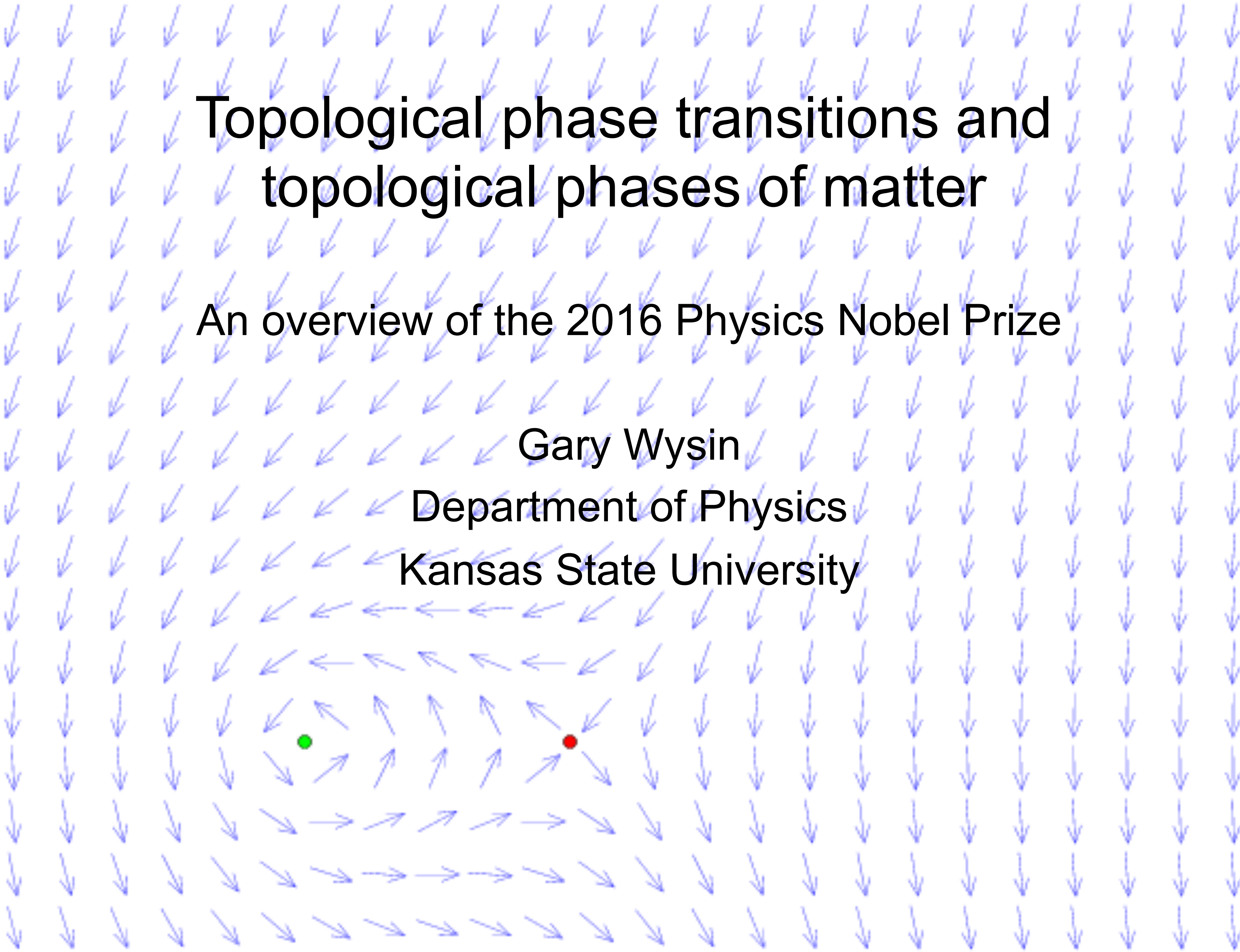


Topological phase transitions and topological phases of matter

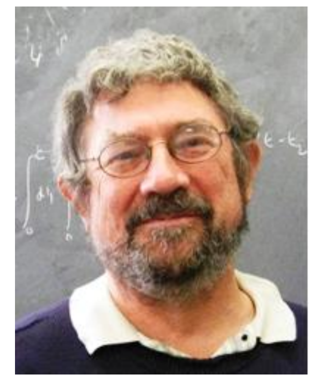
An overview of the 2016 Physics Nobel Prize

Gary Wysin
Department of Physics
Kansas State University

The $1.1 million prize awarded October 4, 2016

"For theoretical discoveries of topological phase transitions and topological phases of matter".

one half to:
David J. Thouless, 82, Univ. of Washington, Seattle
Born: 1934, Bearsden, United Kingdom



the other half to:
J. Michael Kosterlitz, 73, Brown University, Providence
Born: 1942, Aberdeen, United Kingdom

and

F. Duncan M. Haldane, 65, Princeton University
Born: 1951, London, United Kingdom

The prize goes for theories of condensed matter – materials composed from large numbers of atoms or molecules, whose organized behavior leads to new properties that change in steps.

The headline in Wired Magazine.

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NOBEL PRIZE IN PHYSICS GOES TO ANOTHER WEIRD THING NOBODY UNDERSTANDS

John Michael Kosterlitz, one of the three Nobel Prize recipients in Physics, at Aalto University on October 4, 2016.

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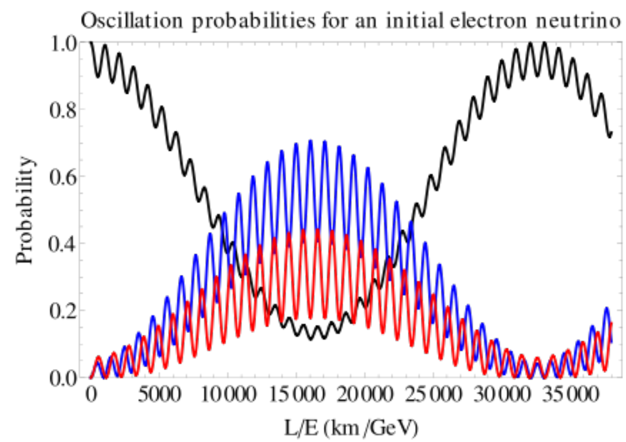
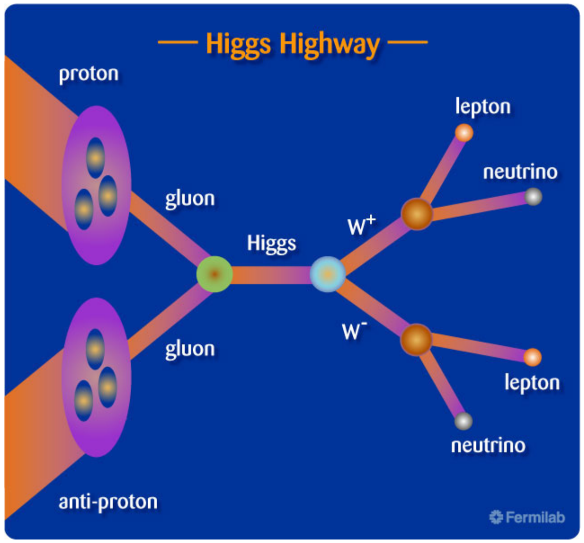
The most recent previous physics Nobel prizes:



2015 - for the discovery of neutrino oscillations, which shows that neutrinos have mass, to Kajita and McDonald.

2014 - for the invention of efficient blue LEDs which has enabled bright and energy-saving white light sources, to Akasaki, Amano and Nakamura.



2013 - for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider, to Englert and Higgs.

Other notable Nobel prizes in physics related to the 2016 prize:

2003 - for pioneering contributions to the theory of superconductors and superfluids, to Abrikosov, Ginzburg and Leggett.

(coherent macroscopic quantum states)

2001 - for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates, to Cornell, Ketterle and Wieman.

(phase transitions)

1985 - for the discovery of the quantized Hall effect, to Klaus von Klitzing.

(coherent macroscopic quantum states)

1962 - for his pioneering theories for condensed matter, especially liquid helium, to Lev D. Landau.

(phase transitions)

What is the importance of:

"theoretical discoveries of topological phase transitions and topological phases of matter" ??

- Understanding of how phase transitions take place in two dimensions. (applications to superconductors, superfluids and magnets in 2D)

- Many condensed materials have topological phases: states of matter where some important properties only change in integer steps, even when dirty or disorganized.

- Results will lead to new electronic materials (better, faster, smaller) with special topological properties.

Example = topological insulators, sheet materials that conduct strongly on their edges but are insulating in their interiors.

“theoretical discoveries of topological phase transitions and topological phases of matter” ??

“phases” – determined by how the atoms are correlated to their neighbors.

plasma = rapid separated ions + electrons

gas = nearly independent fast molecules

liquid = cohesion among nearby atoms or molecules and strong correlations

solid = atoms have small motions around well-defined locations (crystals)

quantum condensate = a large number of atoms or molecules very strongly correlated, maybe even in the same quantum state, with unusual properties.

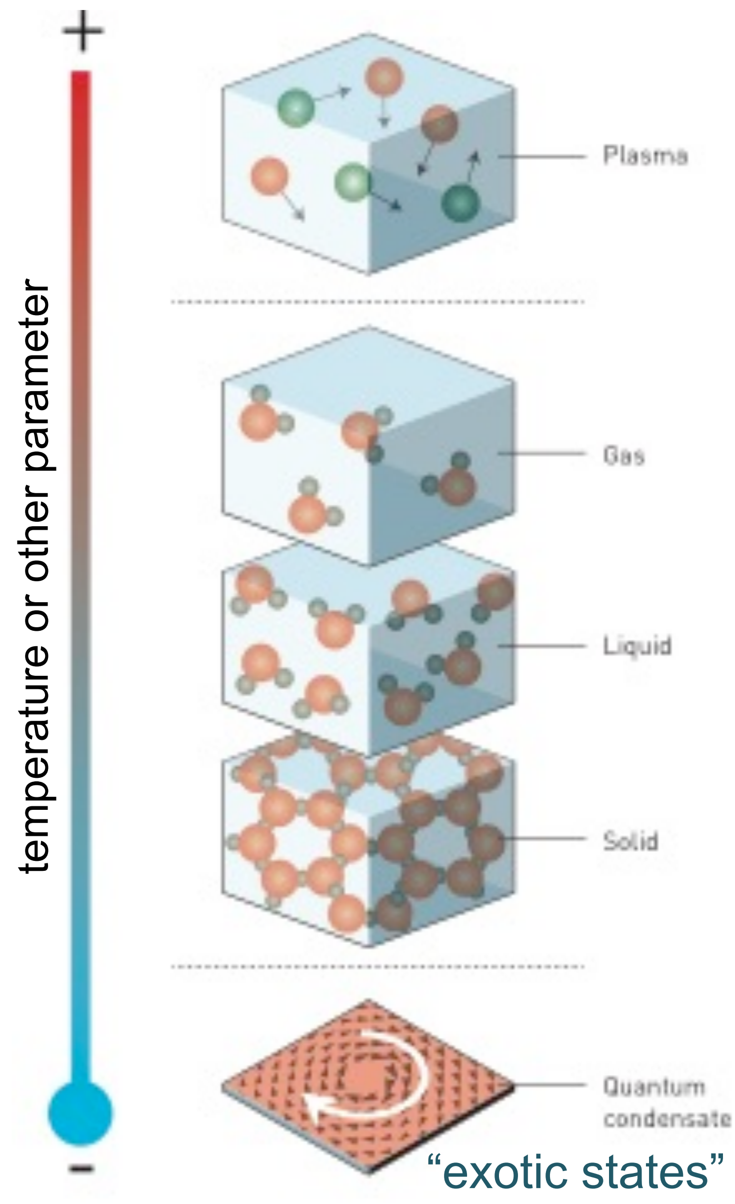


Fig. 1 Phases of matter. The most common phases are gas, liquid and solid matter. However, in extremely high or low temperatures matter assumes other, more exotic states.

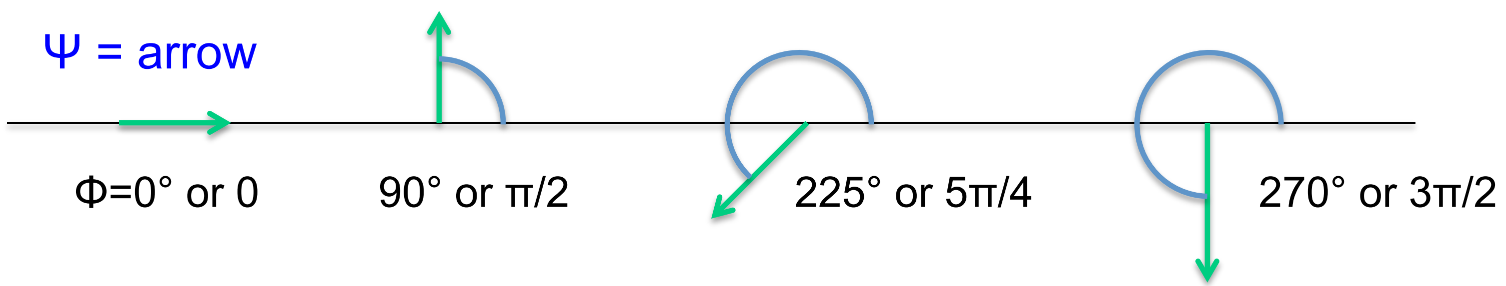
exotic states of matter

- very cold, not more than a few degrees above absolute zero (-273 C) so there is little disturbance due to thermal motion of atoms.

- “flatlands”. two or even one-dimensional systems, where the geometry limits atomic interactions and liberty.

- very large number of *indistinguishable* atoms, molecules, or electrons whose quantum waves overlap on top of each other.

- the quantum waves $\Psi = A \exp(i\Phi)$ have amplitude A and phase angle Φ. exotic behaviors are due mainly to variations of the phase angle.



Three problems where exotic states emerge

1. Two-dimensional quantum fluids. Phase transitions in two-dimensional superfluids, superconductors or magnets – are driven by “vortex unbinding”.

2. Quantum Hall effect -- in a flat conductor exposed to a magnetic field the conductivity changes in integer steps.

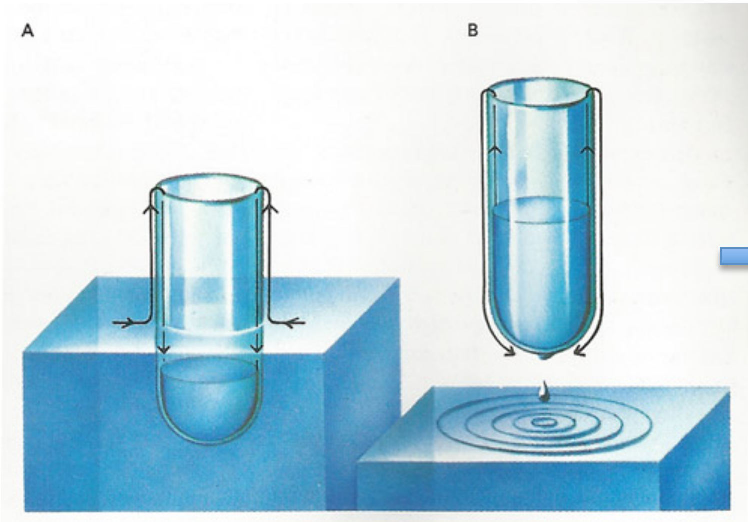
3. Chains of magnetic atoms – the lowest energy excitations are different depending on whether the quantum spins are half-integer (S=1/2, 3/2,…) or integer (S=1, 2,…). “Haldane phase”.

Problem 1. Two-dimensional quantum fluids.

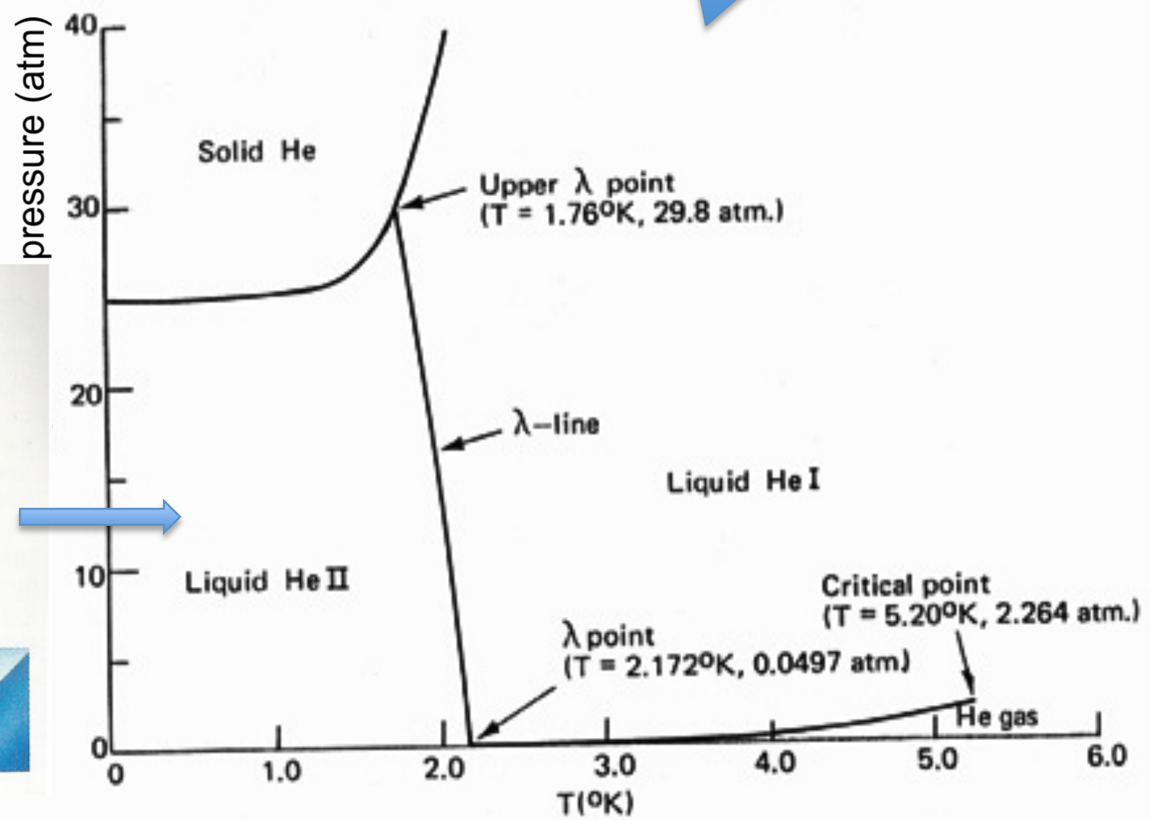
superfluid helium (^4He)
each atom has (2 protons + 2 neutrons) + 2 electrons

Helium walks into a bar. Bartender says, "We don't serve noble gasses here!" He doesn't react.

superfluidity at T < 2.17 K
viscosity = 0, super-flow!



liquid He-II

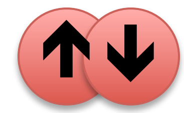


The phase diagram of He^4.

About elementary particles – two types.

Fermions. Half-integer quantum spin S=1/2, 3/2, etc., where 2S+1 counts the number of values its angular momentum may take around any axis.

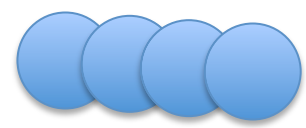
Fermions are prohibited from occupying identical quantum states. (known as the Pauli exclusion principle!)



Protons, neutrons, electrons are all fermions because they have S=1/2.

Bosons. Integer quantum spin S=0, 1, 2, etc., also with 2S+1 values of angular momentum around any axis.

Any number of bosons may occupy the identical quantum state!

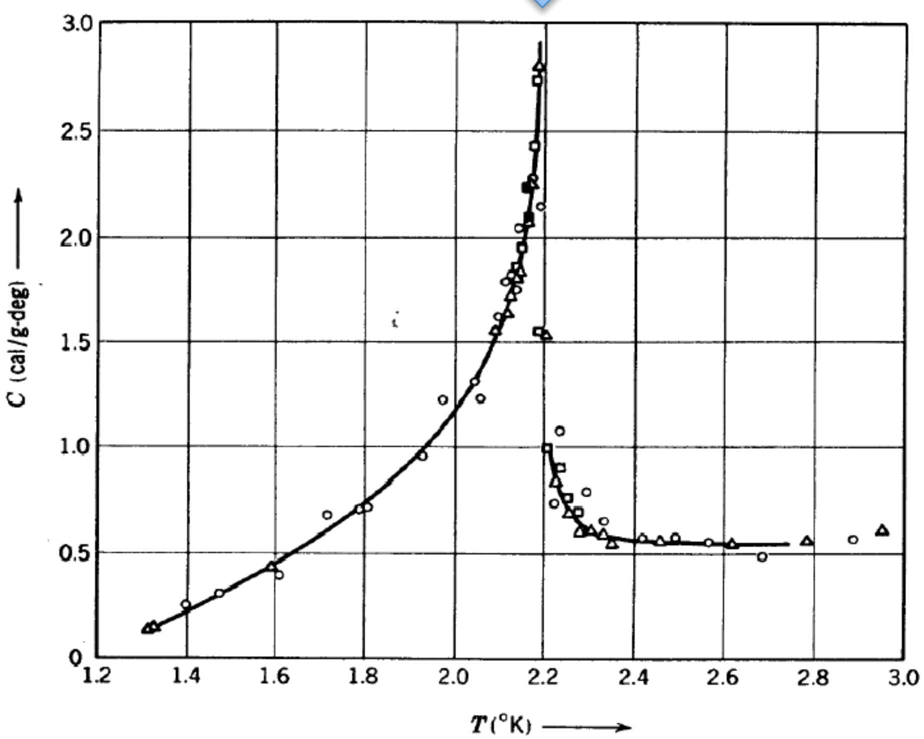


Photons (S=1), Higgs particle (S=0), and ^{4}He atoms (S=0) are bosons.

^{4}He atom = made from 6 fermions.
^{4}He atoms are <u>bosons</u>, they can occupy identical quantum states.

specific heat of helium vs. Temperature, at atmospheric pressure

lambda point T_λ=2.17 K (critical temperature)



superfluid ← | → normal fluid

This is a phase transition.

The helium is 3-dimensional (3D), unless it flows on surfaces.

In 3D, this phase transition is a form of Bose-Einstein condensation for $T < T_\lambda$, where **many** of the He atoms fall down into the same lowest possible quantum state for an atom. They can do that because they are bosons.

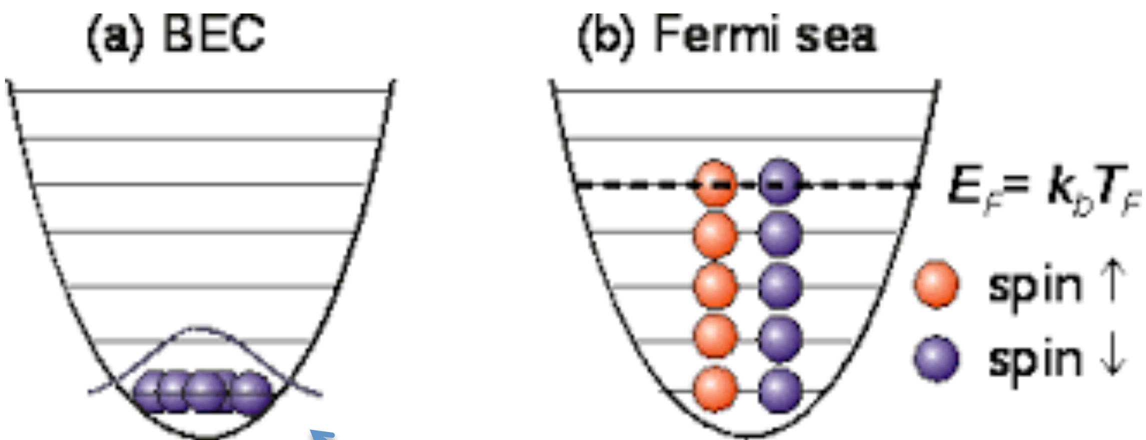
The superfluid phase is a *macroscopic quantum state*.

There is a "quantum order parameter" (like a wave function) for a macroscopic number of atoms in the condensate!

Ψ = A exp(iΦ) = amplitude $\times$ phase factor

phase angle

bosons vs S=1/2 fermions



energy

Bose-Einstein condensate,
described by order parameter:

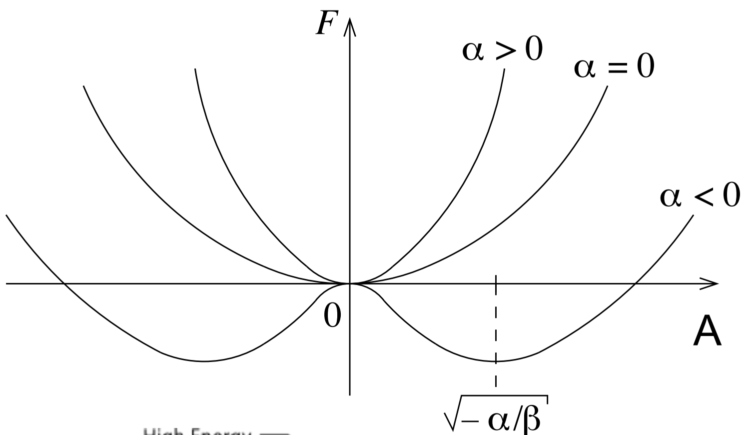
Ψ = A exp(iΦ) = amplitude $\times$ phase factor

Pauli exclusion,
fermions avoid each other

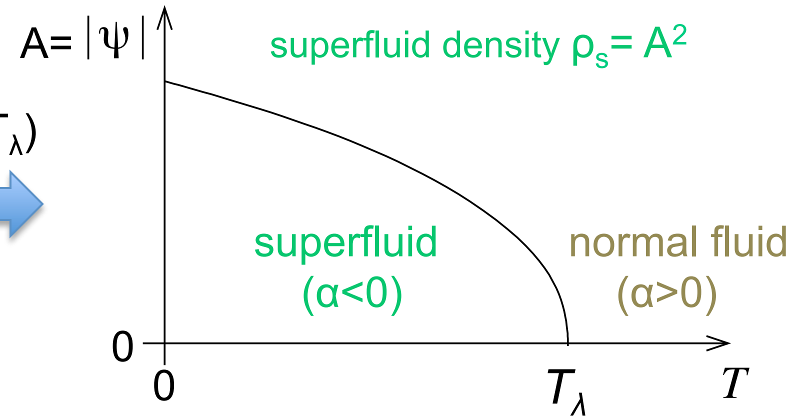
Quantum fluid -- condensate order parameter = $\Psi = A\exp(i\Phi)$ = amplitude x phase factor

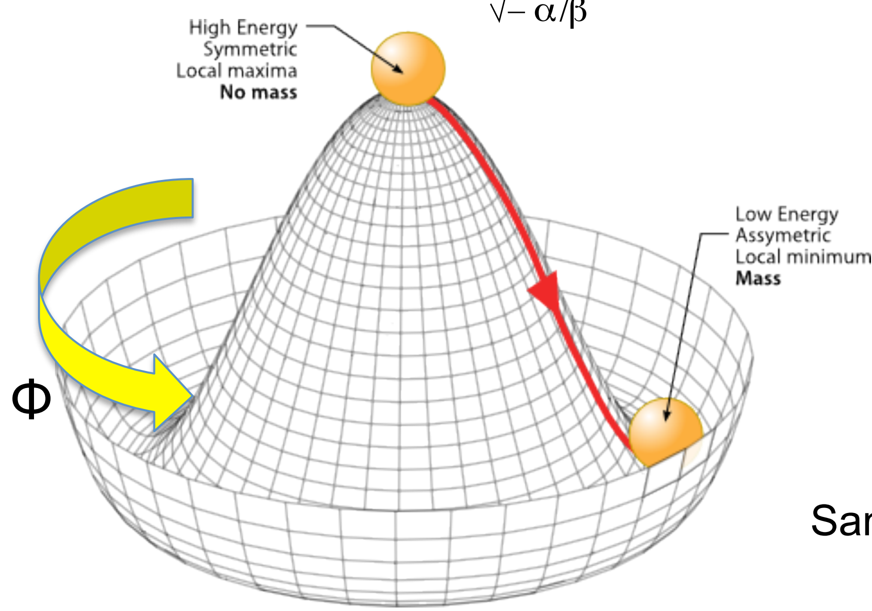
In Landau's theory for phase transitions (Nobel prize 1962!).

a free energy $F = \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4 = \alpha A^2 + \frac{1}{2}\beta A^4$ = (superfluid phase potential energy)



$\alpha = a(T - T_\lambda)$





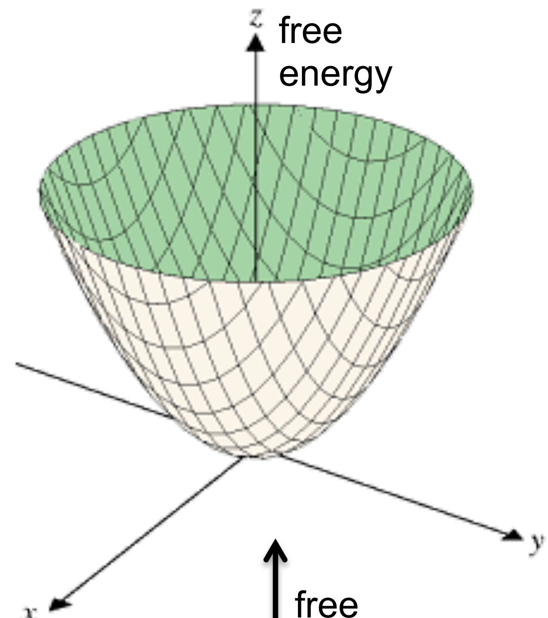
$T < T_\lambda$

Spontaneous symmetry breaking!

The phase angle Φ (0—360°) is arbitrary! It can choose any value, all at the SAME ENERGY.

Same as Higgs mechanism (Nobel prize 2013!)

What makes the condensate become a super fluid!?



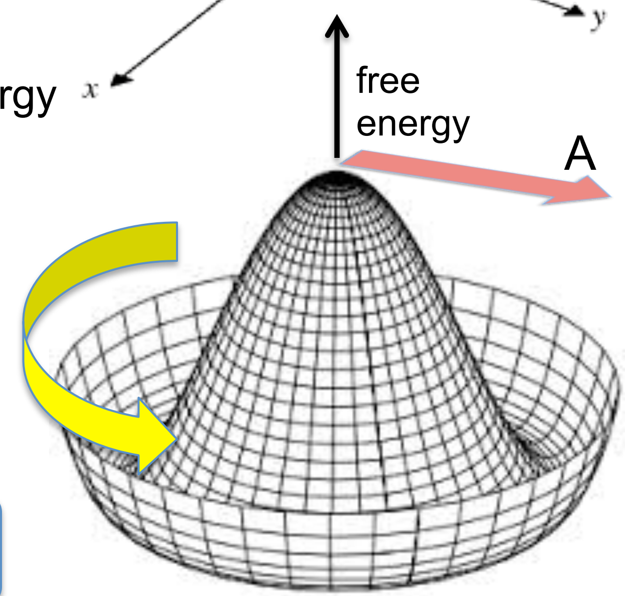
$T>T_\lambda$ Any change in ψ costs energy.

There are two “gapped or massive modes”.

Needs energy input to move this *normal fluid*.

$T<T_\lambda$ Changing A (related to superfluid density) costs energy (a gapped or massive mode).

Changing Φ costs very little energy!
(an un-gapped or massless mode).



Changing Φ slowly over a distance corresponds to the superfluid velocity!

add KE=½ mv_s^2

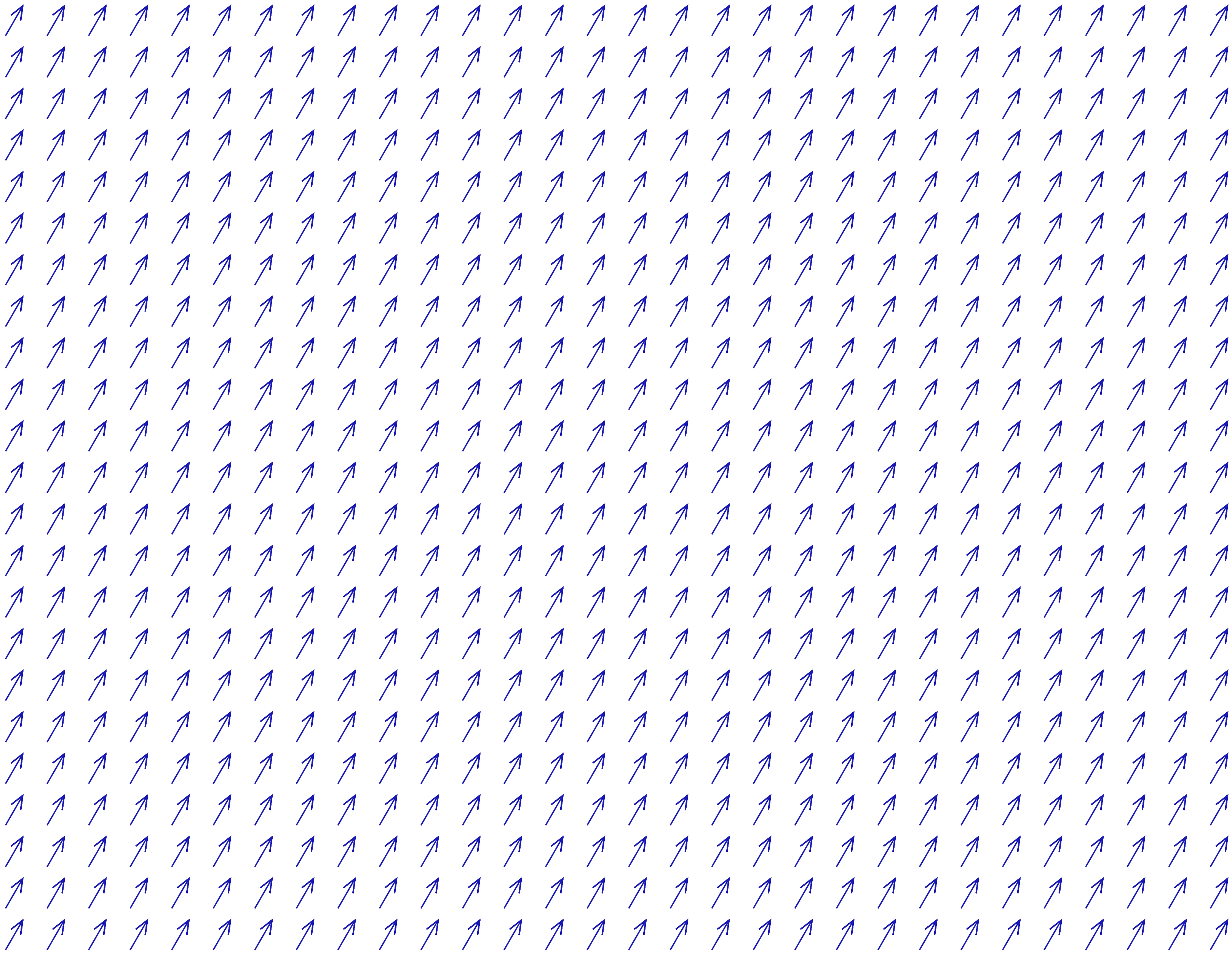
$$v_s = (\hbar/m)\ \text{gradient}(\Phi)$$

$\hbar$ = Planck’s constant/2π, m = mass of He-atom

What about flatlands?? (2D quantum fluids)
Arrows show Ψ (amplitude A and phase angle Φ) at points in the fluid. ALL Φ=0.

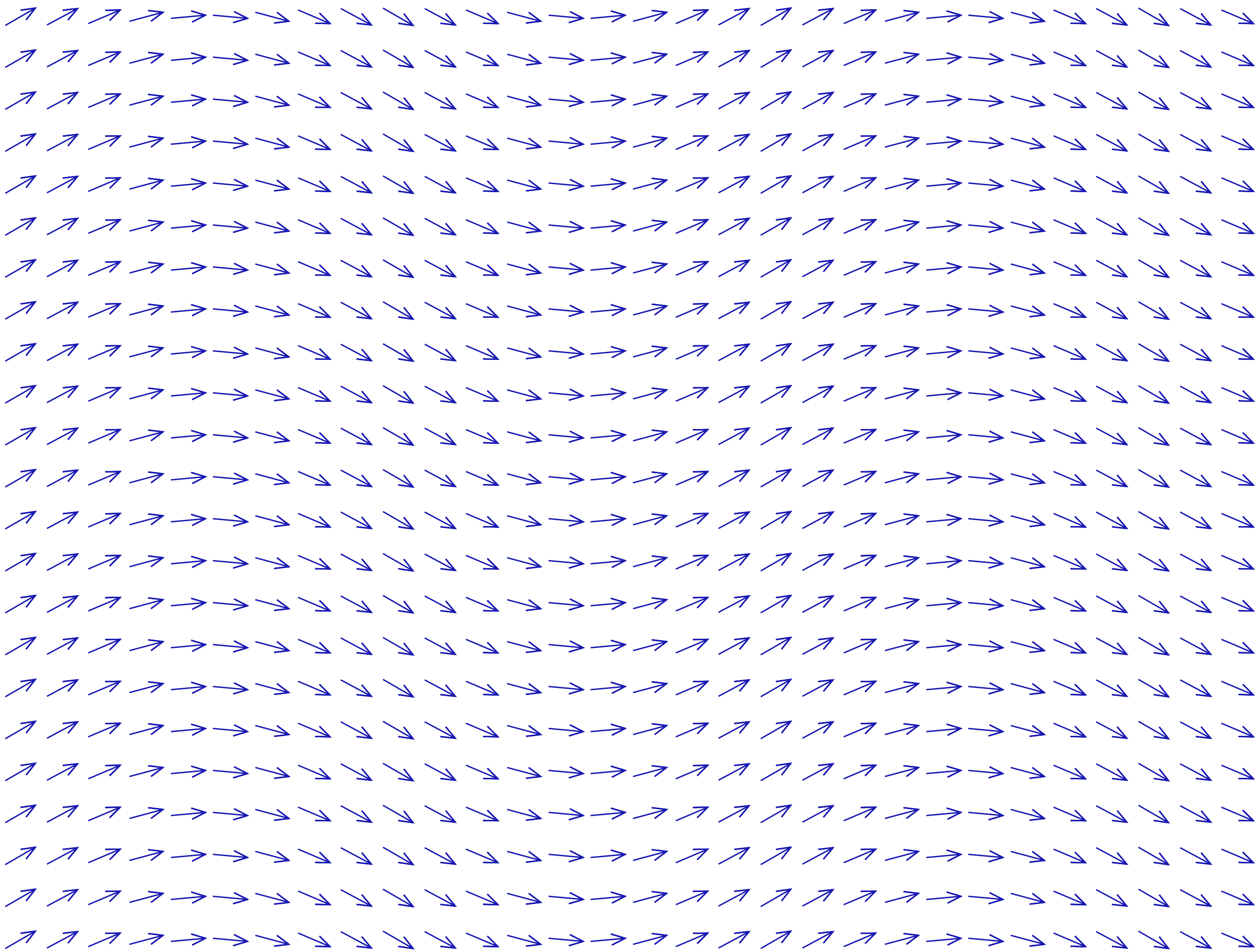
What about flatlands?? (2D quantum fluids)

Arrows show Ψ (amplitude A and phase angle Φ) at points in the fluid. ALL Φ=60°.



What about flatlands?? (2D quantum fluids)

Arrows show Ψ (amplitude A and phase angle Φ) at points in the fluid. A wave in Φ.



(2D quantum fluids in a thermal bath)

OLD THEORY before V. Berezinskii & Kosterlitz & Thouless:

NO PHASE TRANSITION in a 1D or 2D system.

Thermal fluctuations in Φ (random small waves) are large, in A are small.

The Φ fluctuations would destroy the superfluid state at any temperature!

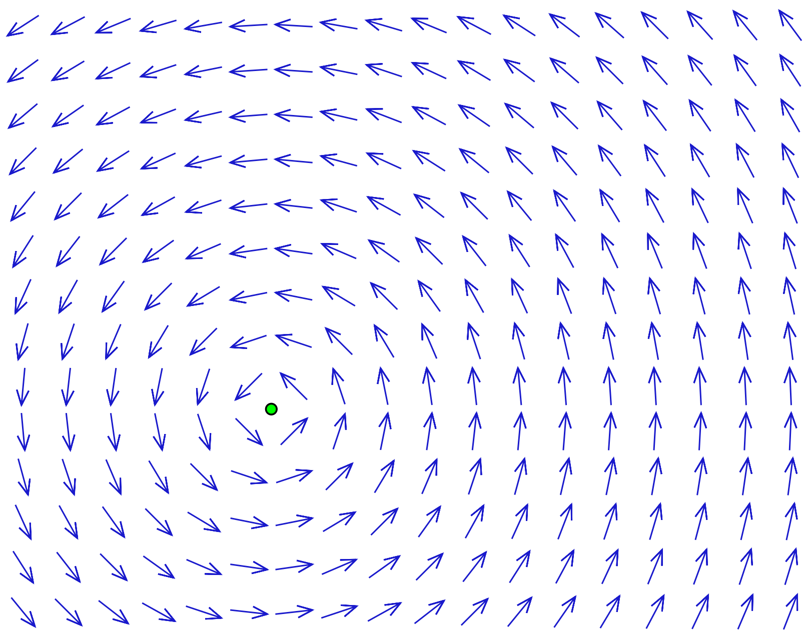
System would always be "normal" with A=0 at all temperatures.

Also, correlation function from small fluctuations has wrong form for high temperature:

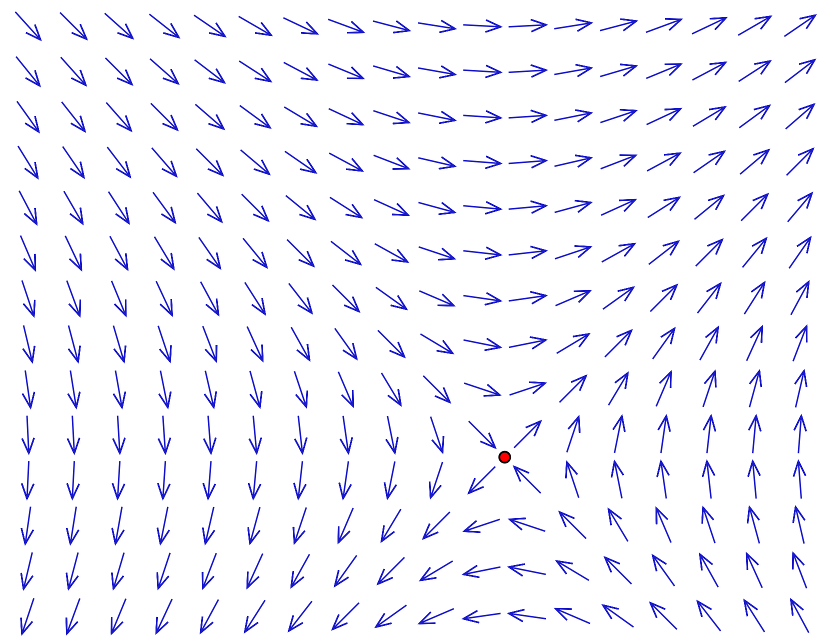
$$\left\langle \psi^*(0)\psi(r) \right\rangle \cong A^2 \left(\frac{r}{r_0} \right)^{-\eta}, \qquad \eta = -k_B T \frac{\beta}{\alpha} \frac{2\pi m}{h^2}$$ (should be exponential at high T).

(2D quantum fluids in a thermal bath)

New BKT theory: The superfluid transition CAN take place in 2D films because there are ALSO TOPOLOGICAL EXCITATIONS that cost large energy BUT once formed, are extremely stable.



VORTEX, q=+1 (winding #)



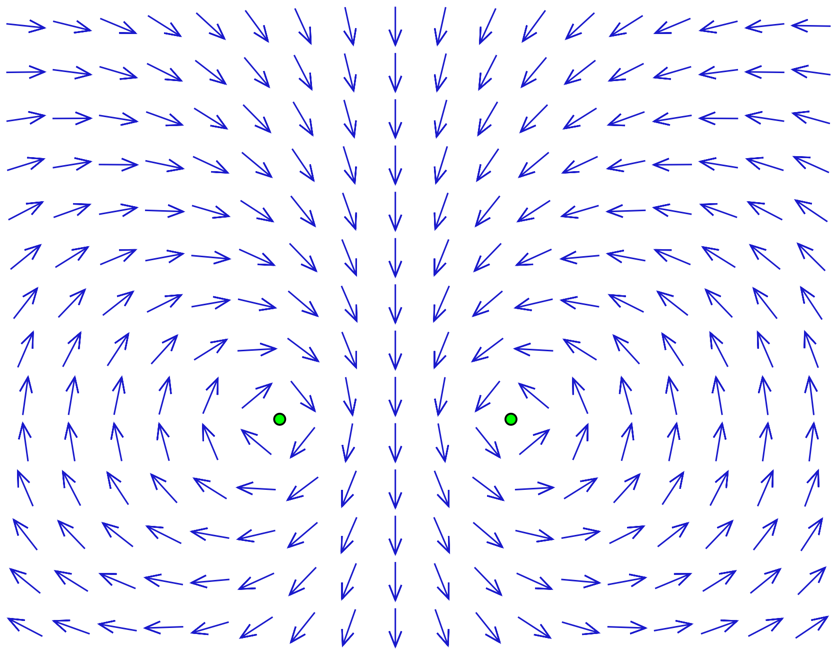
ANTI-VORTEX, q=-1 (winding #)

For low temperature ($T<T_\lambda$), a superfluid state with A > 0 is possible.

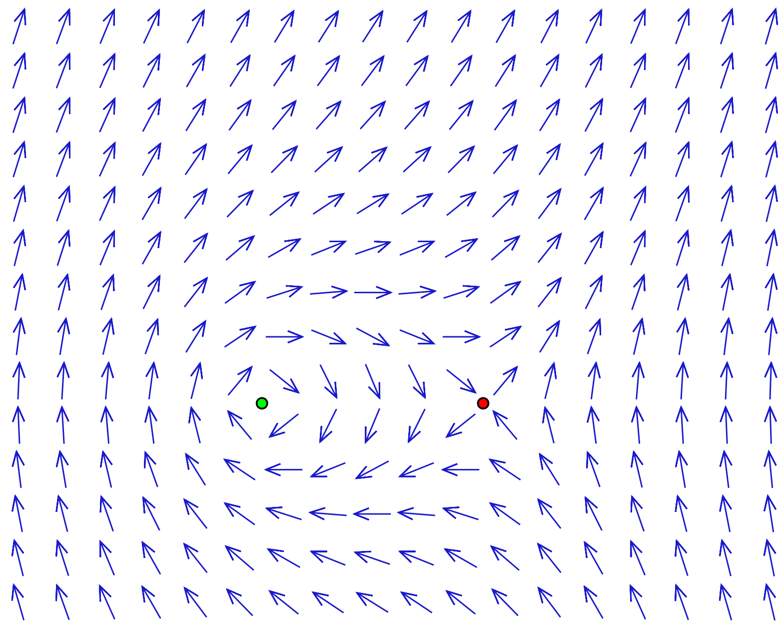
Berezinskii & K&T found that vortices and antivortices form bound pairs at low temperature.

Thermal fluctuations break up the VA-pairs, which destroys the superfluid state!

(2D quantum fluids in a thermal bath)



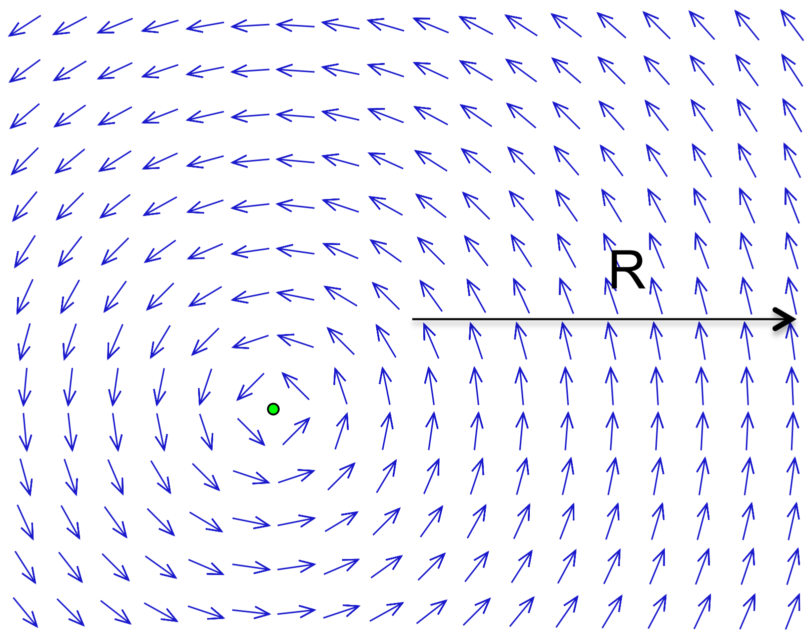
2 VORTICES, (winding # q = +2)

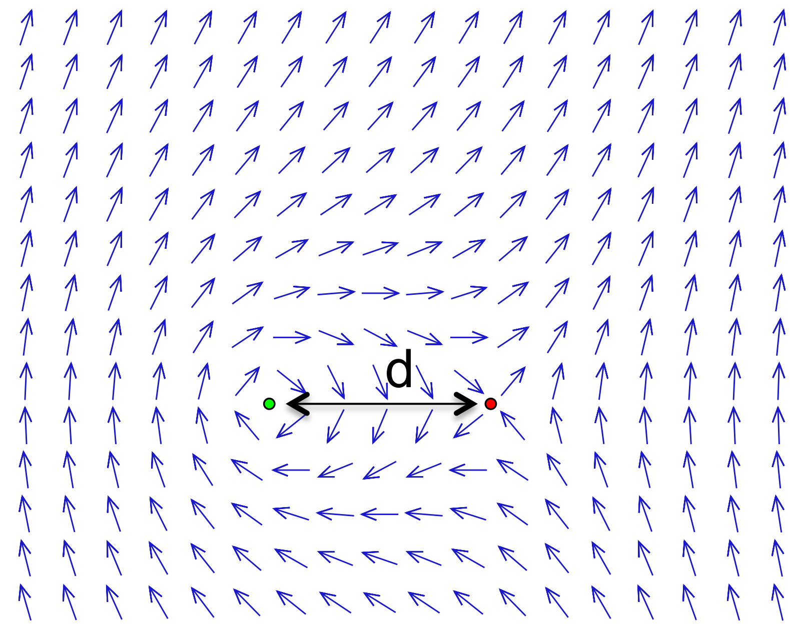


VORTEX-ANTIVORTEX-PAIR, (winding # q = 0)

The influence of the excitations extends to the boundary. They act GLOBALLY.
Addition of 1 vortex or antivortex changes the Boundary.

Energetics -- energy unit = J = $\rho_s (\hbar/m)^2$, quantized circulation = $2\pi r\, v_s$ = qh/m





VORTEX, (winding # = +1)
large energy

$E_1 = E_{core} + \pi J\, q^2 \ln(R/r_0)$, ($r_0$=vortex core size)

VA-PAIRS, (winding # = 0)
smaller energy, easier to form

$E_{pair} = 2E_{core} + 2\pi J\, q_1 q_2 \ln(d/r_0)$

---

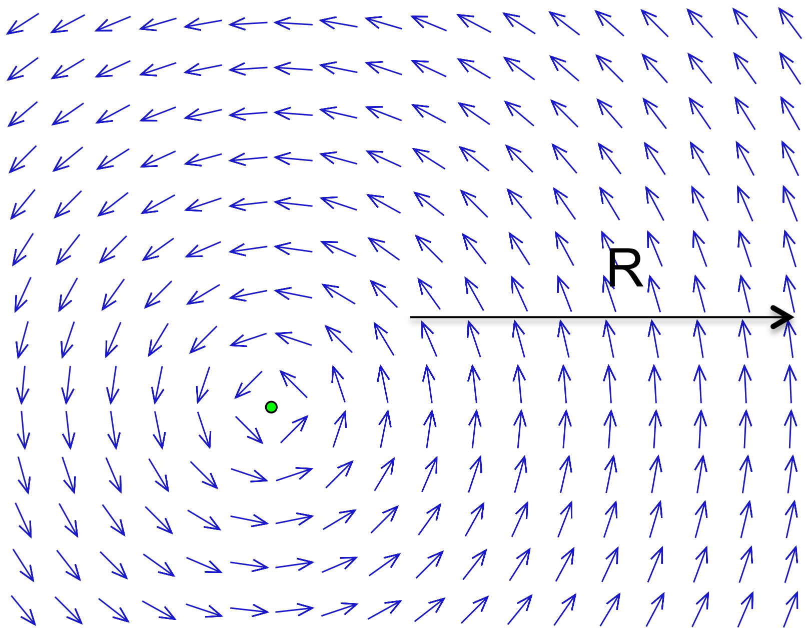
single vortex probability = $p_1 \sim \dfrac{e^{-E_1/k_B T}}{r_0^2} \sim \left(\dfrac{R}{r_0}\right)^{-\frac{1}{2\eta}}$

number of vortices = $N_v \sim p_1 R^2 \sim \left(\dfrac{R}{r_0}\right)^{2-\frac{1}{2\eta}}$

The phase transition!

N_v = LARGE when η >1/4

Entropy arguments



VORTEX, (winding # = +1)
large energy

$E = E_{core} + \pi J\, q^2 \ln(R/r_0)$, ($r_0$=vortex core size)

$J = \rho_s\, (\hbar/m)^2$

# of vortex locations $\mathcal{N} = (R/r_0)^2$

vortex entropy = $\mathcal{S} = k_B \ln(\mathcal{N})$

free energy = $F = E - T\mathcal{S}$

$F = (\pi J - 2k_BT) \ln(R/r_0) < 0$??

vortex production becomes favorable when $F < 0$ or

$$\boxed{k_BT > \tfrac{1}{2}\pi J}$$

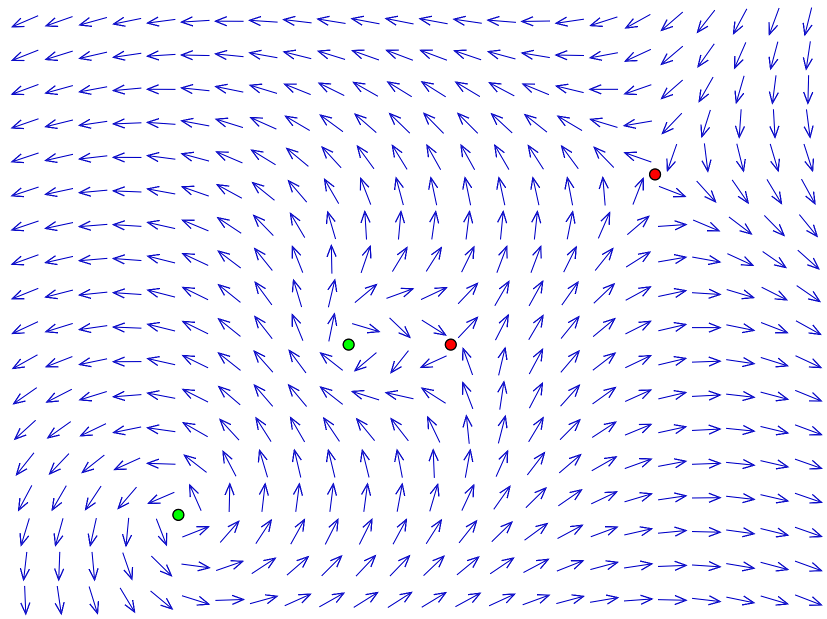
(approximate estimate of T_λ).

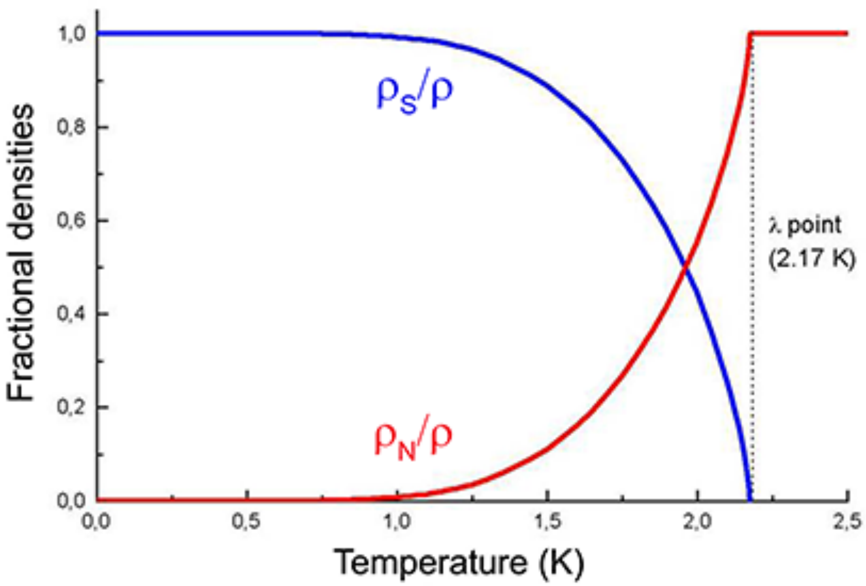
At high enough temperature, entropy (many allowed vortex positions) makes it favorable to produce vortices or anti-vortices even though they cost high energy.

Really K&T did an analysis of the probability to create VA-pairs.

Close pairs *renormalize* or interfere with the interactions of distant vortices.

K&T (1973) used *renormalization group theory* (see the 1982 Nobel Prize!) to get the “flow” equations that lead to T_λ.

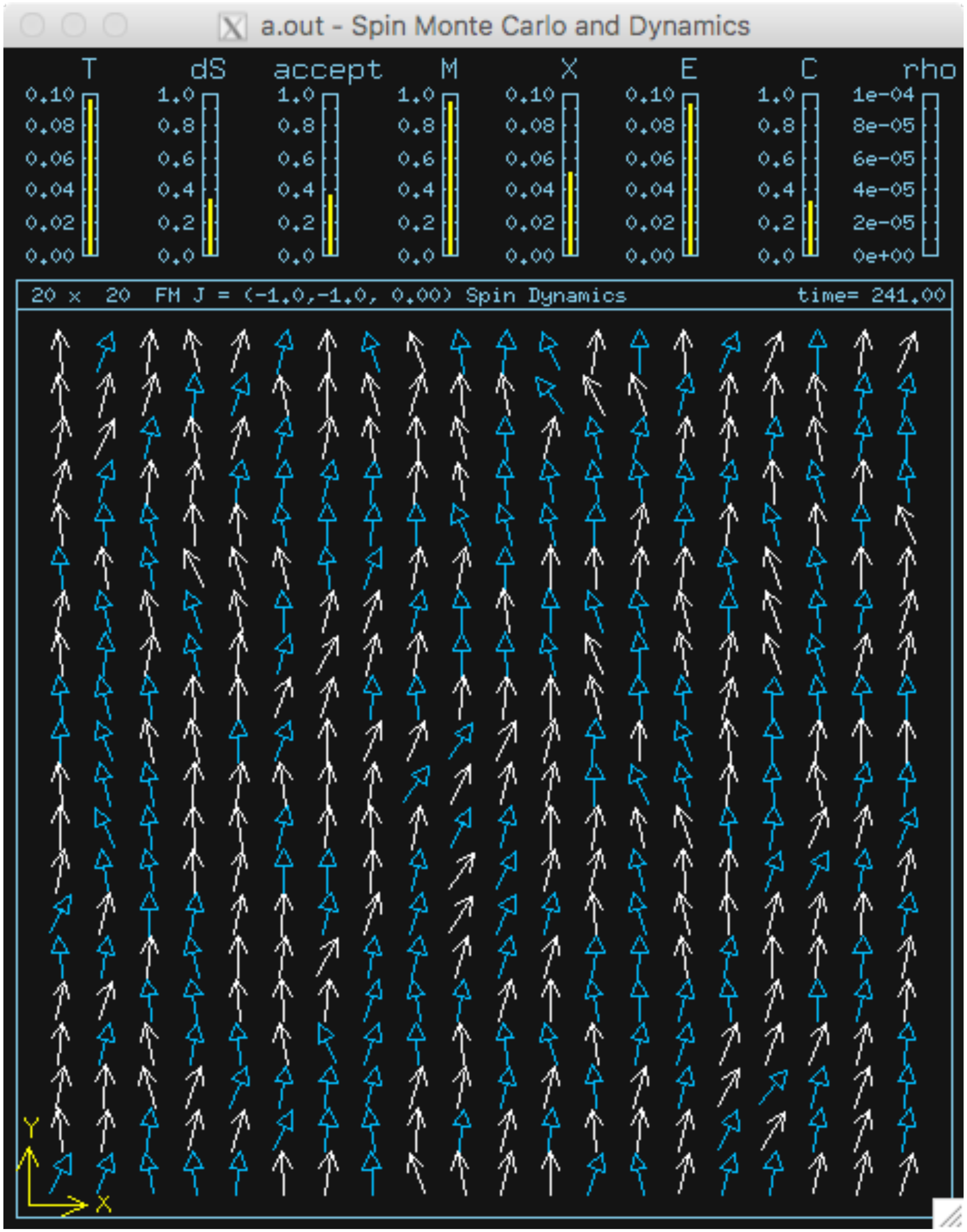




$\rho_s = A^2$ = density of the superfluid part

$T < T_\lambda$, vortices are bound in pairs.

$T > T_\lambda$, vortices become unbound!



2D Magnets

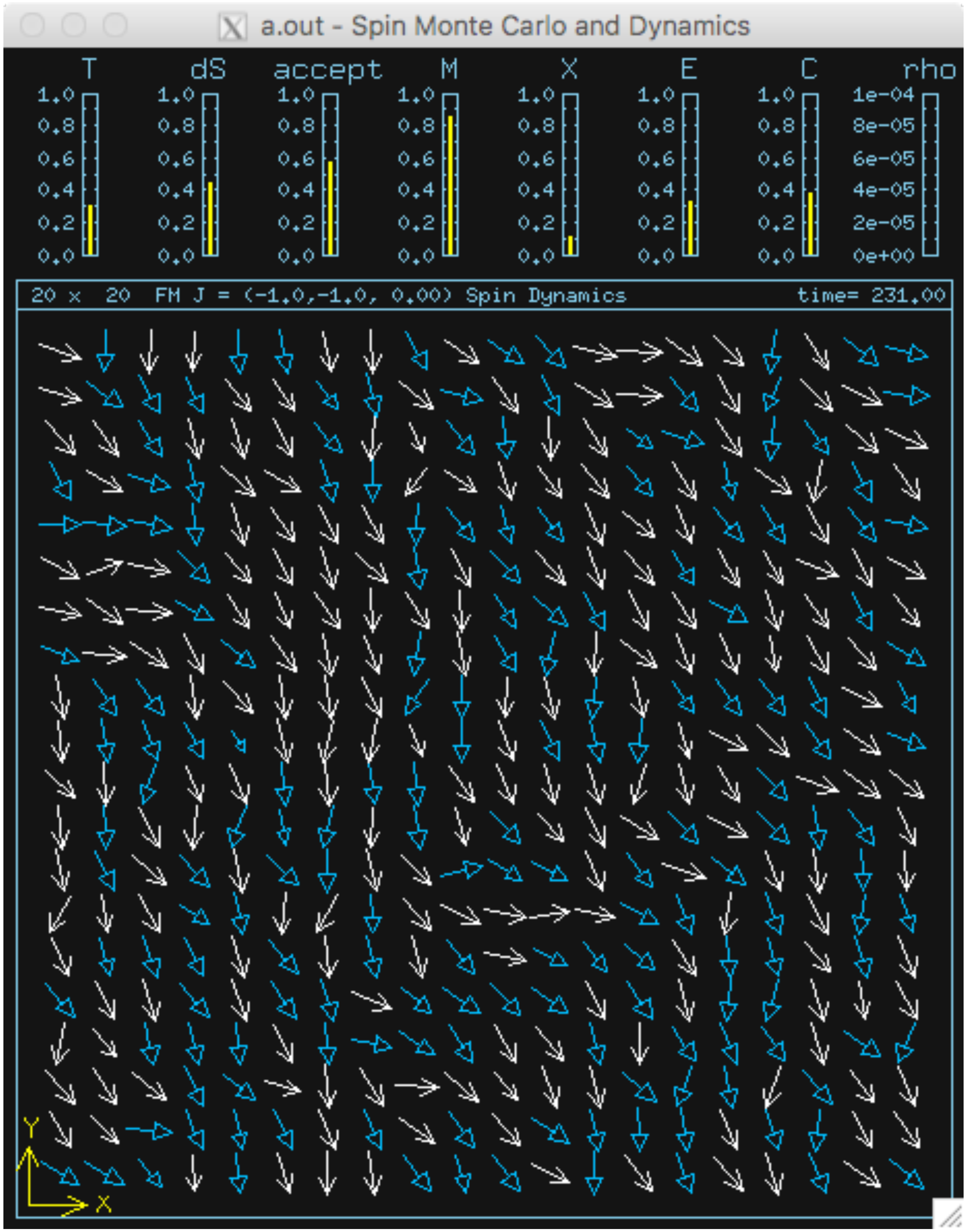
XY model

low T.

$k_B T/J = 0.1$

No vortices.

Ordered.



2D Magnets

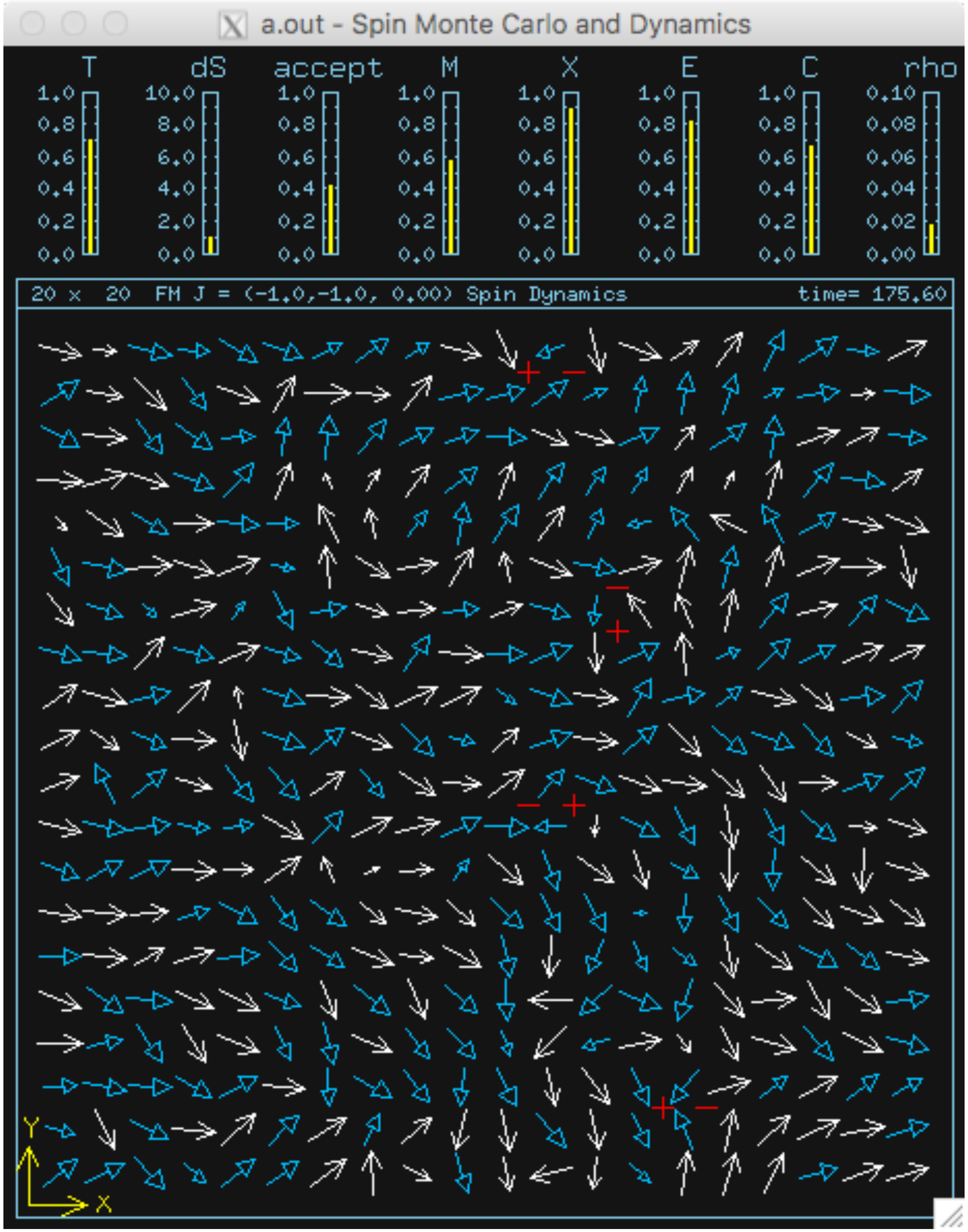
XY model

low T.

$k_BT/J = 0.3$

No vortices.

Less ordered.



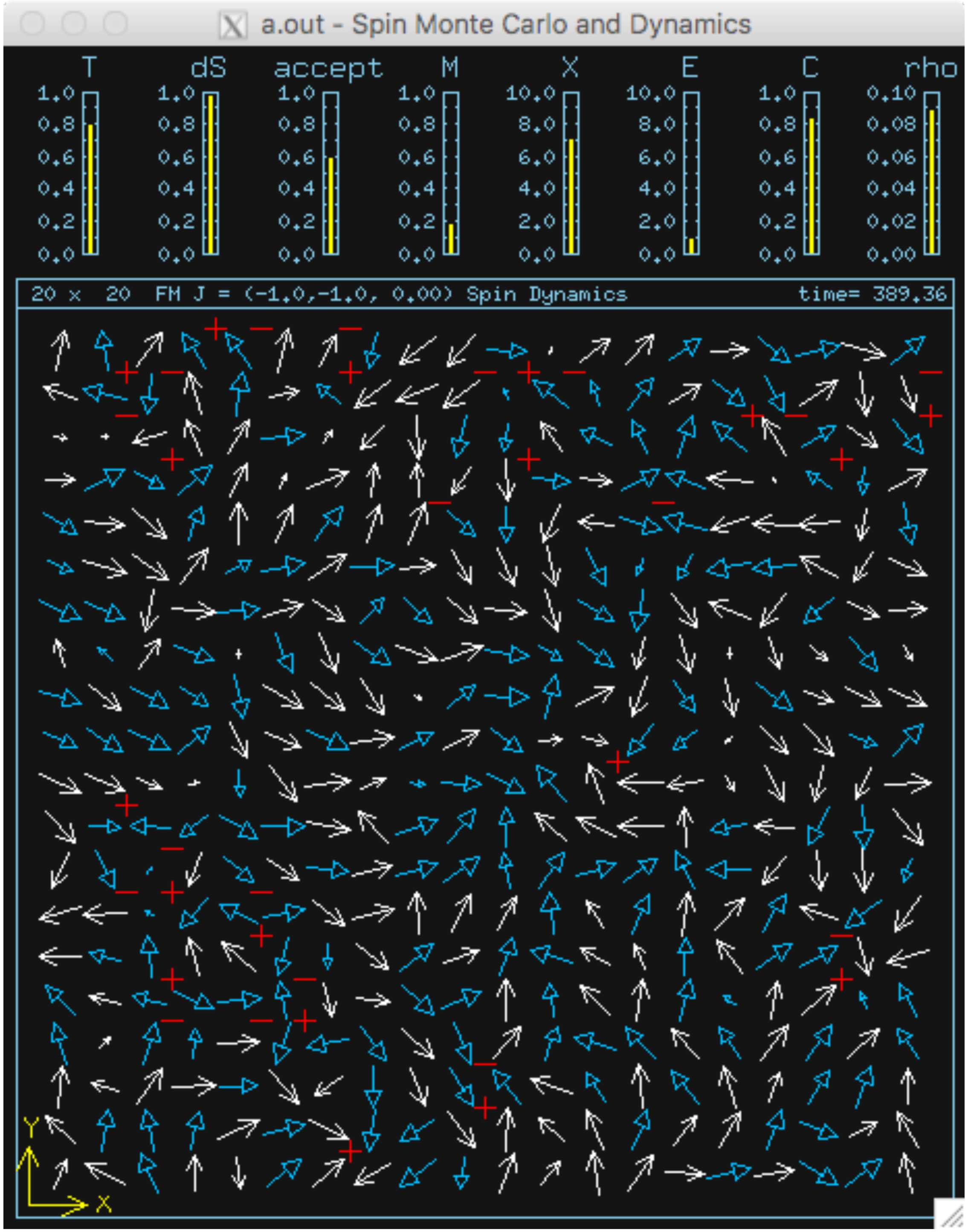
XY model

T near T_λ

$k_BT/J = 0.70$

A few close pairs.

Some disorder.



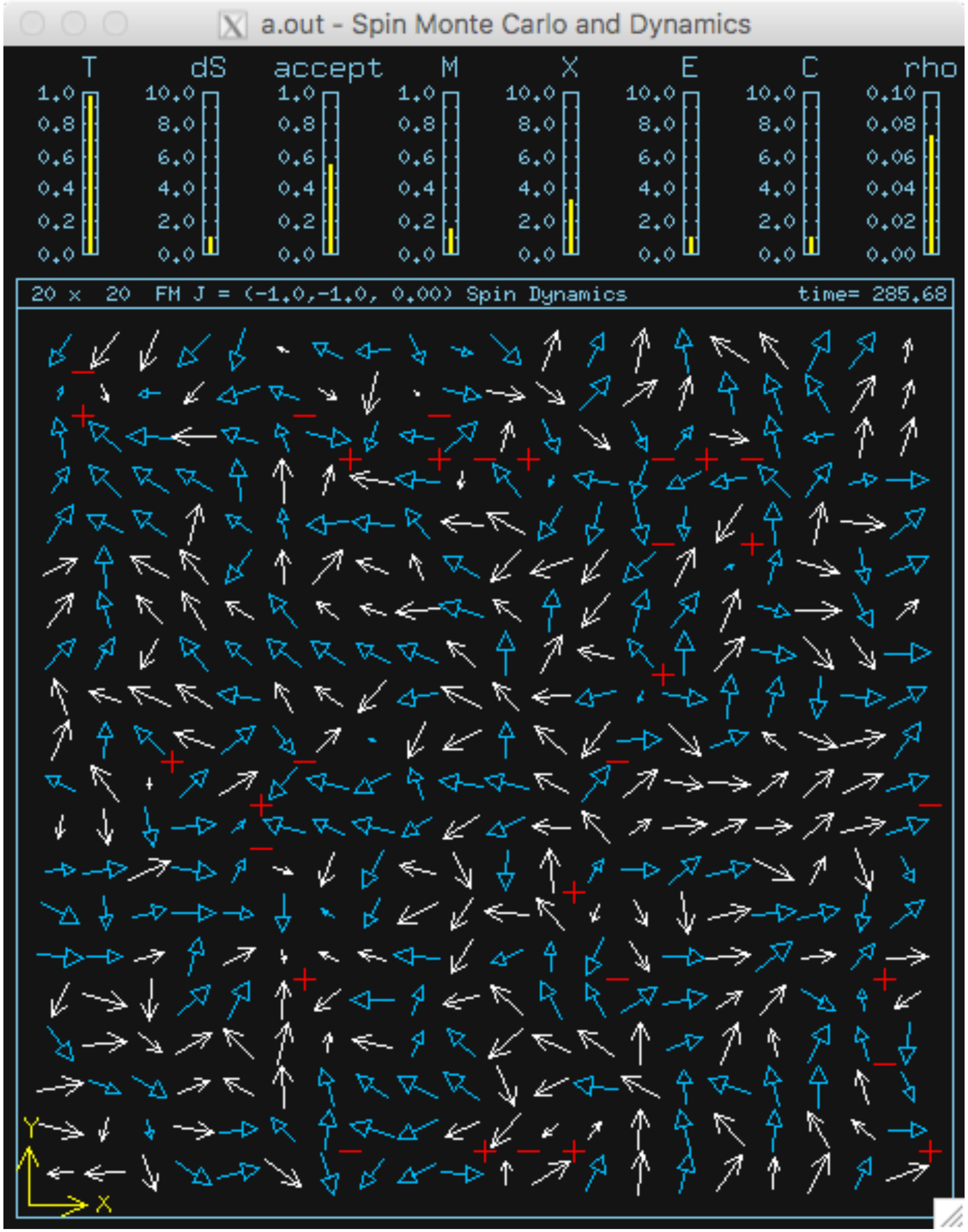
XY model

T just above T_λ

$k_BT/J = 0.80$

Pairs and more.

Disorder and free vortices.



XY model

high T.

$k_B T/J = 1.0$

Free vortices.

Disordered.

Problem 2. Quantum Hall Effect – electrical conduction in a 2D electron gas with a magnetic field.

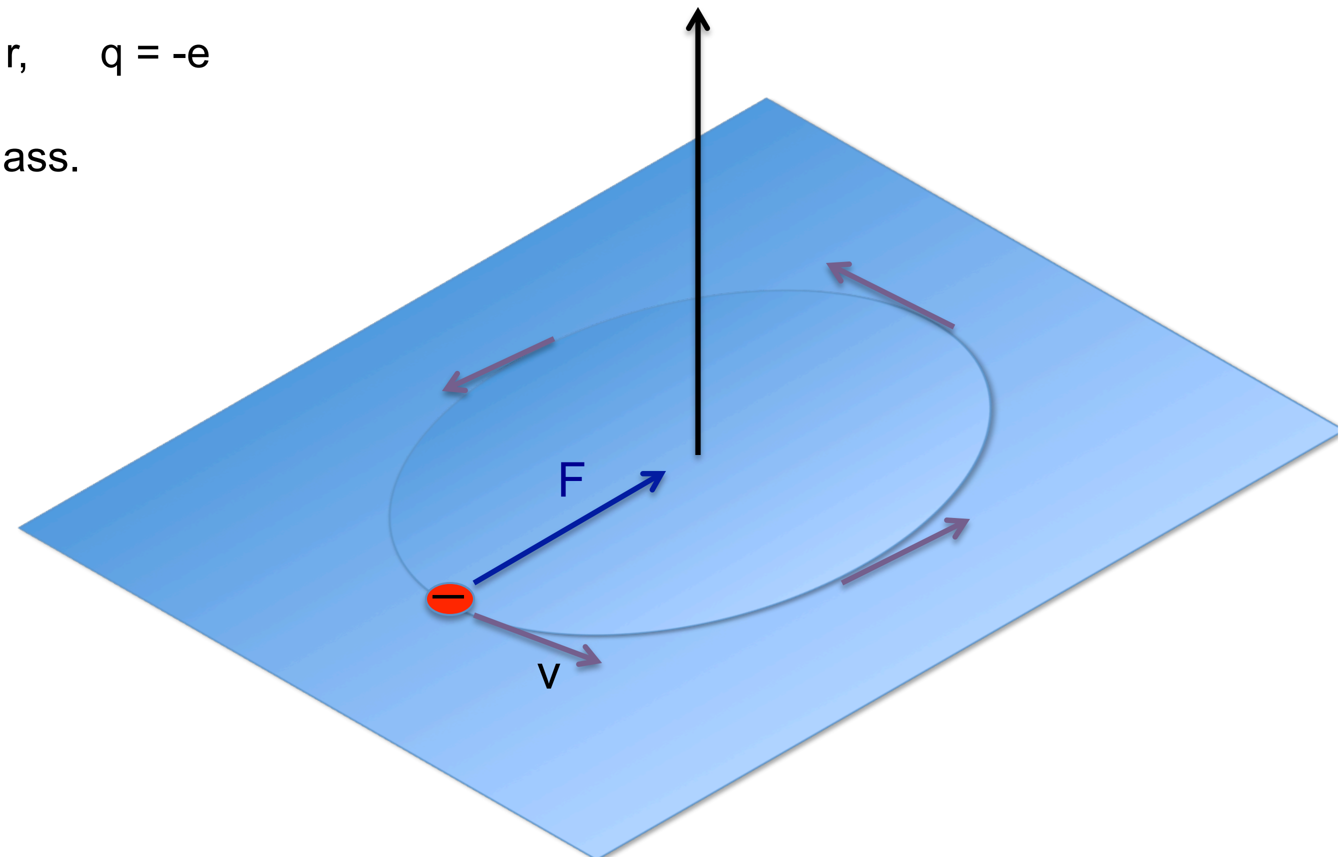
Electrons experience a Lorentz magnetic force which causes circular cyclotron motion.

$F_{magnetic} = q\,v\,B = m^*\,v^2 / r, \qquad q = -e$

m^* = electron effective mass.

cyclotron frequency:

$\omega = 2\pi f = eB/m^*$



Minimum kinetic energy:

$\tfrac{1}{2}\, m^*\, v^2 = \tfrac{1}{2}\, \hbar\omega$

Minimum orbital radius:

($\hbar$ = Planck's constant/2π)

$r_0 = \sqrt{\dfrac{\hbar}{eB}}$ = 25.7 x 10^{-9} m when B = 1.0 tesla.

does not depend on mass!

Problem 2. Quantum Hall Effect – electrical conduction in a 2D electron gas with a magnetic field.

Analyze Classical Hall Effect first.

Electron current along +x
due to electron drift velocity v_d along –x.

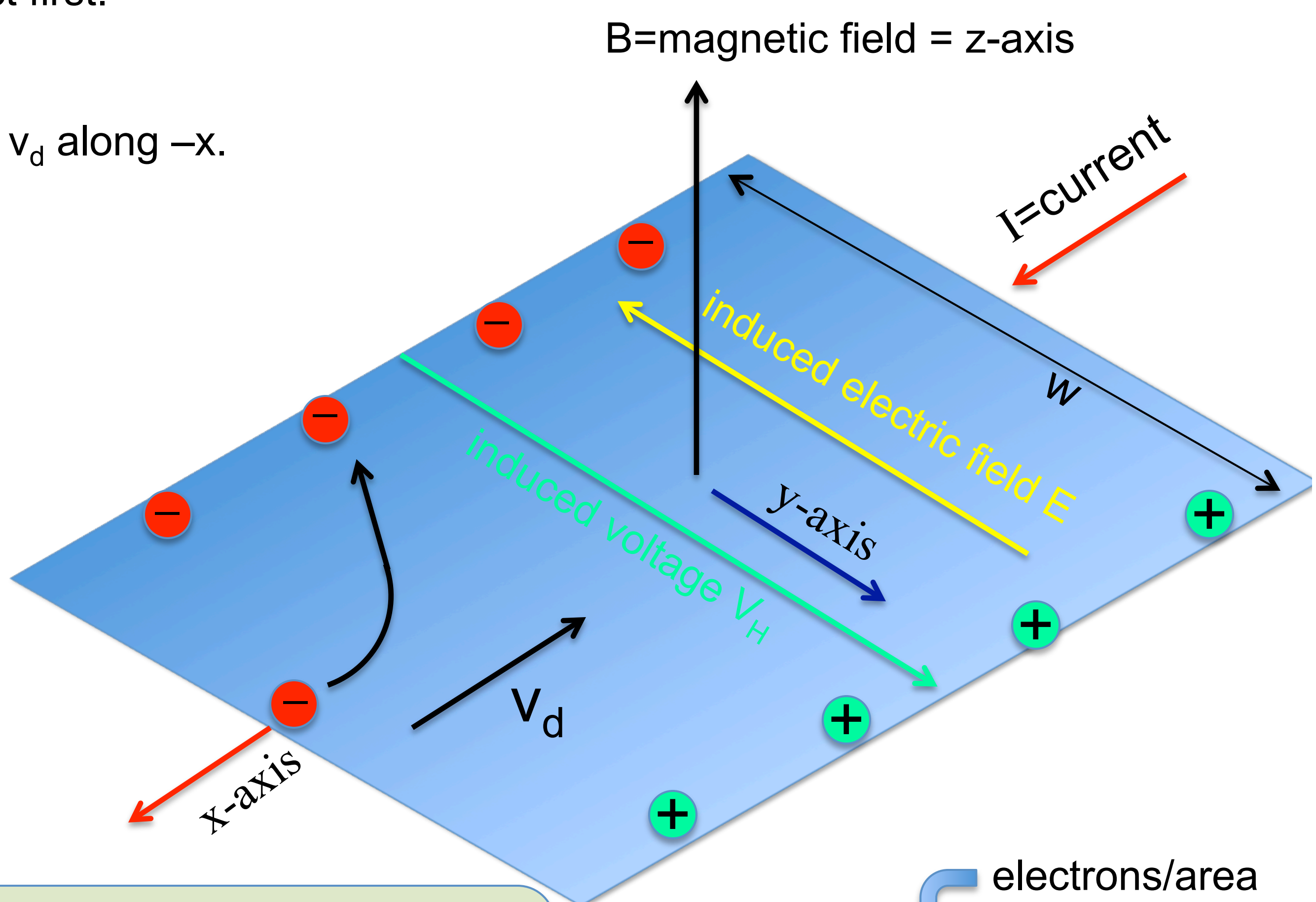
$F_{electric}$ = q E to the right

$F_{magnetic}$ = q v_d B to the left

Then $E = v_d B$

Transverse (Hall) voltage is

$V_H = -w E = -w v_d B$



Transverse resistance: $R_{xy} = V_H / I = -B/ne$ (ohms)

Hall coefficient = $R_H = R_{xy}/B = -1/ne$ (ohms/tesla)

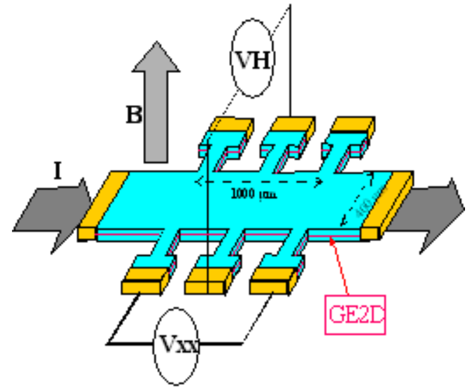
electrons/area
width

current $I = n\, w\, (-e)\, (-v_d)$

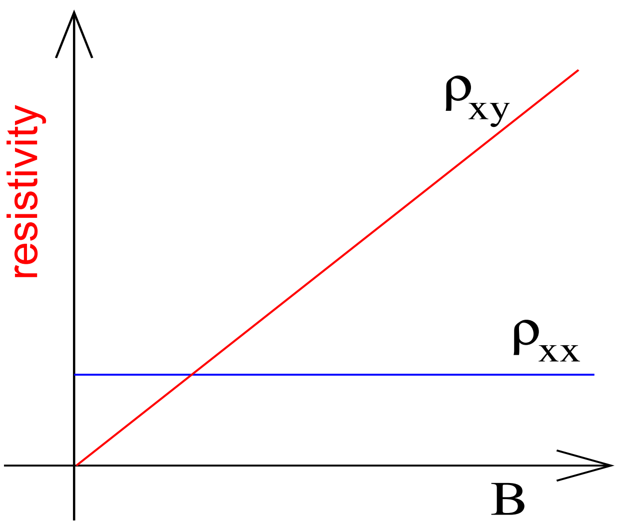
electron charge

Resistance is $R = \rho\, L/A$, where ρ is called resistivity, L=length, A=cross-sectional area.

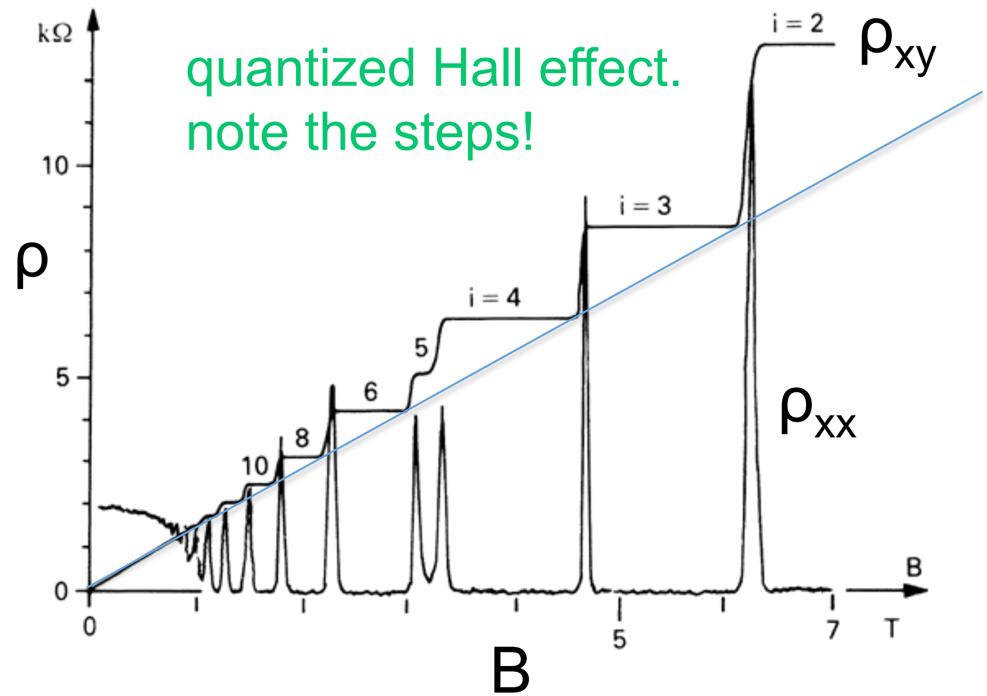
In 2D, *transverse* $R_{xy} = \rho_{xy}$ because $L=L_y=w$ and $A=A_x \rightarrow w$.



classical results:



$\rho_{xy} = B/ne$
$\rho_{xx} = m^*/(ne^2\tau)$=constant



low-T experiments, (T < 4 K)
2D electron gas in a MOSFET,
Klaus von Klitzing,1985 Nobel Prize

von Klitzing found quantization!
(h = Planck's constant)

$$\rho_{xy} = \frac{h}{e^2}\frac{1}{\nu}$$

$\nu=1,2,3\ldots$ = "filling factor"

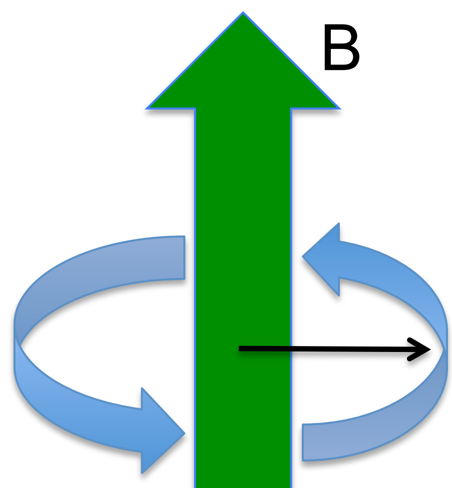
with a natural unit of resistance that is a universal resistance standard:

$$\frac{h}{e^2} = 25812.8075\ \Omega = R_K$$

Thouless found a theoretical explanation. The integer ν is a topological number. It is connected to the state of the electrons in a magnetic field.

<u>Landau levels (quantum states)</u>

$E = (n_E+½)\ \hbar\omega$
$n_E=0,1,2,\ldots$ (energy quantum number)



cyclotron frequency $\omega=eB/m^*$

Very similar to a 2D harmonic oscillator.
All states have the same degeneracy N.

$$N = (\text{area of system}) / (2\pi r_0^2) = \frac{BA}{h/e}$$

$$r_0 = \sqrt{\frac{\hbar}{eB}}$$ (electron waves)

(No spin degeneracy here for large B)

quantum of magnetic flux:

$$\Phi_0 = \frac{h}{e} = 4.1357\text{x}10^{-15}\ \text{T}\bullet\text{m}^2$$

Landau level degeneracy = N = BA/Φ_0

At B=1.0 tesla,

Landau orbital radius r_0= 25.7 nm (r_0 decreases with increasing B).

For area of 1000 nm x 1000 nm, degeneracy N = $A/(2\pi r_0^2)$ = 240.

(N increases as B increases).

THEORY. Why are there steps to plateaus in ρ_{xy}?

Electrons occupy the lowest available energy states, consistent with the Pauli exclusion principle. Each Landau level can hold up to N electrons. Suppose a total of ν levels are completely filled, each with N electrons.

Then total electrons/area = n = $\nu N/A = \nu B/\Phi_0$

which leads to the Hall resistivity

$$\rho_{xy} = \frac{B}{ne} = \frac{\Phi_0}{\nu e} = \frac{h}{e^2}\frac{1}{\nu}$$

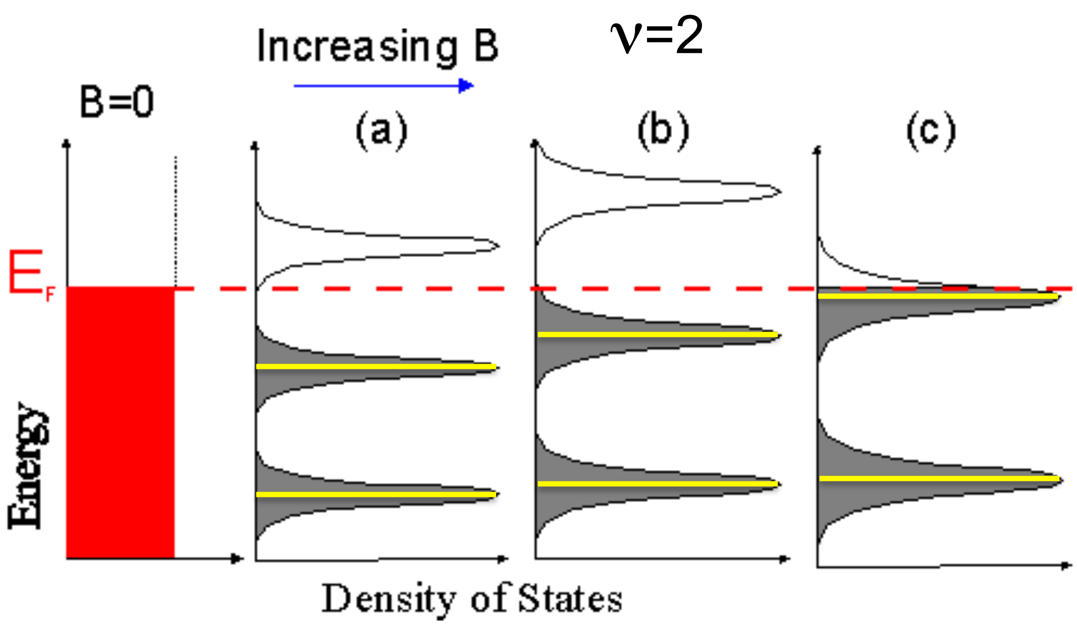
Each STEP in ρ_{xy} empties out another Landau level.

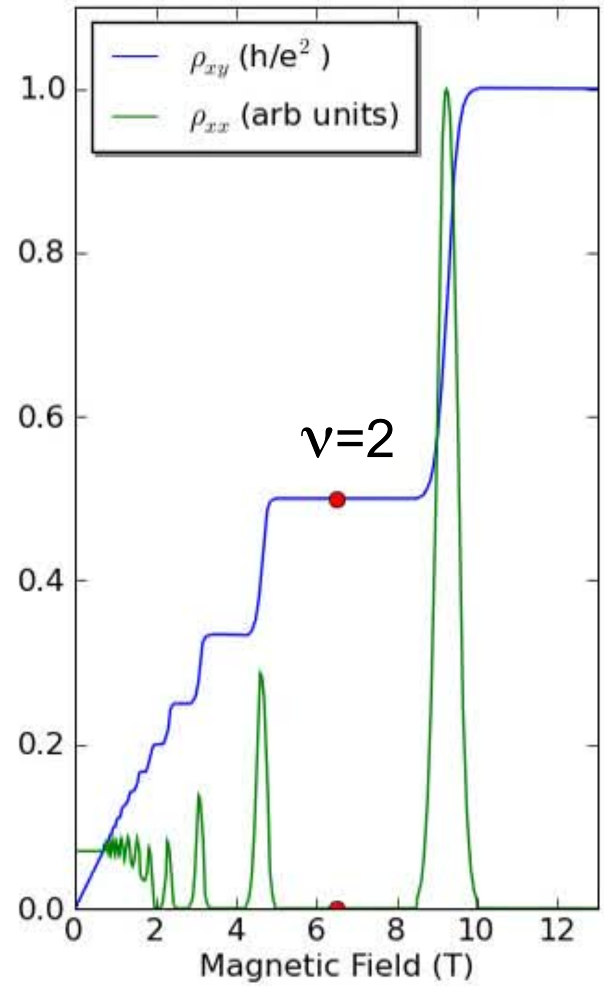
Why the effects on ρ_{xx} and ρ_{xy}?

Due to disorder in the samples (dirt, impurities, etc.),

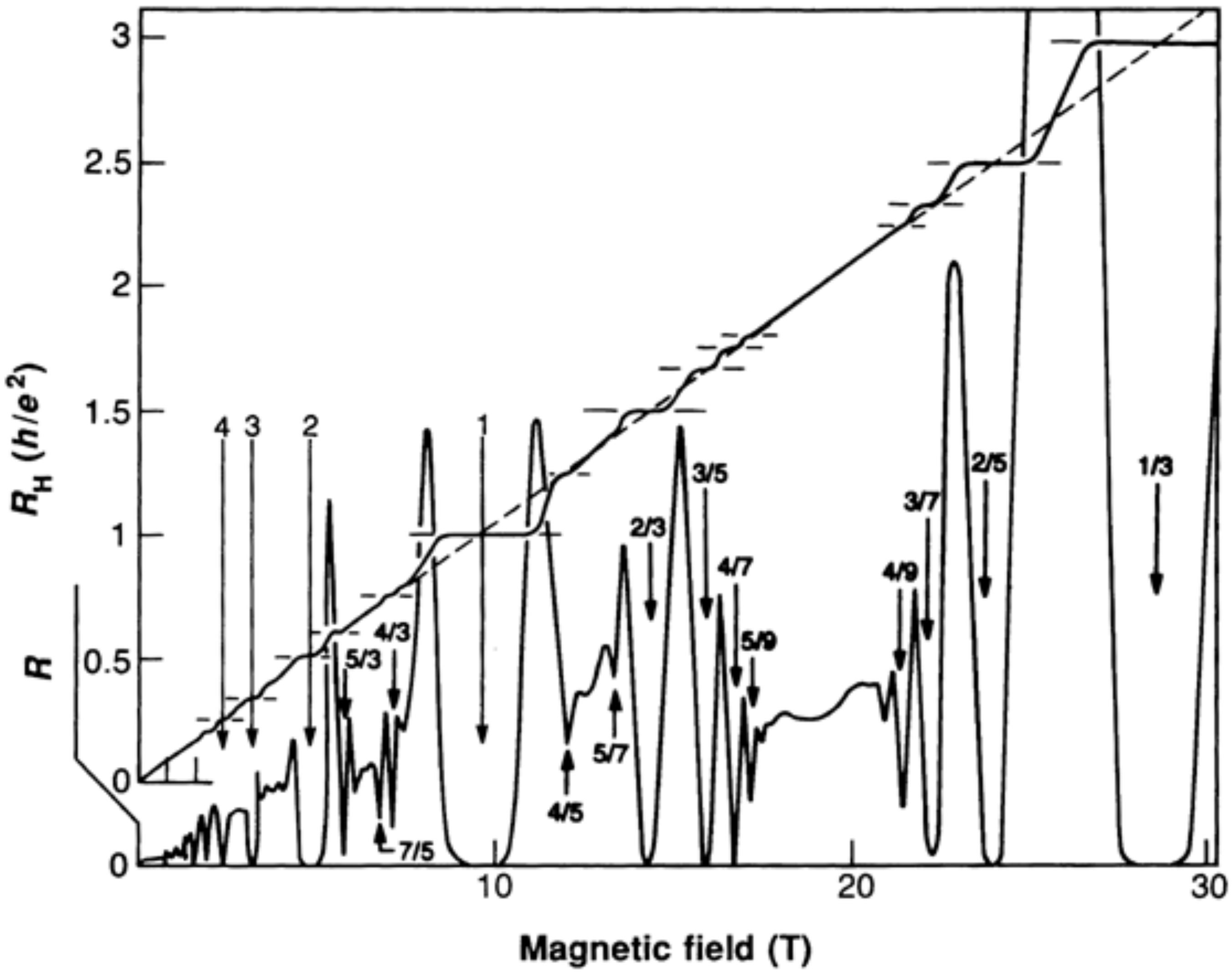
Many states are “localized” -- their electrons don’t move through the sample.
The other states are “extended” – their electrons contribute to conduction.
The energy levels $E = (n_E+½)\hbar\omega$ get spread out in energy

yellow bands = extended conducting states





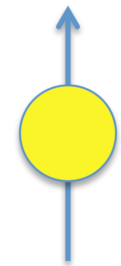
At T< 4K, see the Fractional QHE where for example ν=1/2, 1/3, etc.
The electrons interact with each other to form new types of quasi-particles.



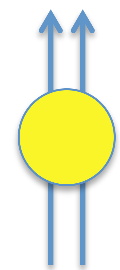
J.P. Eisenstein & H.L. Stormer, Science 1990

QHE.   Where's the topology?!   Electrons pair with the quantized flux vortices.

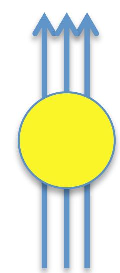
Integer QHE (ν=1,2,3…).
One electron pairs with each quantized magnetic flux vortex.



Fractional QHE (ν=1/2, 1/3, etc.).
The electrons interact with each other and the flux quanta
to form new types of quasi-particles.

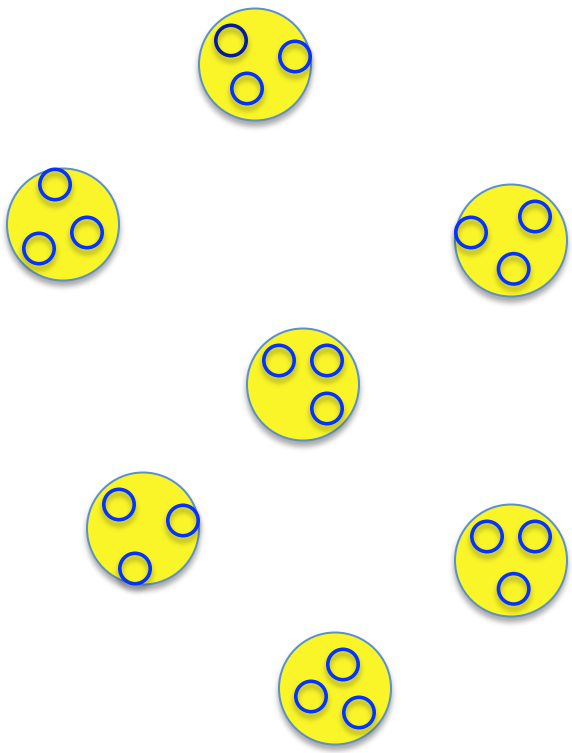


For ν=1/2, an electron pairs with the two flux vortices.
(half-filling).   This is fermion-like. Each vortex charge = -e/2.

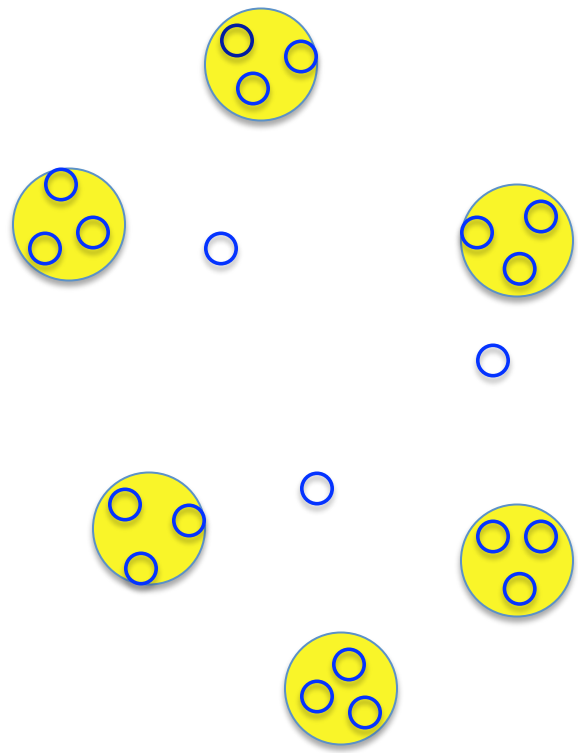


For ν=1/3, an electron pairs with three flux vortices.
(1/3-filling).   This is boson-like.   Each vortex charge = -e/3.

But this is oversimplified! Electrons interact strongly.
The more general theory is that of "anyons",
quasi-particles that are neither bosons nor fermions.

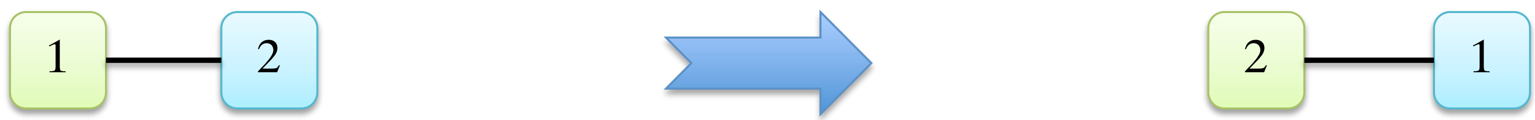


ground state (ν=3).
electrons paired with
flux lines.



removing an electron makes
three quasi-particles, each q = e/3.

QM in 2D, large B.   fermions, bosons, anyons?! Look at two particles.



Ψ_a Ψ_b Ψ_a Ψ_b

$\Psi(a_1,b_2)$ exchange the particles $\Psi(a_2,b_1) = e^{i\theta}\, \Psi(a_1,b_2)$

leads to a phase change

Suppose the particles are indistinguishable.
Schrodinger says: "The probability of the state = $|\psi|^2$ is not affected by the phase change."

Let $\theta=2\pi S$ (angle in radians, 2π same as 360°). $e^{i\theta}=e^{i\pi 2S}=(-1)^{2S}$

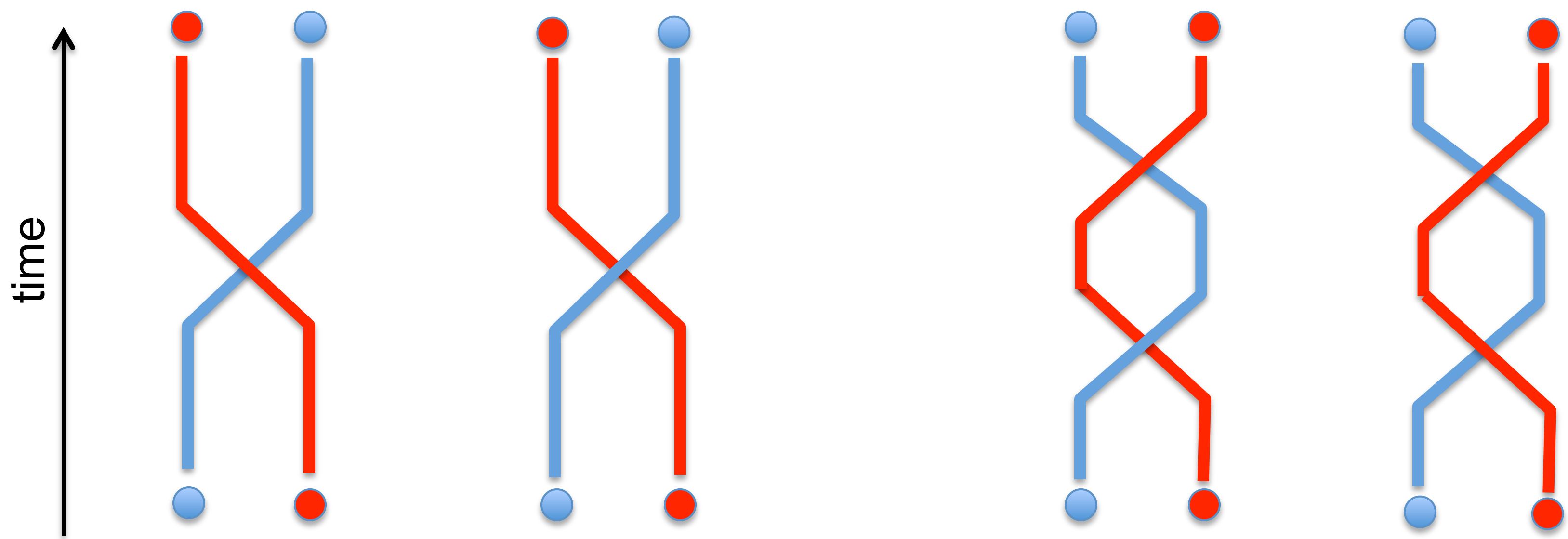
Fermions: S=1/2, 3/2, 5/2….   then $e^{i\theta} = -1$

adequate for 3D.

Bosons: S=0, 1, 2, ….   then $e^{i\theta} = +1$

Anyons: S=other numbers,   then $e^{i\theta}$ = complex numbers!   Needed for 2D.

In 2D, a 180° rotation of particles is not the same as a -180° rotation!



the world lines are braided differently.

they can get entangled, or not.

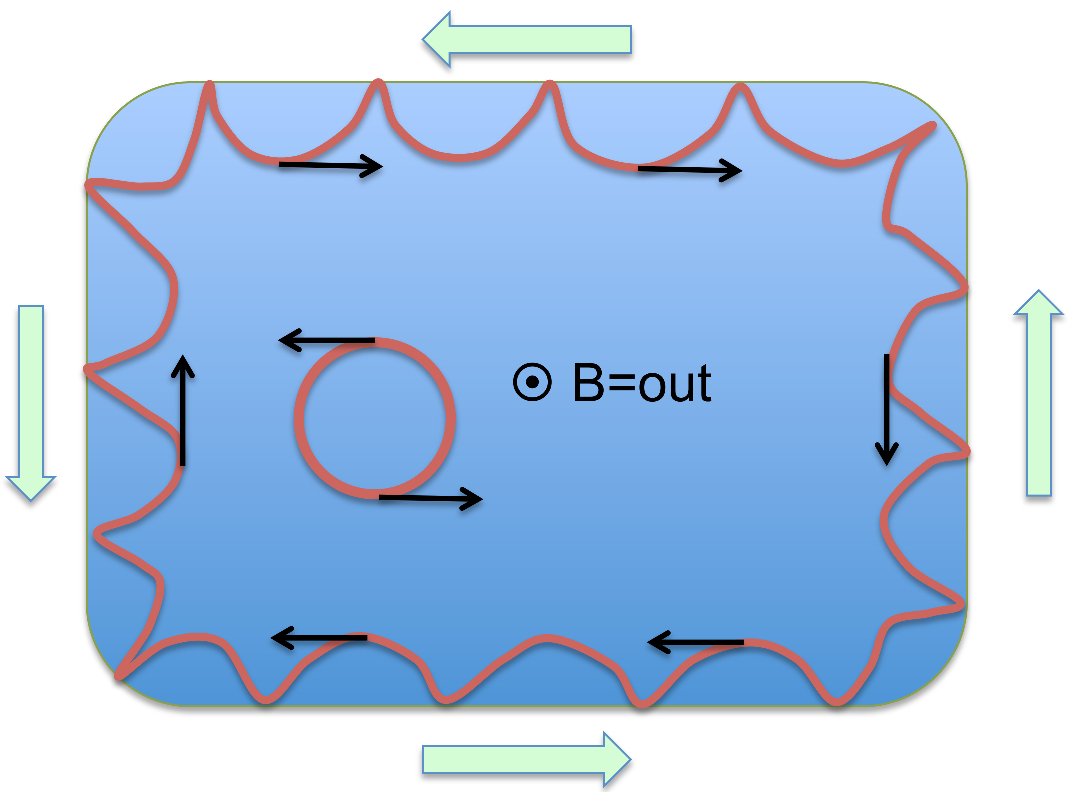
With more particles it is easy to see that different particle exchanges lead to complicated quantum states of indistinguishable particles.

These braids form the mathematical braid group B_N. Basis for "anyon statistics"
The particles being exchanged are the FQHE quasiparticles with charges like e/3.

Another topological effect: edge modes.

cyclotron speed > drift speed.

electrons scatter from edges but move along at little energy cost (un-gapped modes).



chiral electric current at edges.
direction is determined by B.

leads to:

"topological insulators"

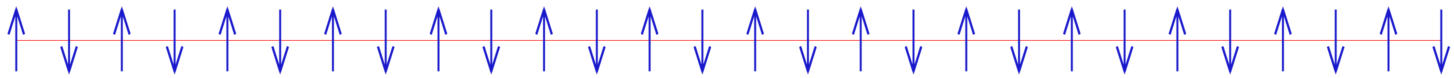
which can conduct mainly on the edges but are insulating in the interior.

bulk excitations = gapped.
edge excitations = ungapped.

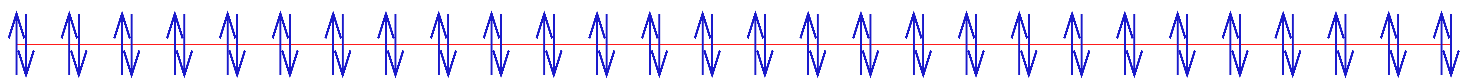
Problem 3. Chains of magnetic atoms.

Duncan Haldane, Princeton Univ. article: discussing his brief career in chemistry as a student,
"After a few experiences in the chemical or biology lab, I decided I should not let myself near any kind of nasty chemicals or radioactive materials, so after having a few spills on myself I decided I was going to be a theorist."

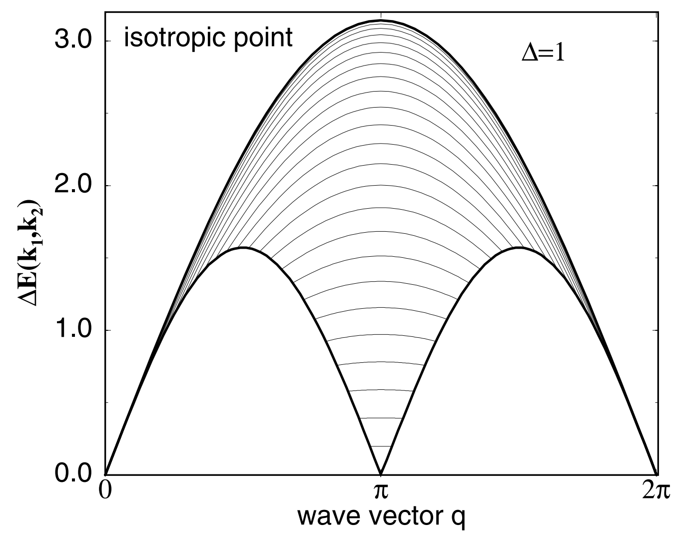
P. Anderson & D. Haldane



Classical Heisenberg antiferromagnetic chain (ground state = alternating up/down spins).
A small-amplitude wave costs little energy.   No energy gap to the first excited state.



Quantum S=1/2 Heisenberg antiferromagnetic chain.
(quantum ground state is not up/down ordered).
Small changes from ground state cost little energy.
Lowest excitations are spinons (domain-wall pairs), massless, with a wide spectrum but no energy gap.
This can be solved exactly (see Bethe Ansatz).



Haldane (1983) – quantum spin S=1 Heisenberg antiferromagnetic chain. Ground state is a "valence bond solid". There is an Energy Gap (0.41 J) to first excited state. It was not expected. Haldane offered his theory as a "conjecture" (very good educated guess!), until it was proven.

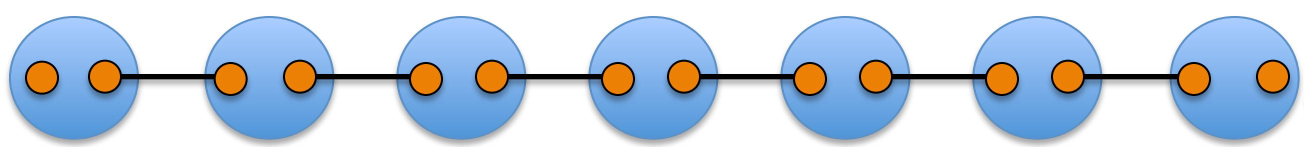
valence bond solid ground state.

S=1

(triplets, S_z=-1,0,+1)

S=½

spin singlets = (↑↓)-(↓↑)
(S_z=0)



fractionalized spins,
S=½ at ends!

Haldane's theory: Mapped the large-S quantum spin model to the
O(3) nonlinear sigma model in 1 space + 1 (imaginary) time dimension!
It corresponds to 3D unit vectors living in a 2D plane.

It includes a nonlocal "topological term" or "Berry phase" that depends on the spin S value = ½, 1, 3/2, 2, etc., which adds a mechanical *action* = 2πħ S Q,

integer
topological charge = $Q = \frac{1}{4\pi}\int d^2x\, \boldsymbol{l}\cdot(\partial_0\boldsymbol{l}\times\partial_1\boldsymbol{l}),$

= # of times the vector $\boldsymbol{l}$ covers a unit sphere in a configuration.

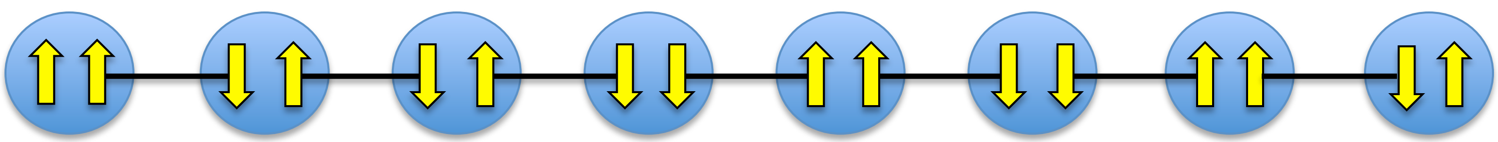
Quantum theory used by Haldane.
QM states get a

phase factor = $e^{i 2\pi S Q}$ = +1 or -1

S=1/2, 3/2, etc.: configurations with different Q interfere destructively!
$e^{i 2\pi S Q}$ = +1 or -1. → no energy gap from ground state to excited states.

S=1, 2, etc.: configurations with different Q interfere constructively.
$e^{i 2\pi S Q}$ = +1 only. → energy gap from ground state to excited states.
The gap gets smaller as S gets larger. "Haldane phase".

S=1 system has spin disorder, but a hidden topological (string) order:
z-components of spins prefer neighboring +1,-1, separated by strings of 0's.
Also known as *diluted antiferromagnetic order*!



S_z = +1 0 0 -1 +1 -1 +1 0

The 0's are diluting the alternating +1-1 antiferromagnetic order.

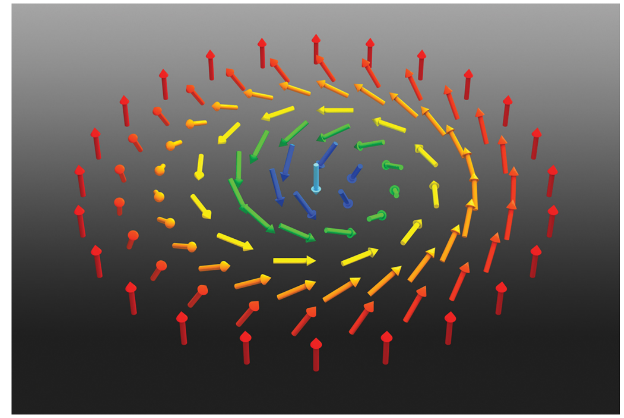
Summary & Future.

Topological excitations are common objects in quantum condensed matter, that are involved in phase transitions and the energy spectrum.

One example is vortices and anti-vortices, whose unbinding in 2D quantum fluids (superfluids, superconductors, magnets) produces the KT-phase transition. Another is fractionalized quasi-particles such as in quantum Hall effects.

Quantum mechanics depends greatly on *phase angles* (interference!) that are themselves determined by hidden topological charges Q as in the Haldane phase of magnets.

Deeper understanding of effects such as edge modes leads to new materials such as “topological insulators”, whose conduction takes place mainly on the edges or surfaces.



Q=1 magnetic skyrmion

topological insulator energy levels