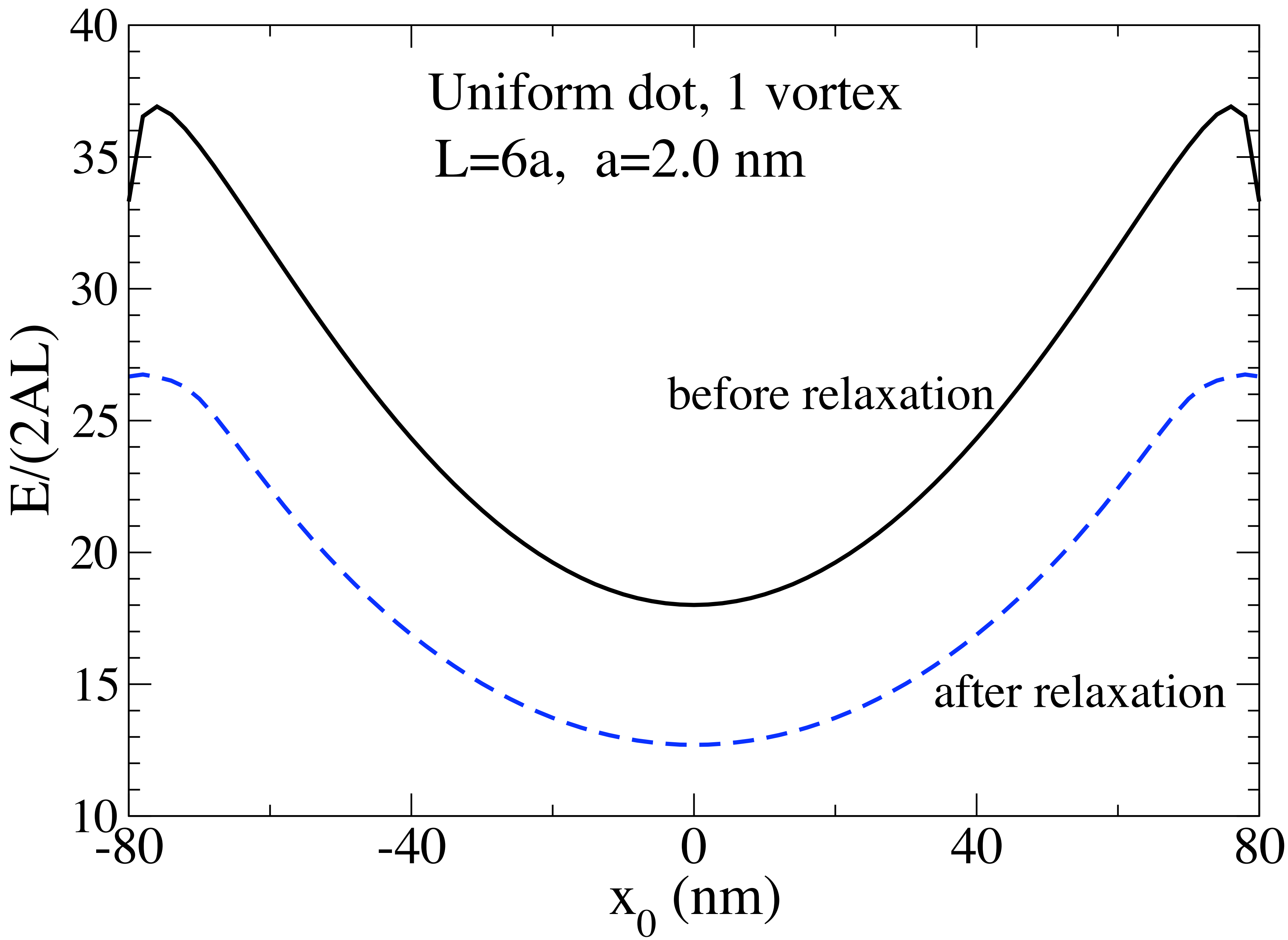
Iterate, placing each dipole along its effective field:

C.
$$\vec{m}_n = m\frac{(F_n^x+\lambda_x)\hat{x}+(F_n^y+\lambda_y)\hat{y}+F_n^z\hat{z}}{\sqrt{(F_n^x+\lambda_x)^2+(F_n^y+\lambda_y)^2+(F_n^z)^2}}$$

(not using Landau-Lifshitz dynamic equations)

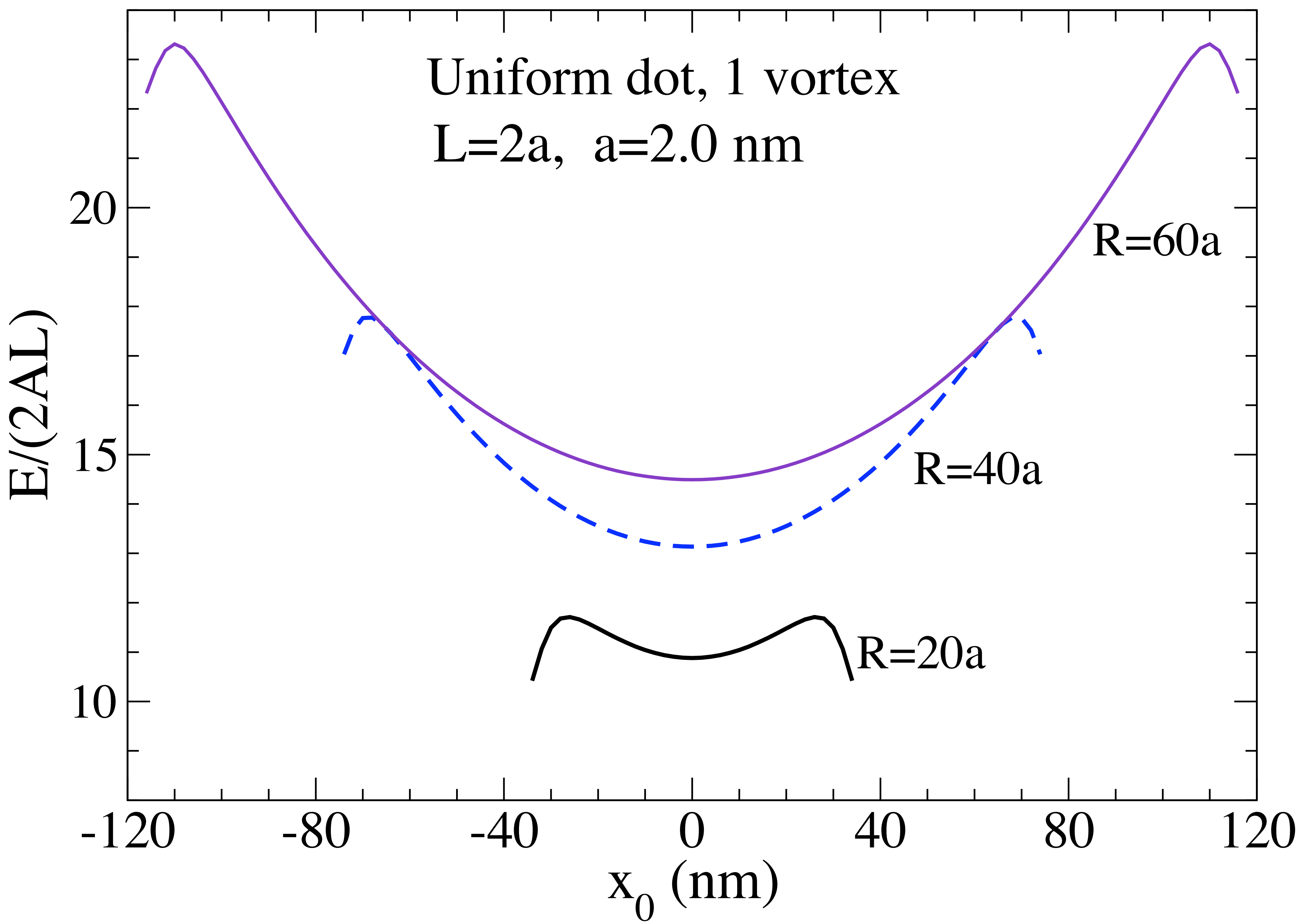
Example A. Vortex-in-dot total energy

a=2.0 nm, λ_{ex}=5.3 nm, L=12 nm, R=80 nm



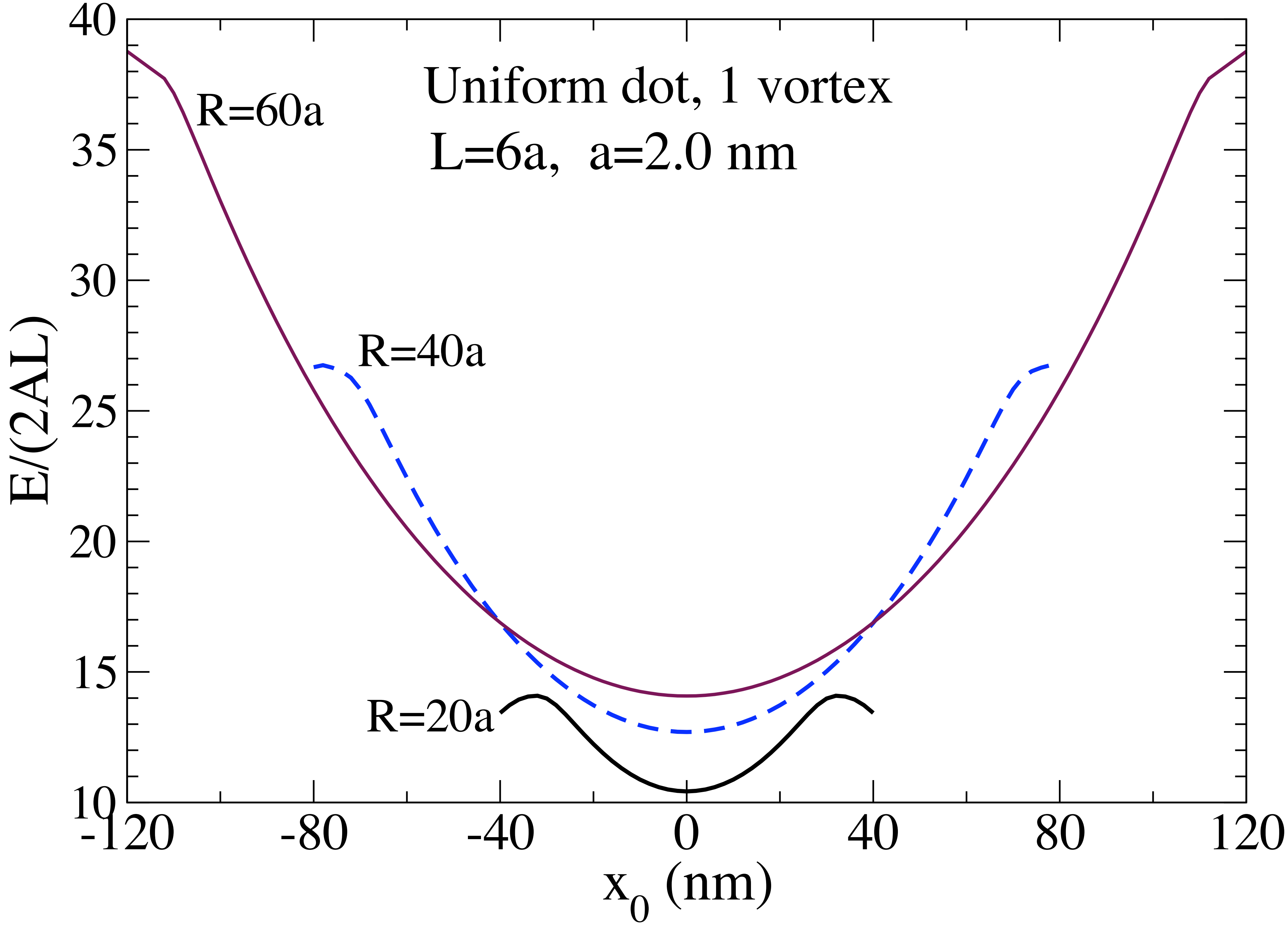
Example A. Vortex-in-dot total energy

a=2.0 nm, λ_{ex}=5.3 nm, L=4.0 nm, R=40, 80, 120 nm



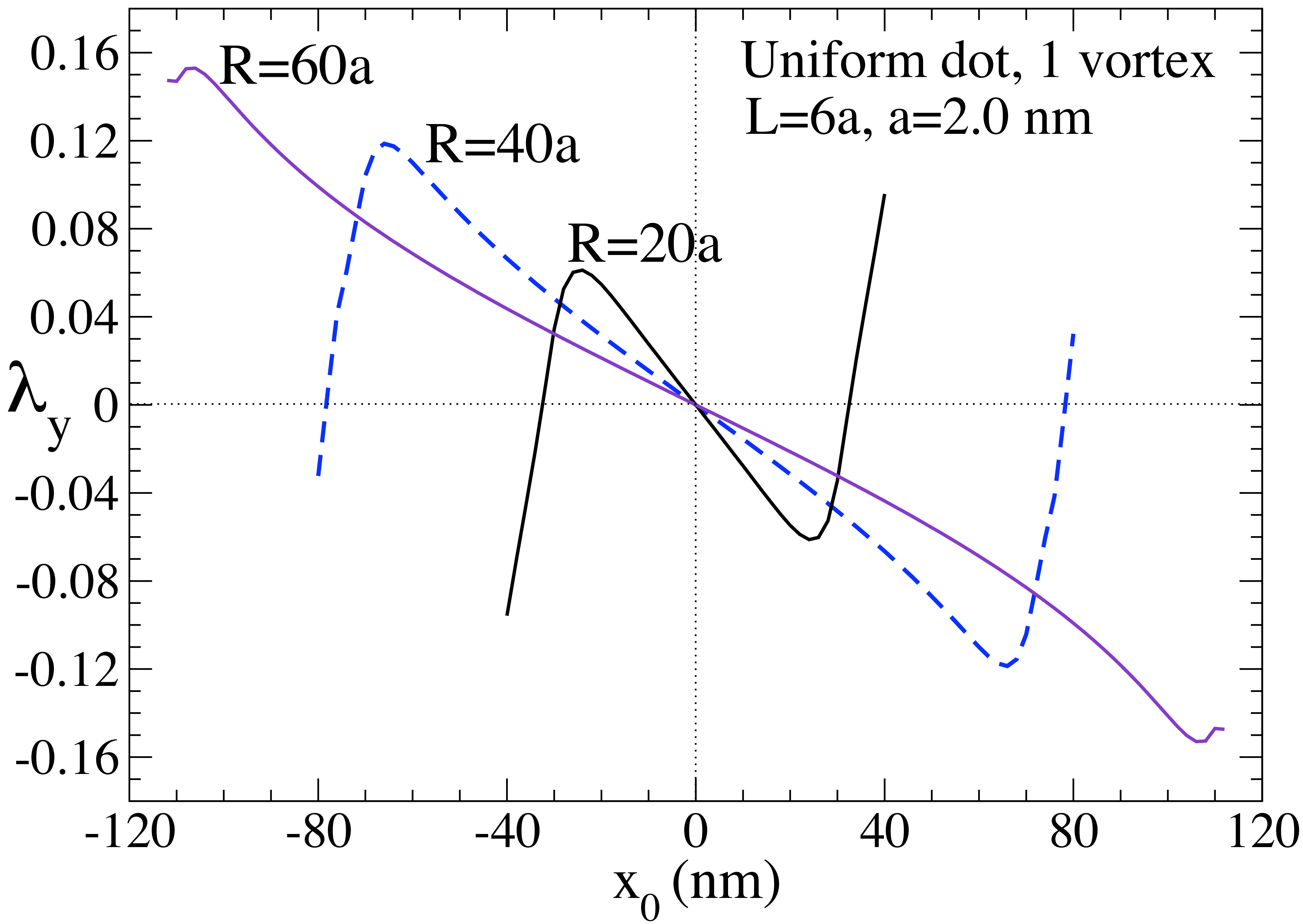
Example. Vortex-in-dot total energy

a=2.0 nm, λ_{ex}=5.3 nm, L=12 nm, R=40, 80, 120 nm



Example A. Vortex-in-dot constraint field, $\lambda=(0,\lambda_y)$

$a=2.0$ nm, $\lambda_{ex}=5.3$ nm, $L=12$ nm, $R=40$, 80, 120 nm



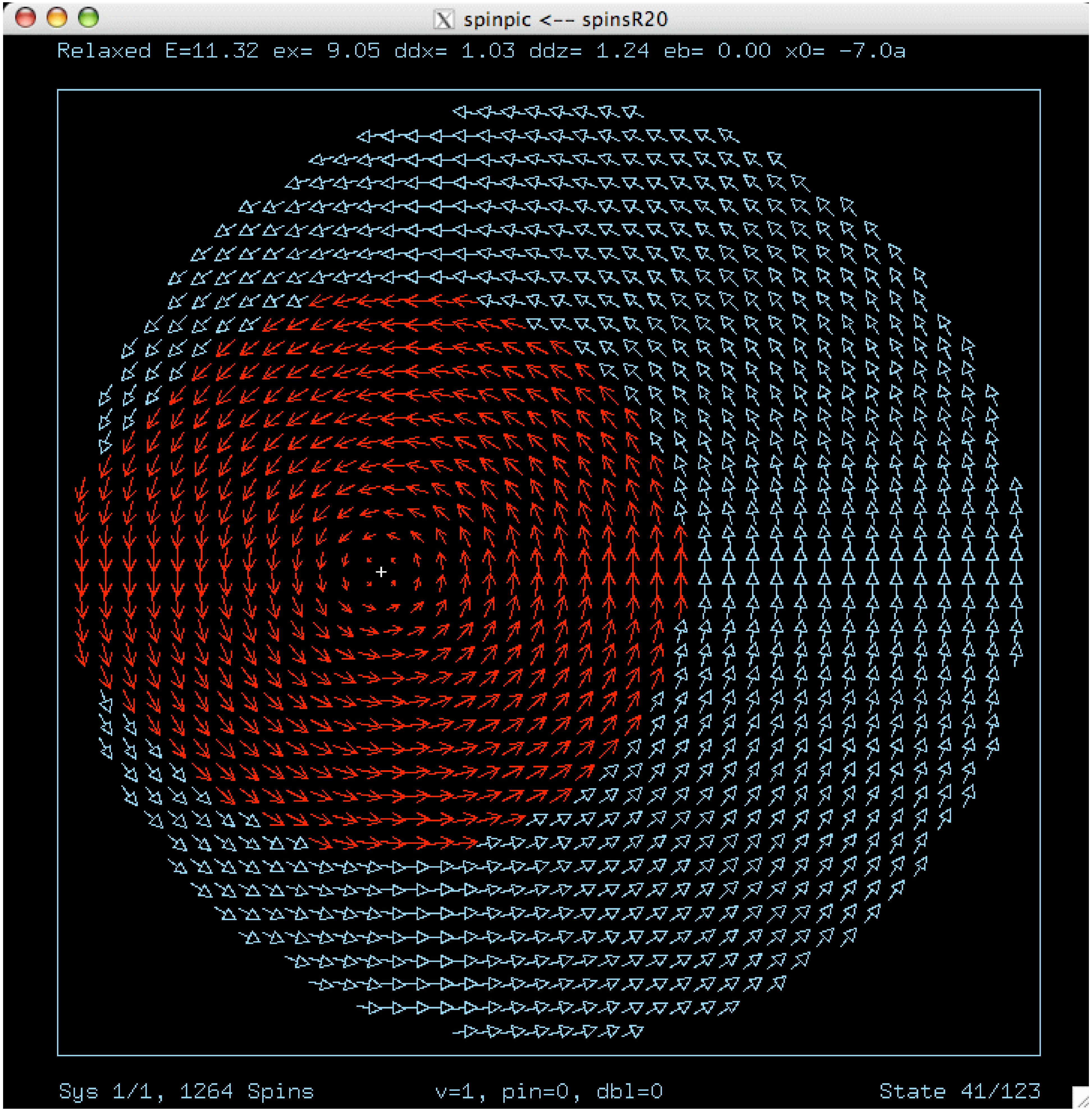
Example A. Vortex-in-dot typical relaxed configuration

a=2.0 nm,

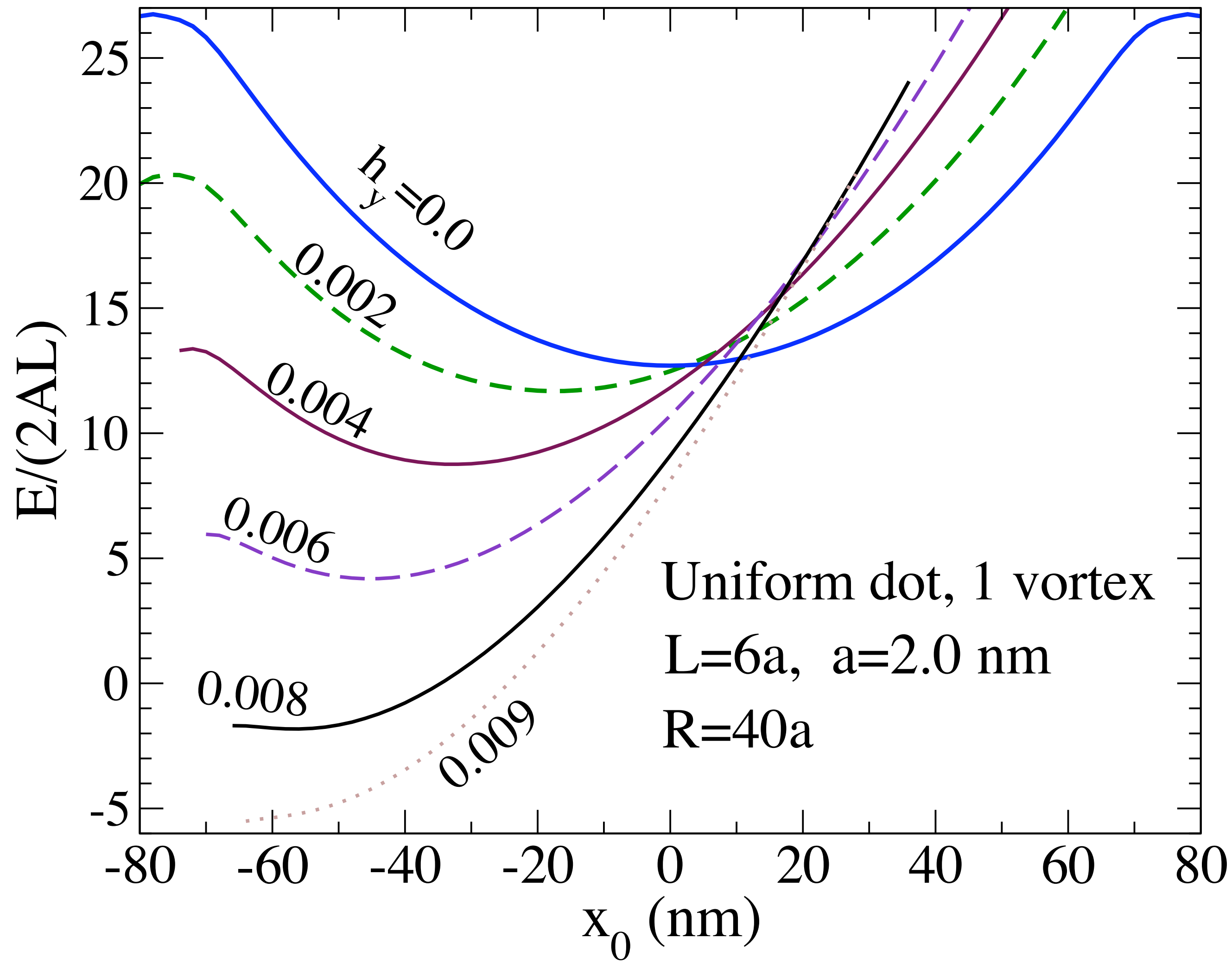
λ_{ex}=5.3 nm,

L=12 nm,

R=40 nm



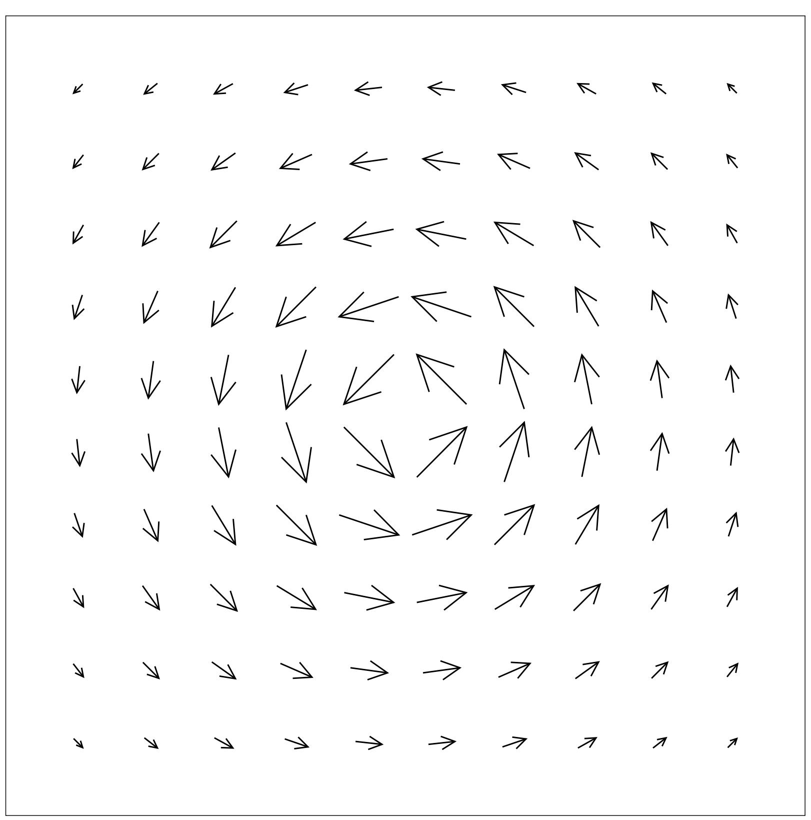
Example B. Effect of an applied magnetic field, Hy

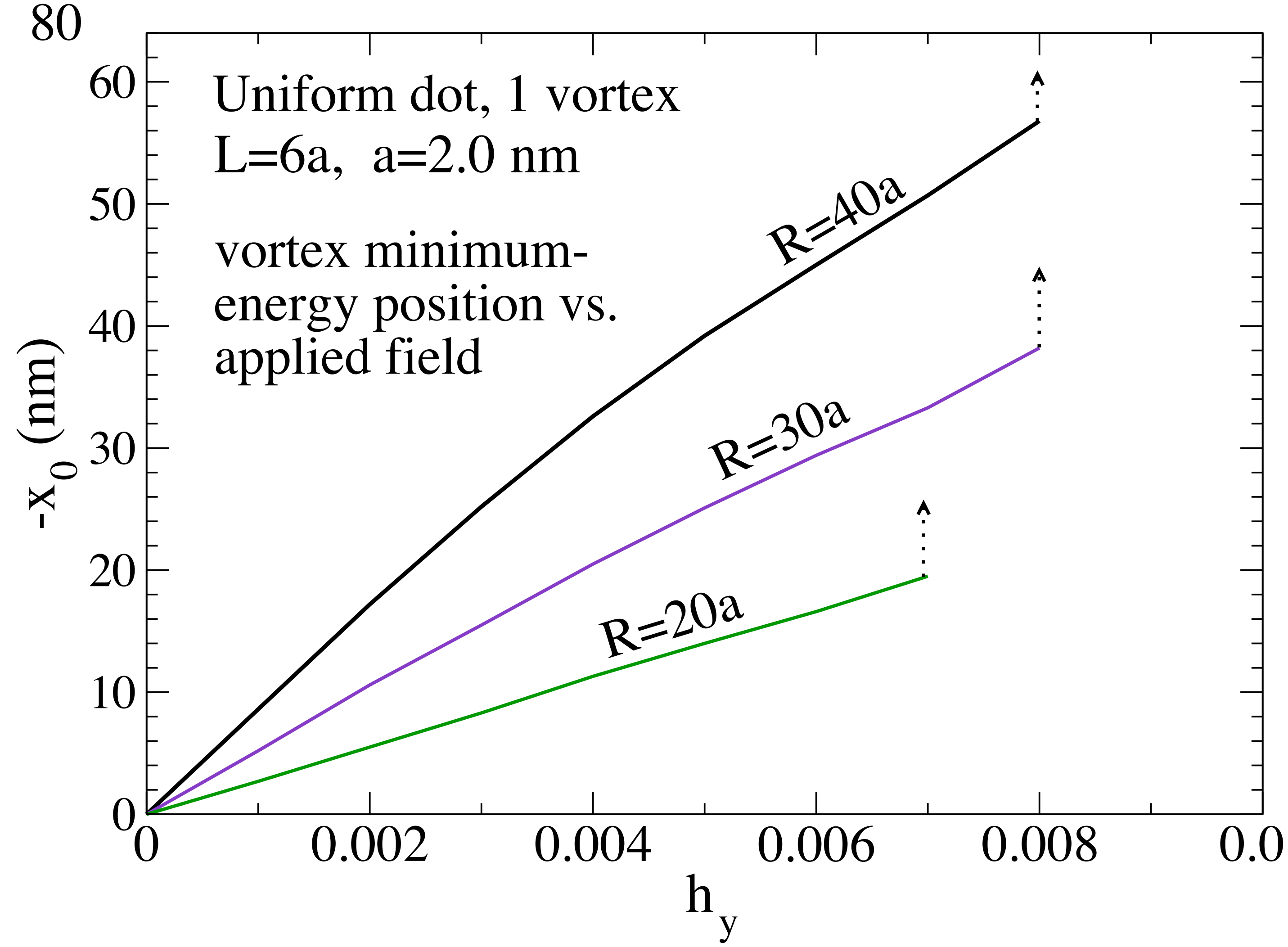


$$\vec{h}_{\rm ext} = \frac{a^2}{\lambda_{\rm ex}^2}\tilde{H}_{\rm ext} = \frac{a^2}{\lambda_{\rm ex}^2}\frac{\vec{H}_{\rm ext}}{M_S}.$$

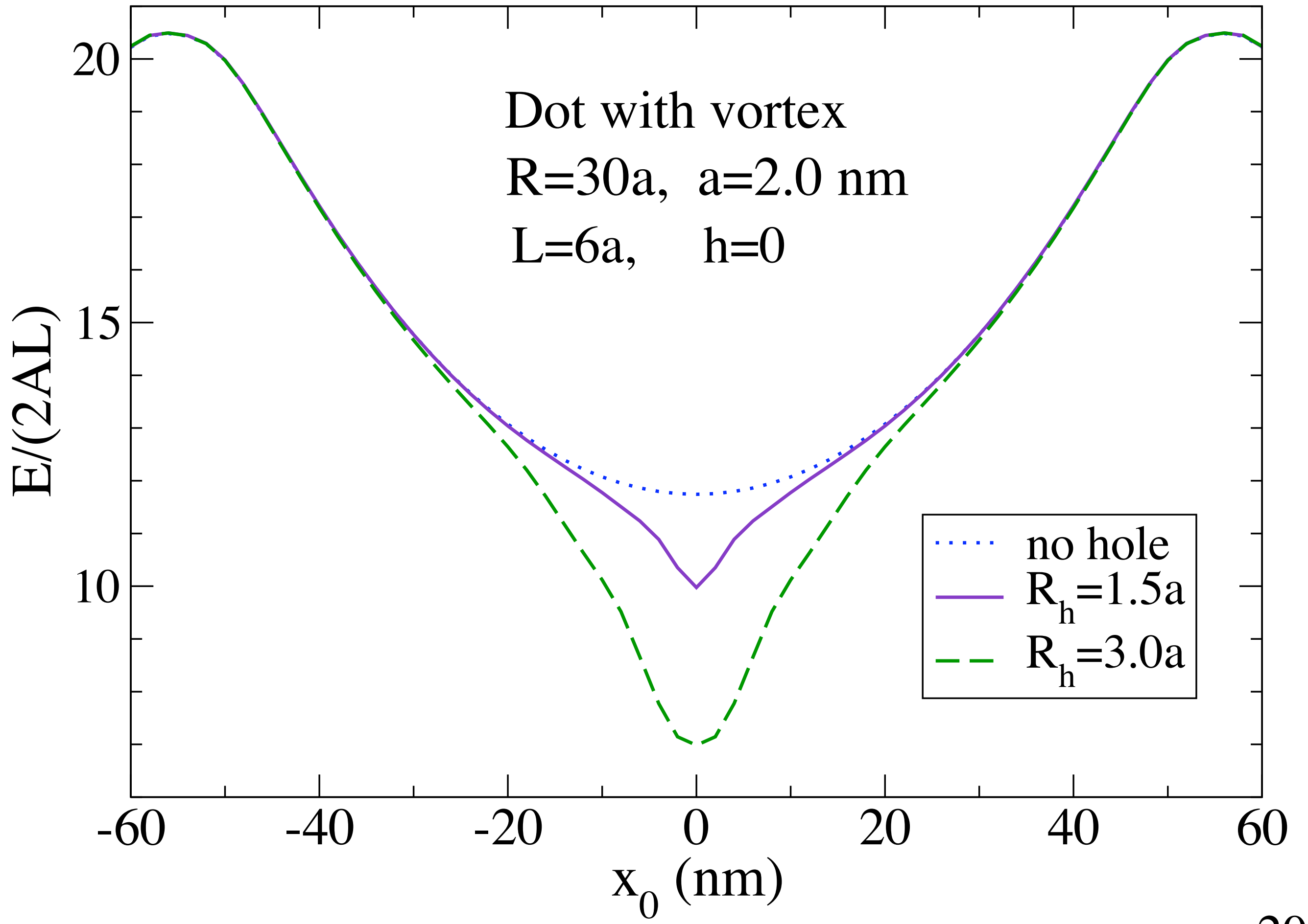
equilibrium position ➘

$$x_0 = -\beta C\frac{a^2}{\lambda_{\rm ex}^2}\frac{H_y}{M_S}$$

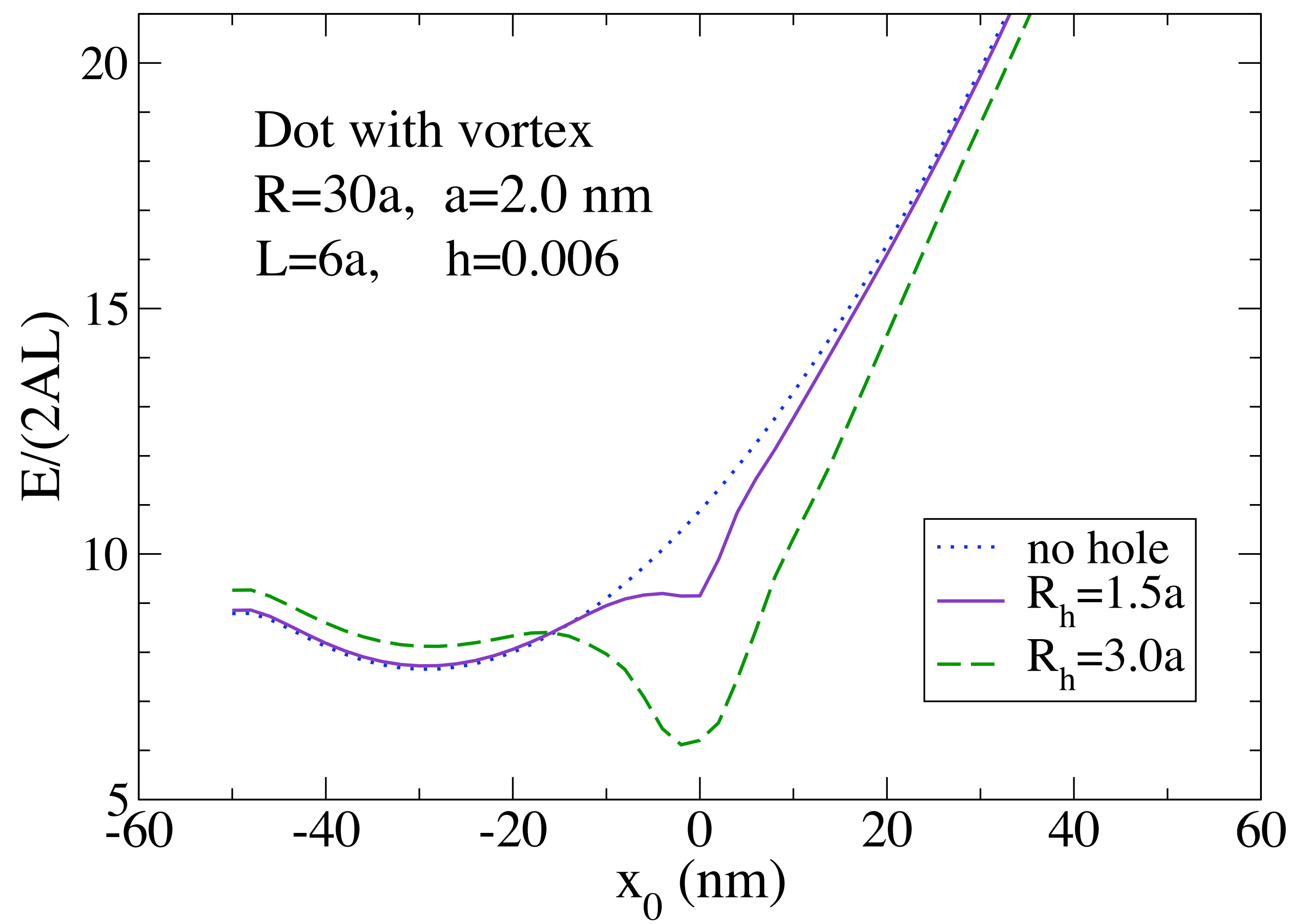




Example C. Effect of a hole together with an applied magnetic field, Hy



Bistable potentials ⇒
(with applied field)



Example C. Dot with 12 nm diameter hole-defect.

Vortex-in-dot with applied h.

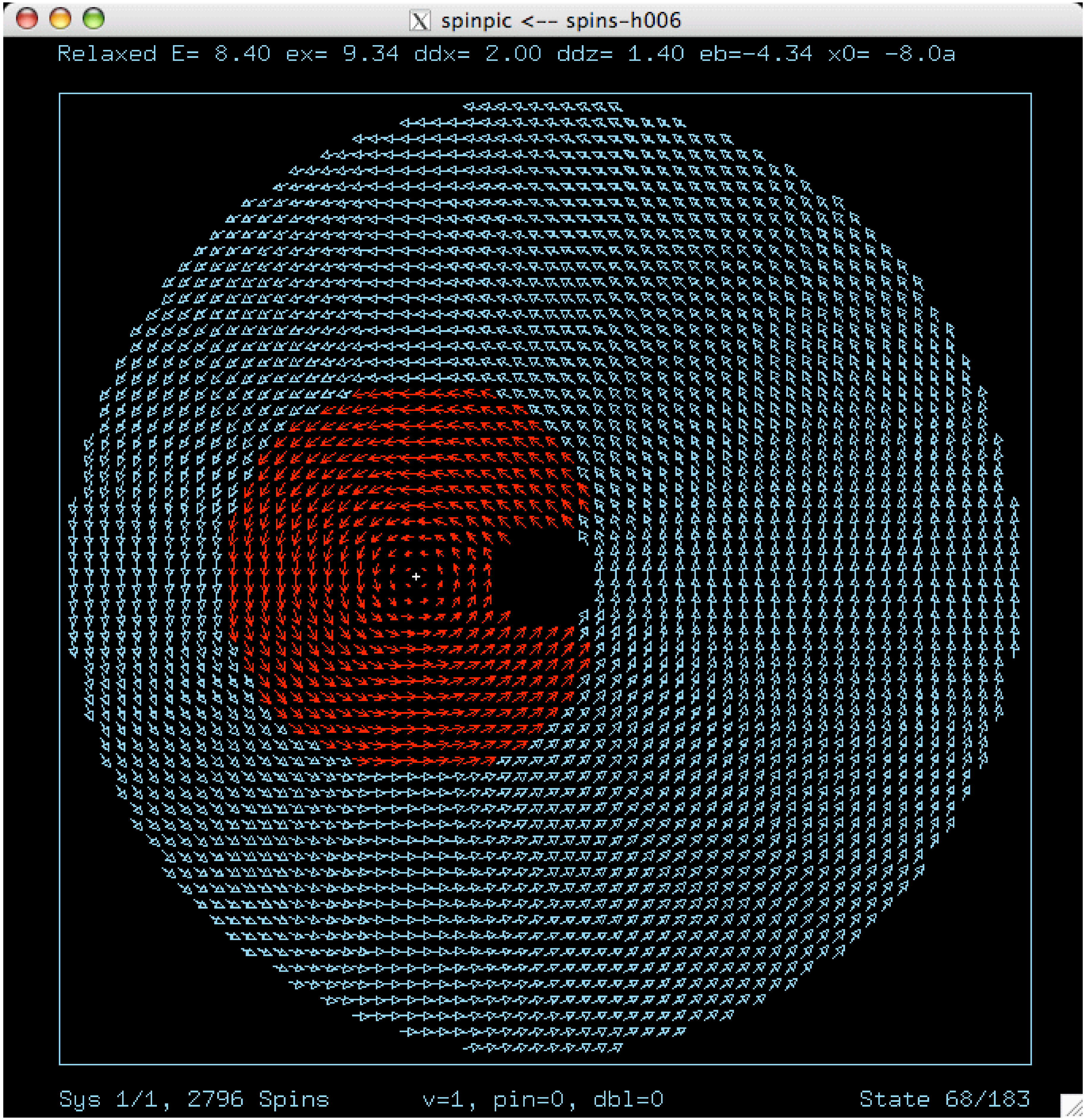
a=2.0 nm,

λ_{ex}=5.3 nm,

L=12 nm,

R=60 nm

(arrows proportional to m_x, m_y)

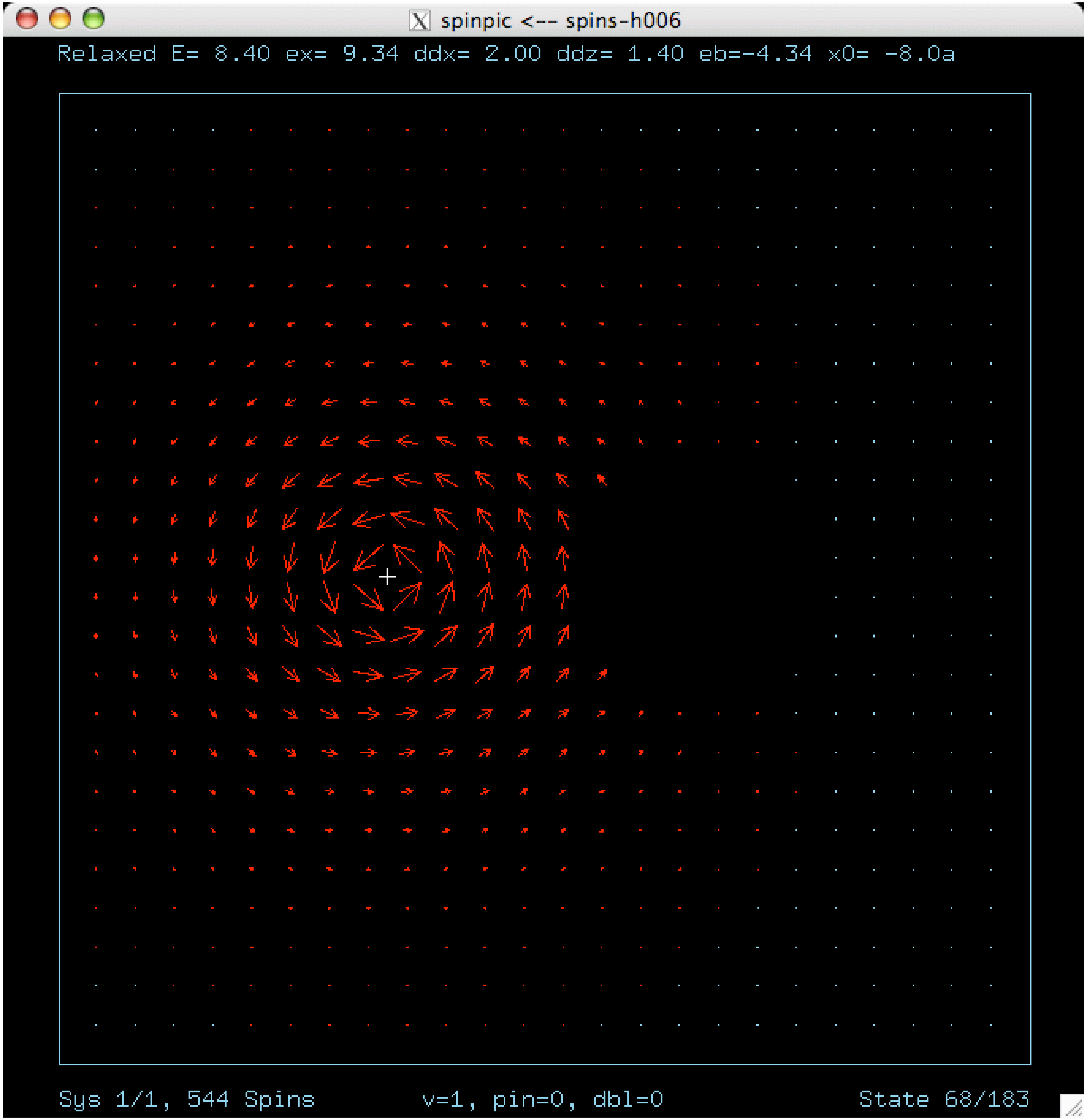


Example C. Dot with 12 nm diameter hole-defect.

Vortex-in-dot with applied h.

a=2.0 nm,
λ_{ex}=5.3 nm,
L=12 nm,
R=60 nm

(arrows proportional to m_z)



Summary

- Used a modified micromagnetics description, demagnetization field found via FFT evaluation with Green's functions for a thin disk.

- A constraining (magnetic) field (λ_x,λ_y) in the vortex core was found using Lagrange's undetermined multipliers.

- Can find the effective potential for vortex motion within a dot, which could be useful for analysis of vortex dynamics.

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