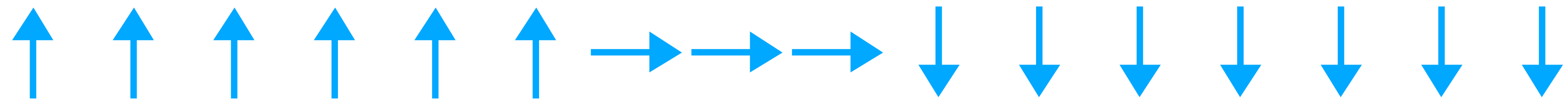
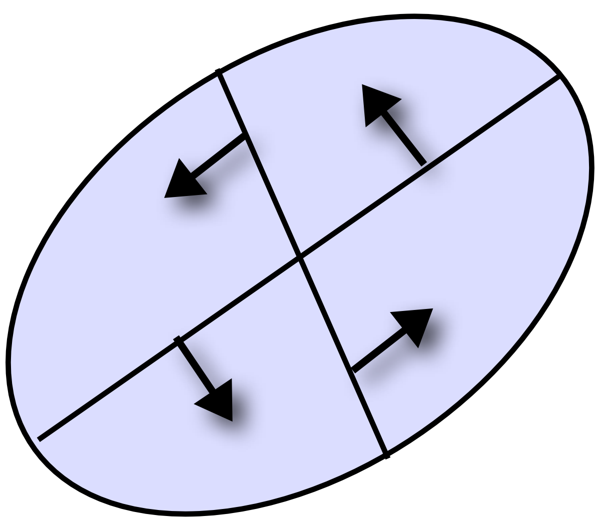


Energy and Dynamics of Vortices in Magnetic Nano-Dots

2012



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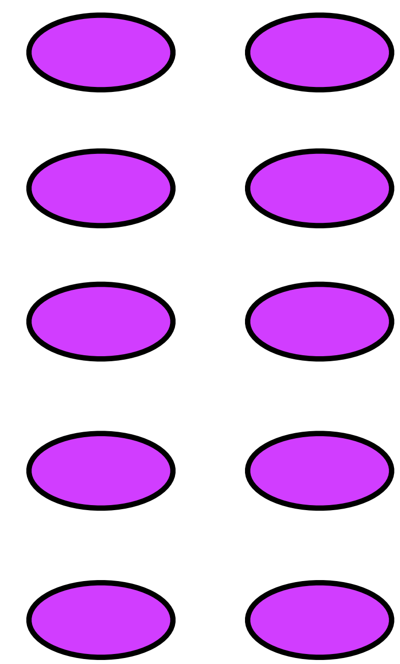

Estudo e Pesquisa em
Física, Kansas State University, EUA

Atomic, molecular, optical physics (ion accelerators, attosecond lasers, high-harmonics, intense laser fields, Bose-Einstein condensation)

Condensed, soft and biological physics (magnetism, electromagnetics, polymers, colloids, proteins, nanoparticles, fractals, nanowires, self-organization, biophysics, cell physics)

Cosmology and particle physics (gravitation, dark matter, neutrinos, high energy accelerators, standard model)

Physics education research (teaching, problem solving, learning transfer)

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UFSC - Physics -
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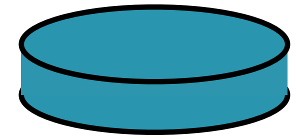
UNIVERSIDADE FEDERAL DE VIÇOSA
1976

Magnetic Nanodots



Approx. 50 nm - 5 μm, individual & in arrays, made from high-permeability soft magnetic materials, grown with techniques of epitaxy & lithography.

Can be islands in a non-magnetic substrate. Form arrays of particles that interact.

They should exhibit new effects due to their small size: (modified spin waves, surface effects, special sensitivity as detectors).

Two principle states:

(1) quasi-single domain; (2) a vortex.

Magnetic nanodots: applications

☛ memory elements, signal processing

☛ non-volatile data storage (magnetic ram)

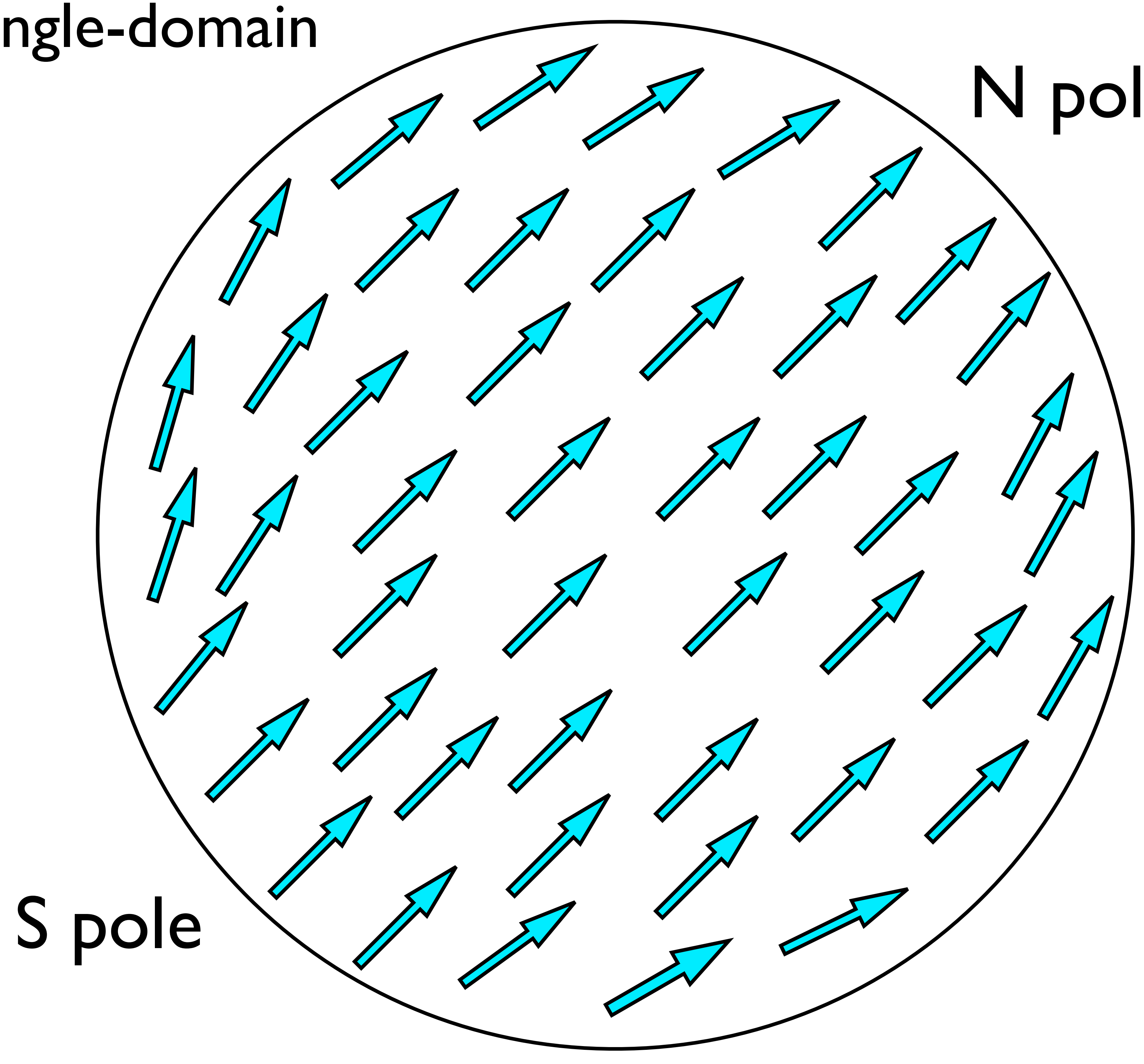
☛ use in sensors of (giant) magneto-resistance (GMR)

☛ integration into spintronics (switching between states via spin polarized currents.)

⊙ a one-vortex state with small stray magnetic field.

Magnetization M in a circular nanodot

(1) Quasi-single-domain

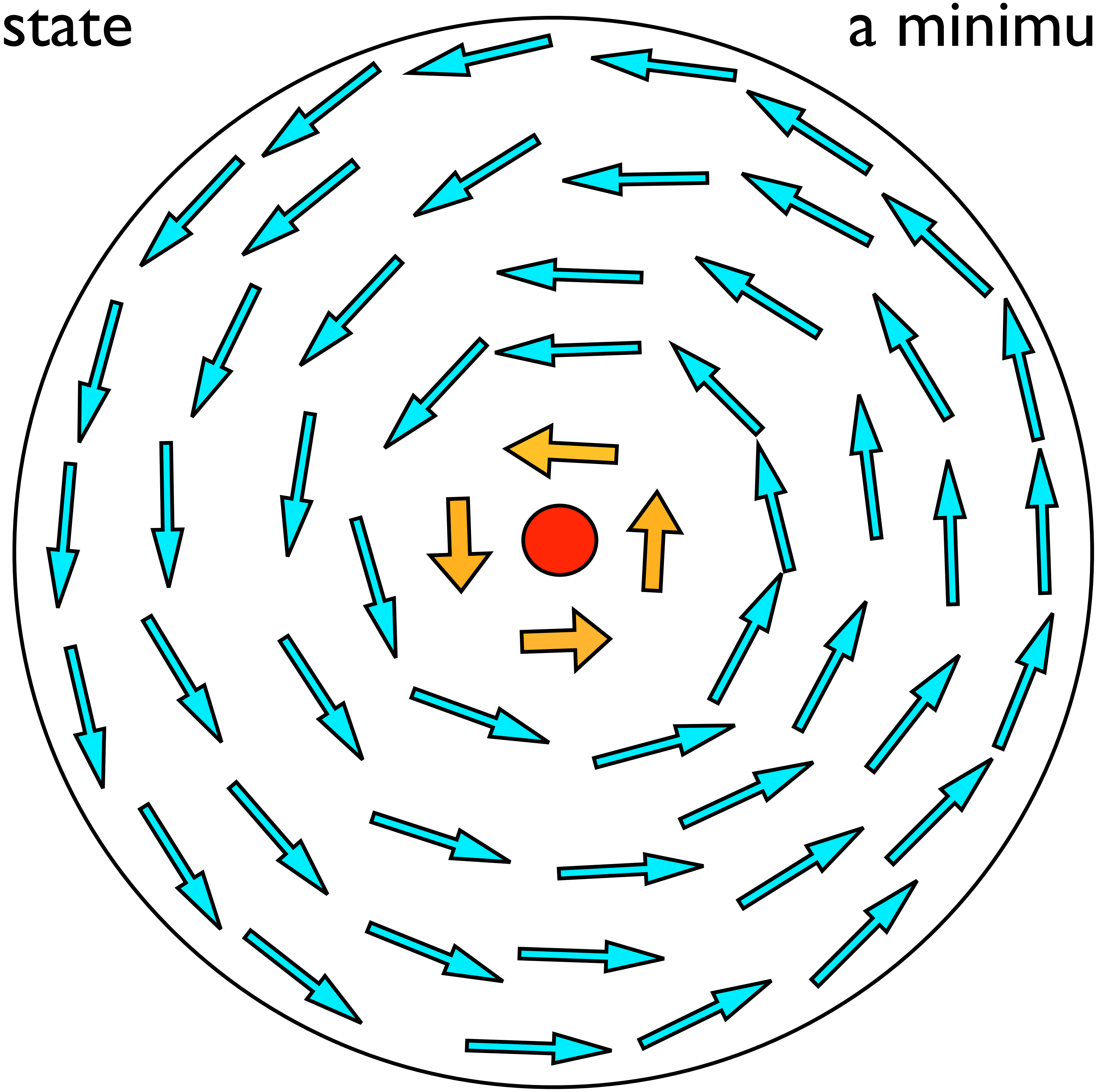


The poles require extra energy.

The energy of ferromagnetic exchange in small.

(2) Single-vortex state

Stable only above a minimum radius



Has poles (±z) only in the core. Their energy is small.

Now the energy of FM exchange is greater.

For the vortex states.

A. Energy & Potential E(X)?

$X=(x_v,y_v)$=position of the vortex center.

B. Dynamics and frequency ω_G of the gyrotropic movement?

$V=(V_x,V_y)$=velocity of the vortex center.

How to study the properties of magnetic vortices inside a cylindrical nanodot?

Define the magnetic energies in a disk of Permalloy, as functions of the magnetization M.

Look at vortices as particles with charges, transitions between internal states, objects to store data, and with interesting dynamics for X(t).

Energy ⇒ potential E(X), using a Lagrange constraint.

Energy ⇒ the dynamics M(t), via a Langevin equation.

Results: Stability, gyrotropic movement, the frequency, as functions of geometry, etc.

Magnetic Vortex Core Observation in Circular Dots of Permalloy

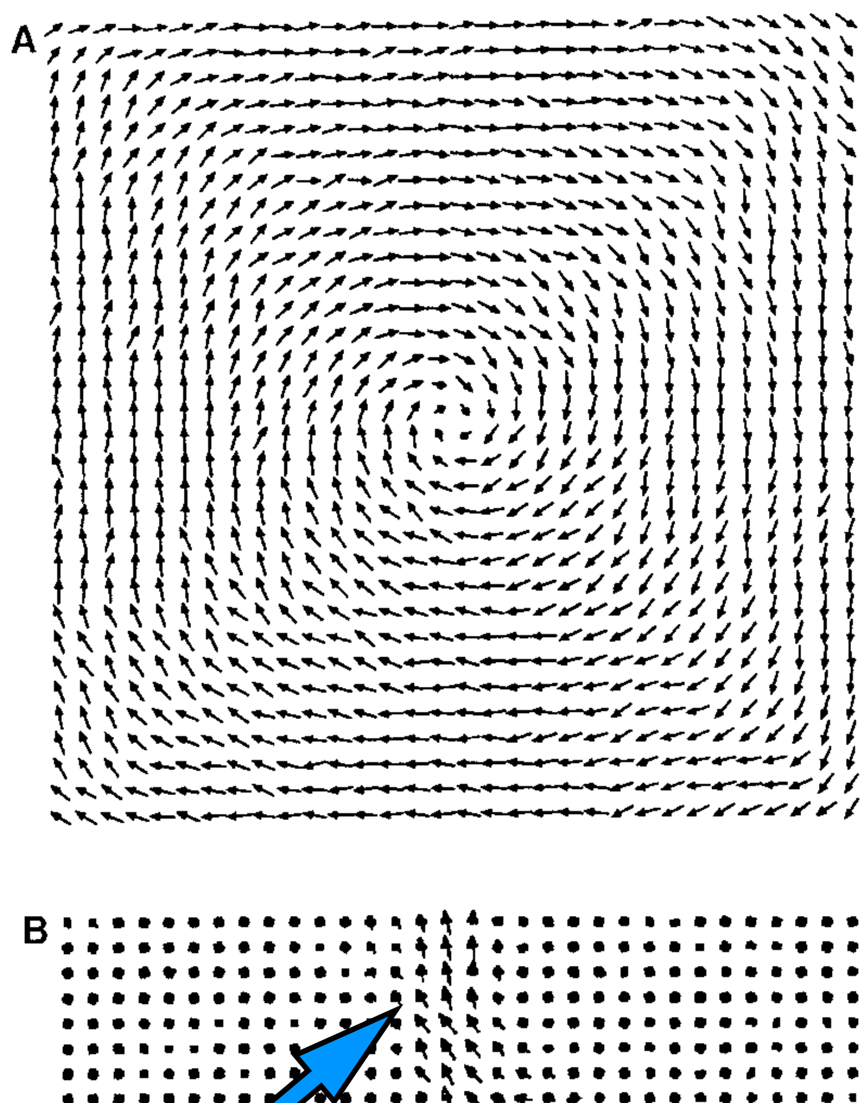
T. Shinjo,[1*] T. Okuno,[1] R. Hassdorf,[1†] K. Shigeto,[1] T. Ono[2]

930 11 AUGUST 2000 VOL 289 SCIENCE www.sciencemag.org

The vortex cores are visible.

Fig. 1. Monte Carlo simulation for a ferromagnetic Heisenberg spin structure comprising 32 × 32 × 8 spins [courtesy of Ohshima *et al.* (*2*)]. (**A**) Top surface layer. (**B**) Cross-section view through the center. Beside the center, the spins are oriented almost perpendicular to the drawing plane, jutting out of the plane to the right and into the plane to the left, respectively. These figures represent snapshots of the fluctuating spin structure and are therefore not symmetric with respect to the center. The structure should become symmetric by time averaging.

M(r)



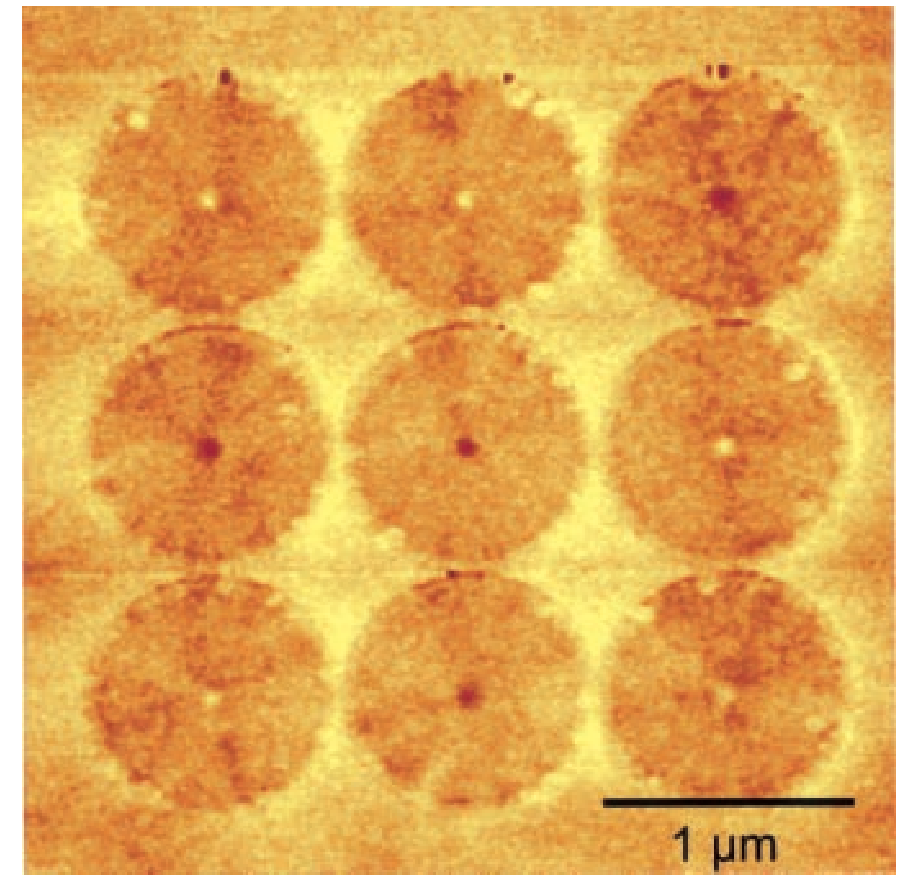


Fig. 2. MFM image of an array of permalloy dots 1 µm in diameter and 50 nm thick.

$M_z \approx \pm M_s$ in the core

We can see up/down Mz = polarity of the core!

Vortex control & switching?

How to control the position, circulation, and polarity of a magnetic vortex in a nanomagnet?

--voids or holes?
--applied fields, currents?
--optical impulses?

Theory: computer simulations of spin energetics & dynamics to study vortex motion and spin reversal.

053902-2 Vavassori *et al.*

JOURNAL OF APPLIED PHYSICS **99**, 053902 (2006)

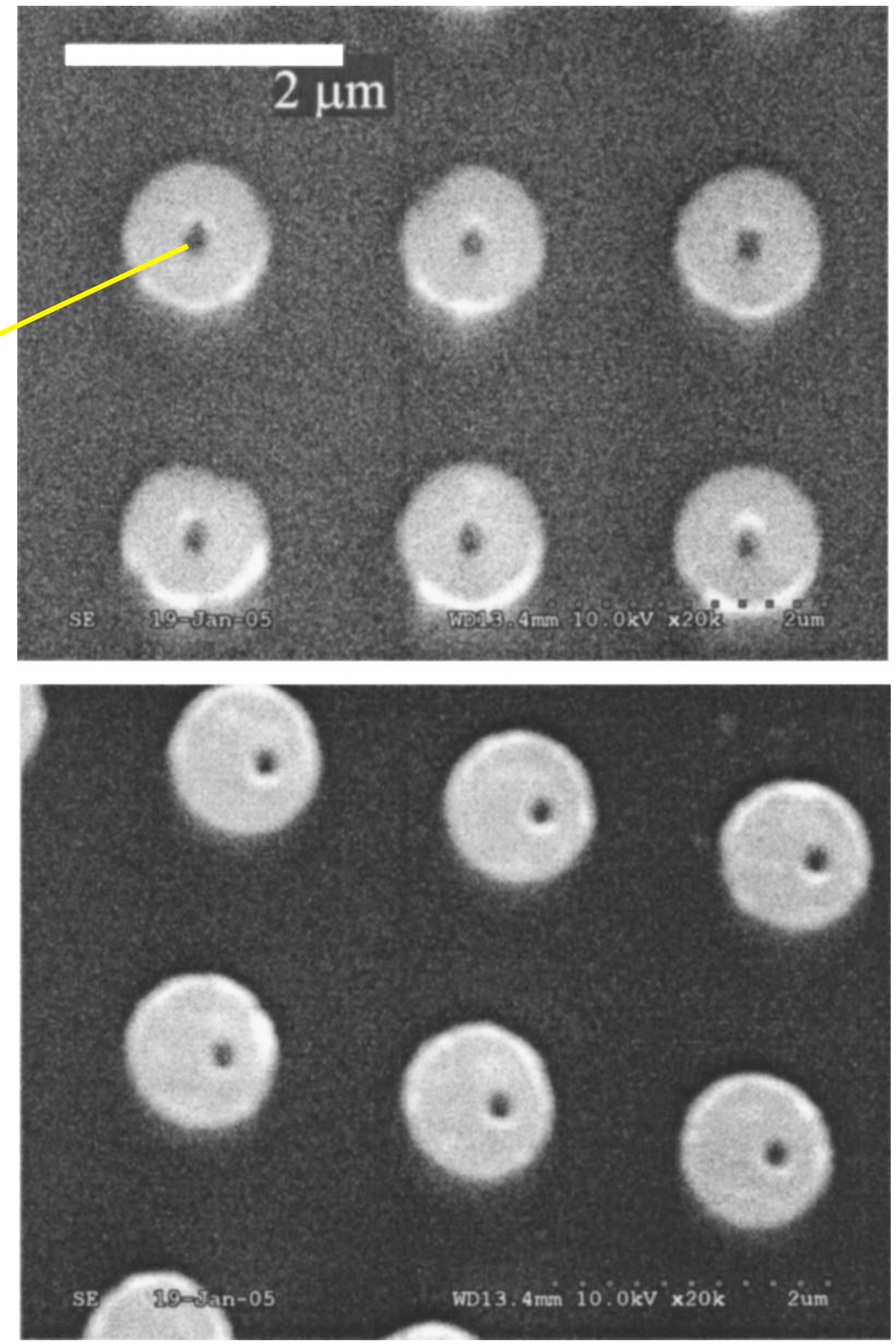
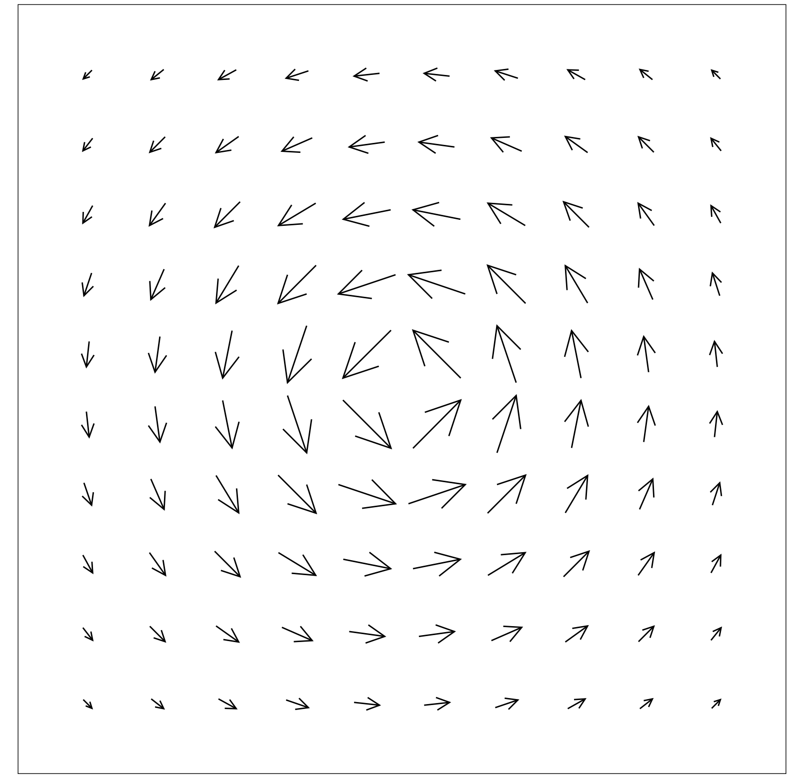


FIG. 1. Scanning electron images of a portion of the two patterns: symmetric rings (upper panel) and asymmetric rings (lower panel).

Vortices:
Particle-like properties



"vorticity charge"

$$q = \frac{1}{2\pi}\oint \vec{\nabla}\phi \cdot d\vec{r} = 0, \pm 1$$

circulation or *curling*

$-1 \leq C \leq +1$

$$C = \frac{1}{N}\sum_i \hat{\sigma}_i \cdot \hat{\phi}_i \qquad \hat{\sigma}_i = \vec{\mu}_i/\mu.$$

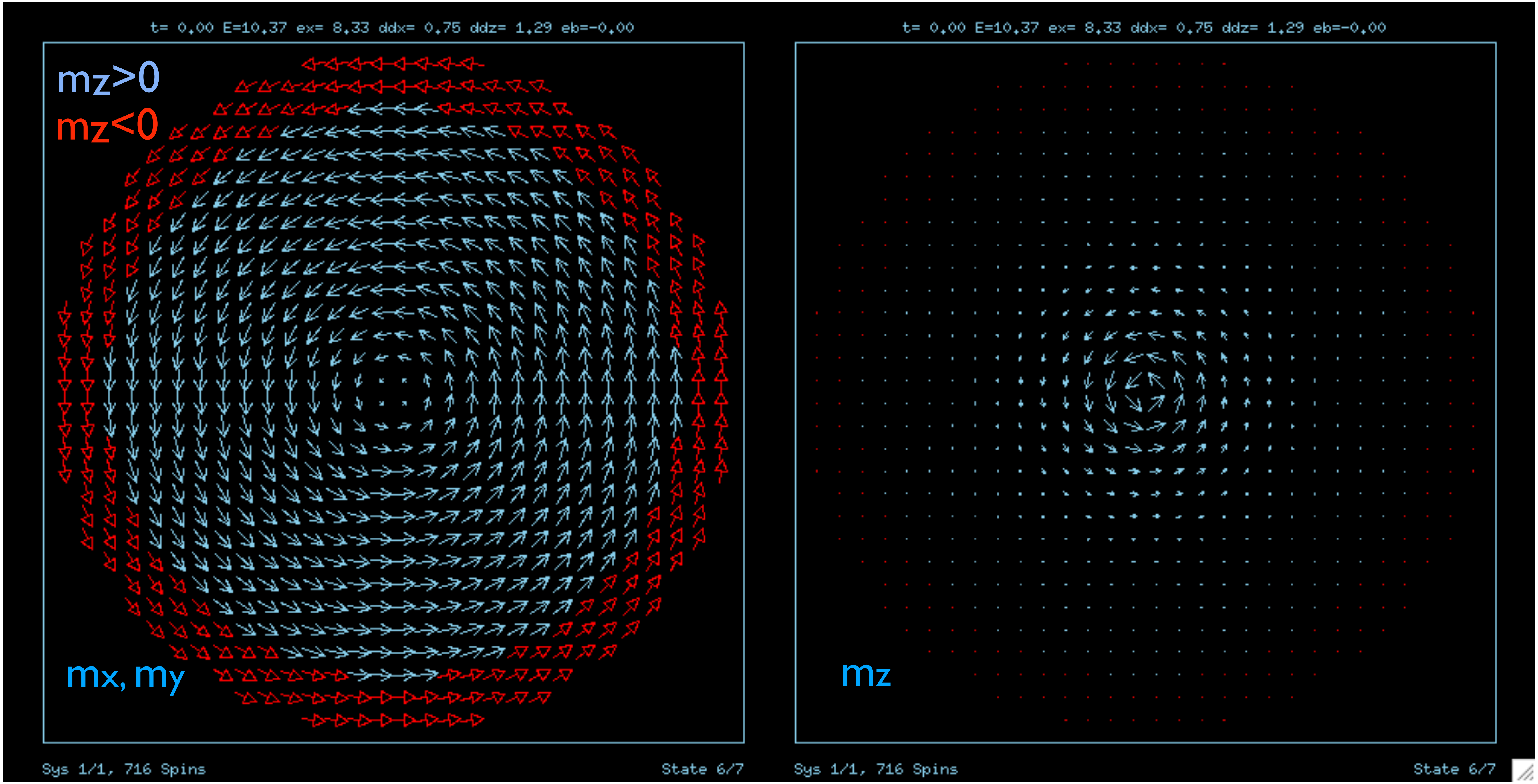
polarization

$p = m_z = \pm 1$ in the nucleus

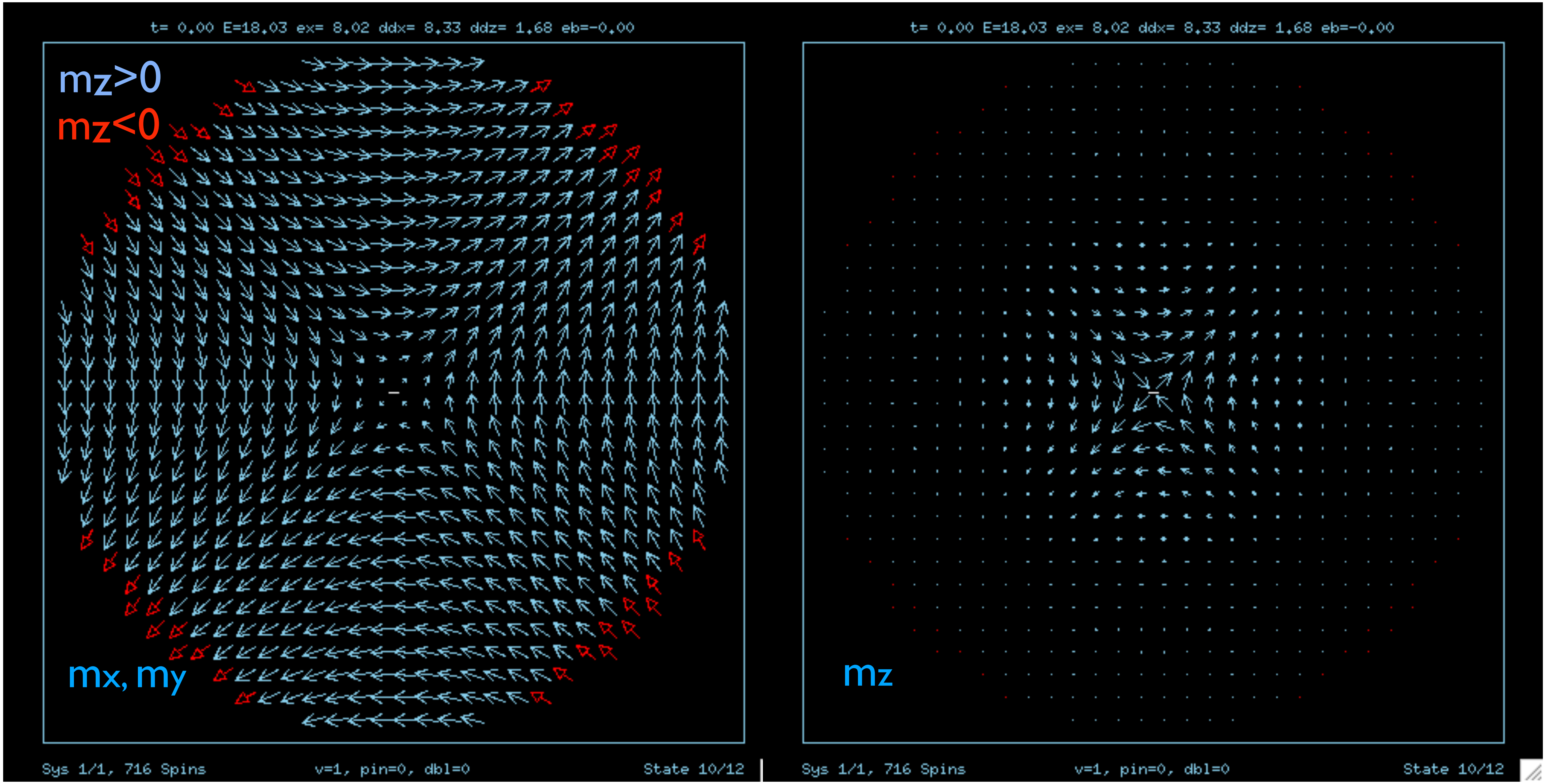
"topological charge
= gyrovector"

$G = 2\pi pq$ = solid angle mapped out by all the spins

Vortex, q=+1, p=+1
R=30nm, L=8nm



Anti-Vortex, q=-1, p=+1
R=30nm, L=8nm



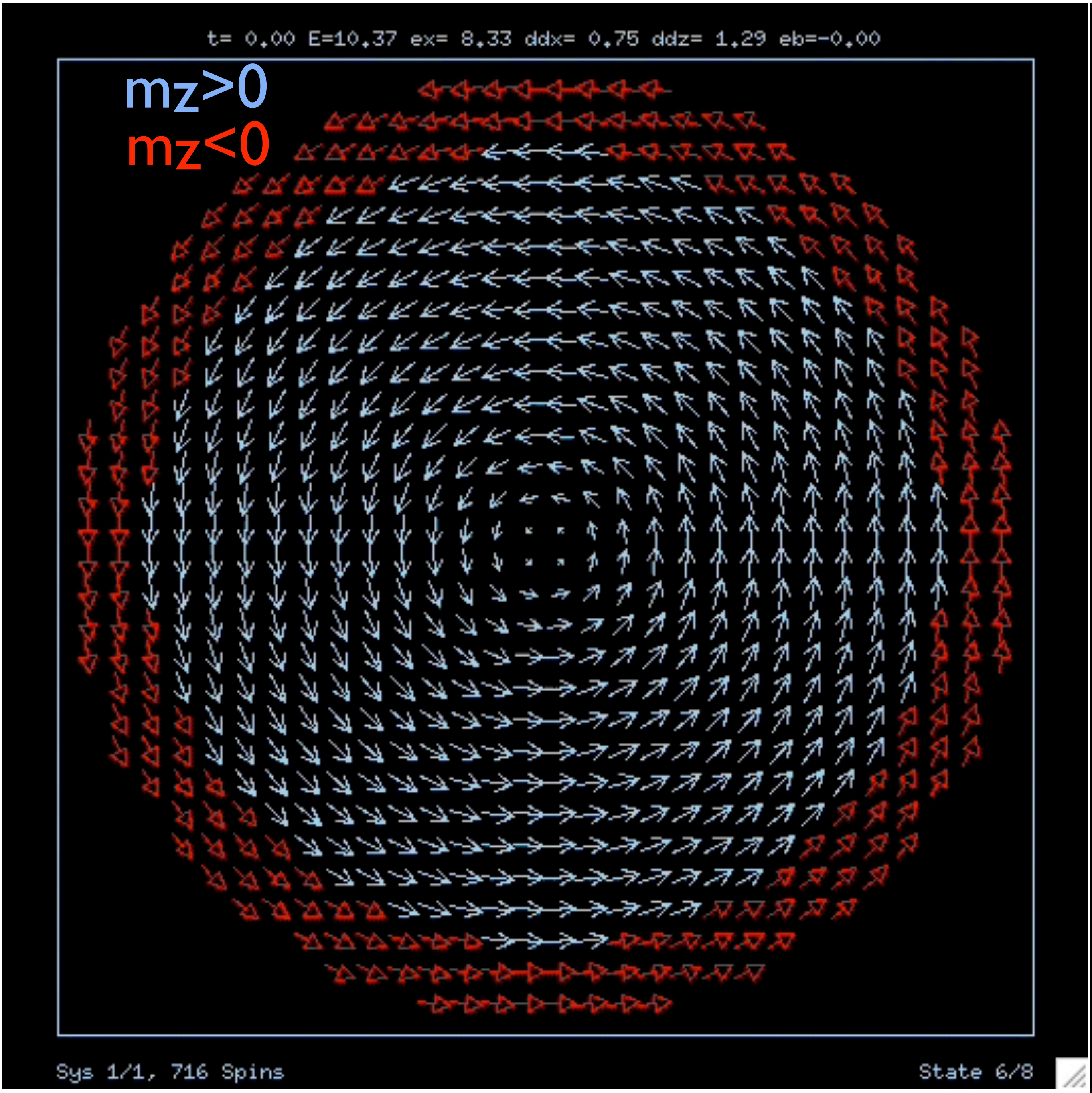
(Unstable)

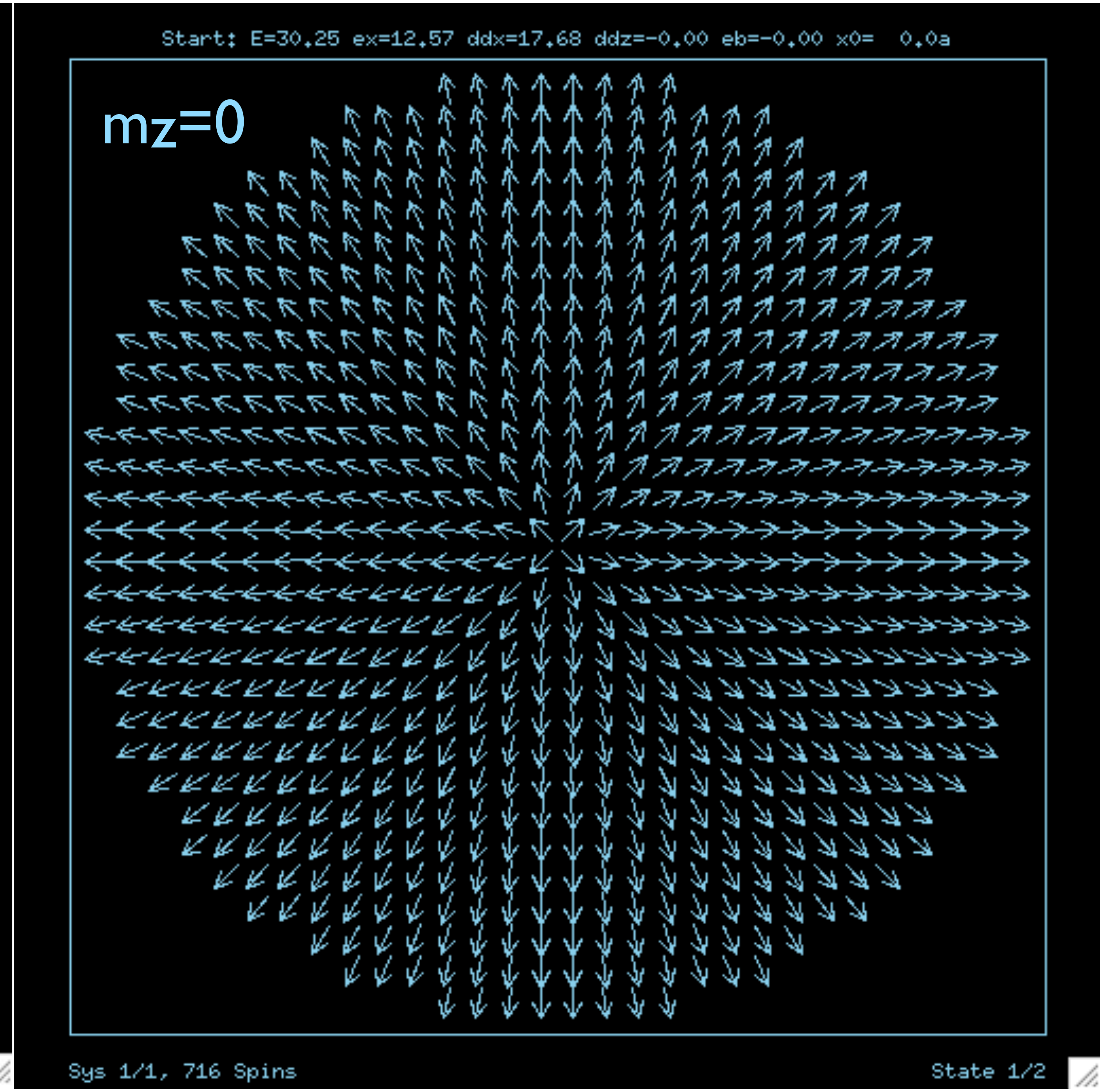
Bistability using the charges?

changing C=+1/-1

vorticity q=+1 does not change
energy E=10.37 does not change.

(unstable) planar vortex
state: E=30.25



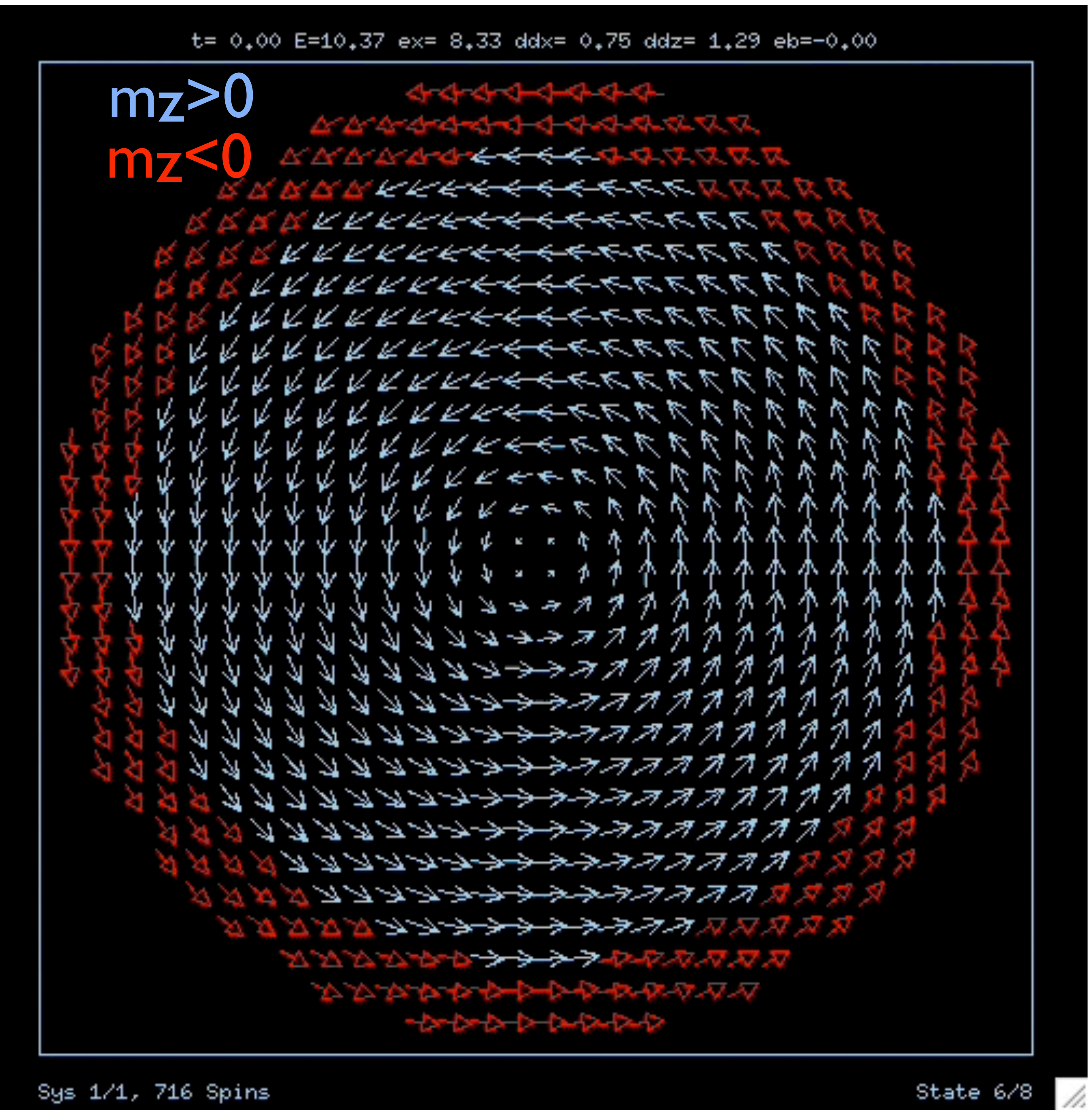


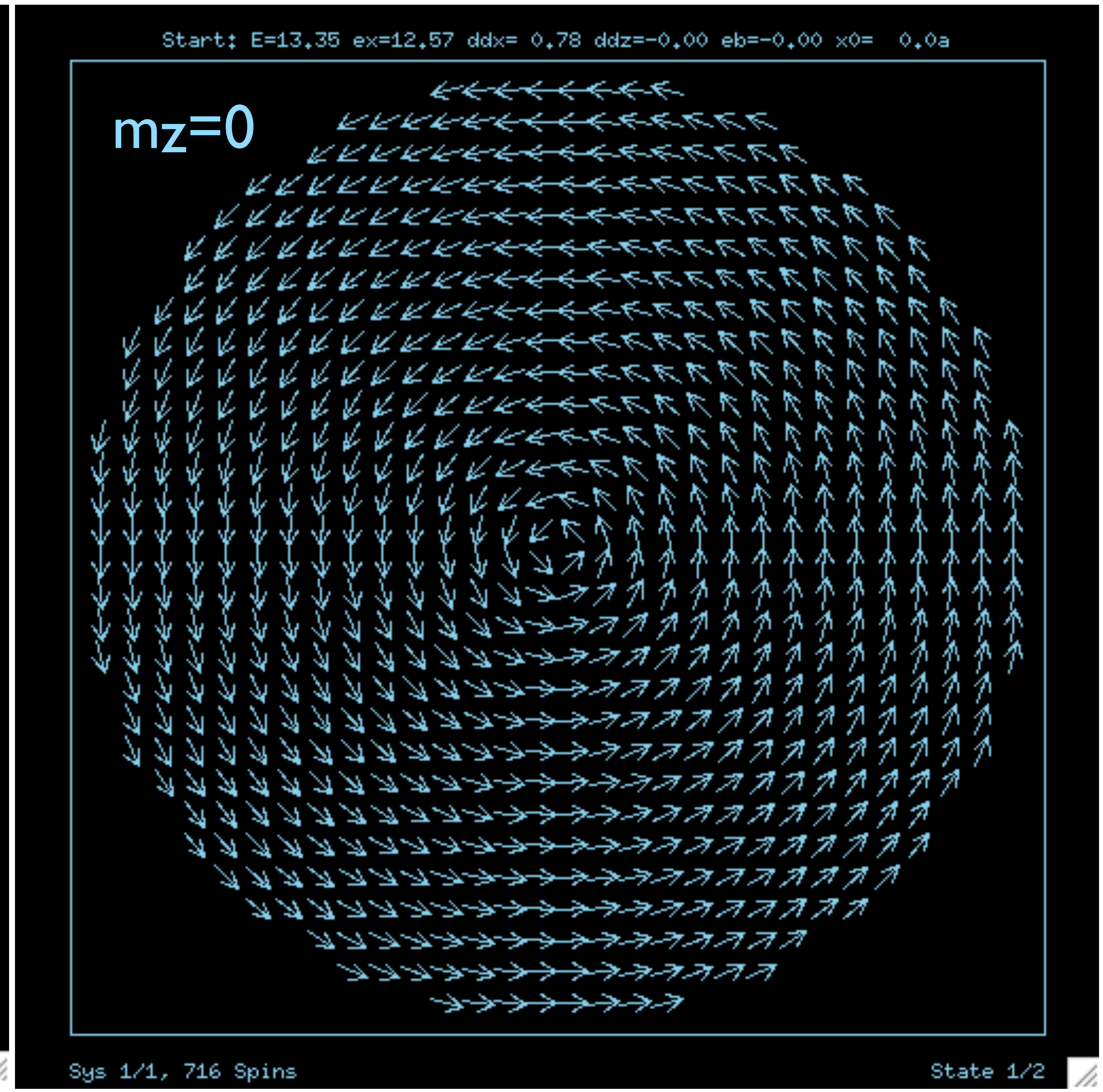
Bistability using the charges?

changing p=+1/-1

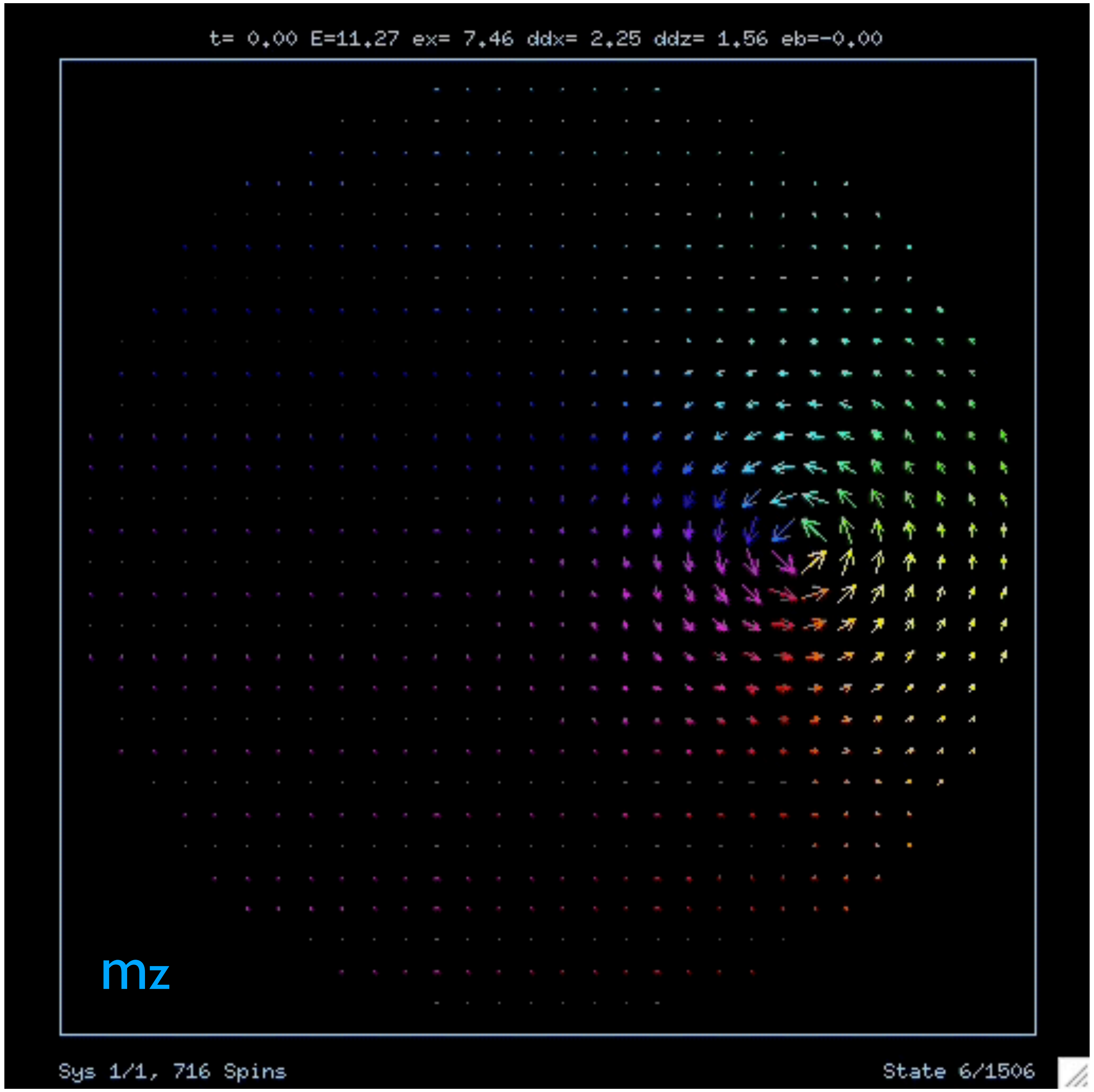
vorticity q=+1 does not change
energy E=10.37 does not change.

(unstable) planar vortex
state: E=13.35





Gyrotropic movement



R=30 nm,
L=10 nm,
cells a=2.0 nm

Vortex,
q=+1, p=+1

The arrows are proportional to Mz, out of the plane.

Atomic theory. Model for interacting atomic dipoles.

Hamiltonian: $H = H_{ex} + H_{dd} + H_B$

$$\mu_{\text{atom}} = g\mu_B S$$

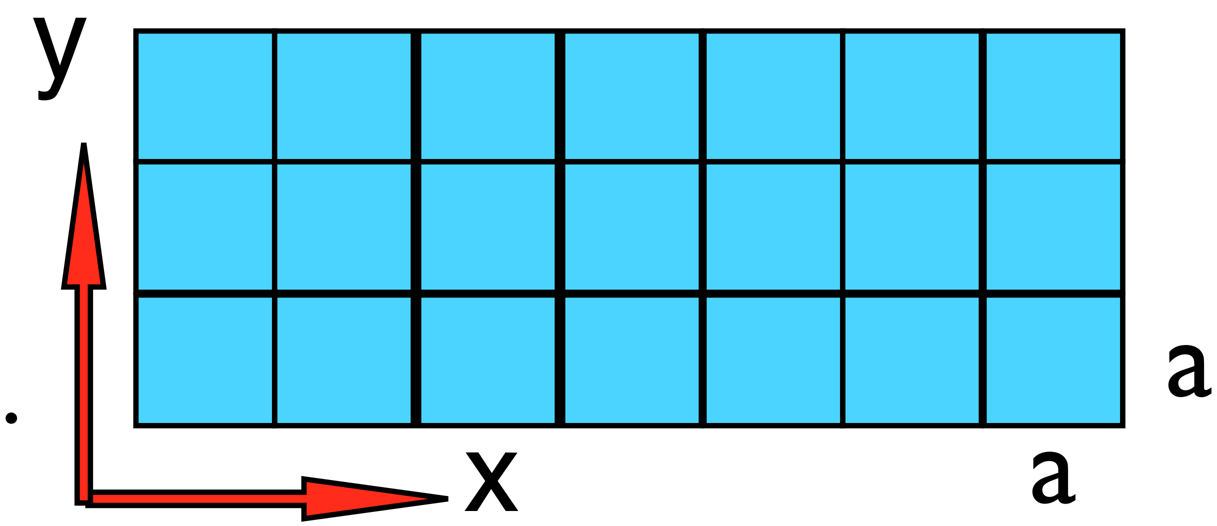
exchange: $$H_{\text{ex}} = -J \sum_{(i,j)} \vec{S}_i \cdot \vec{S}_j$$

dipole-dipole: $$H_{\text{dd}} = -\left(\frac{\mu_0}{4\pi}\right) \sum_{i>j} \frac{[3(\vec{\mu}_i \cdot \hat{r}_{ij})(\vec{\mu}_j \cdot \hat{r}_{ij}) - \vec{\mu}_i \cdot \vec{\mu}_j]}{r_{ij}^3},$$

applied field: $$H_{\text{B}} = -\sum_i \vec{B} \cdot \vec{\mu}_i$$

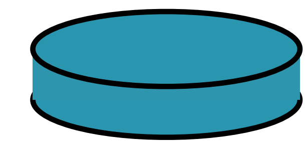
Problem: Too many atoms to calculate in a typical nanodot.

Micromagnetics.
A technique for studying a continuous system.



Each cell contains a magnetic dipole:

$$\hat{m} = \vec{M}/M_S.$$



- Model for a cylindrical nanodot, radius R, height L.

- Divide the sample into cells of size a x a x L.

- Assume that the magnetization is saturated (M_S) inside each cell: |m|=1. Only the directions vary between cells.

- The cells interact as dipoles, with exchange energy between neighbors & with the demagnetization field.

Hamiltonian: $H = H_{ex} + H_{demag} + H_B$

exchange: $$\mathcal{H}_{\rm ex} = A \int dV \; \nabla \hat{m} \cdot \nabla \hat{m},$$

demagnetization: $$\mathcal{H}_{\rm dd} = \mathcal{H}_{\rm demag} = -\frac{1}{2}\mu_0 \int \; dV \; \vec{H}_M \cdot \vec{M}$$

applied field: $$\mathcal{H}_B = -\mu_0 \int \; dV \; \vec{H}_{\rm ext} \cdot \vec{M}$$

Statics: minimize the energy ⇒ stable configurations.
Dynamics: equation of motion ⇒ periodic configurations.

Difficulties: (i) Calculating the desmagnetization field H_M;
(ii) Enforcing a desired position, X, of the vortex⇒E(X).

Scale energies by the exchange between cells:

$$J_{\text{cell}} = \frac{2Av_{\text{cell}}}{a^2} = 2AL.$$

"magnetic exchange length"

$$\lambda_{\text{ex}} = \sqrt{\frac{2A}{\mu_0 M_S^2}}$$

demag. field:

$\vec{H}_M = M_S \tilde{H}_M$

Hamiltonian on the grid of cells:

$$\mathcal{H}_{\text{mm}} = -J_{\text{cell}} \left\{ \sum_{(i,j)} \hat{m}_i \cdot \hat{m}_j + \left(\frac{a}{\lambda_{\text{ex}}}\right)^2 \sum_i \left(\tilde{H}_{\text{ext}} + \frac{1}{2}\tilde{H}_M \right) \cdot \hat{m}_i \right\}$$

Need $\left(\frac{a}{\lambda_{\text{ex}}}\right)^2$ less than 1 for reliable solutions.

(cells smaller than exchange length)

Finding the demagnetization field via Green/FFT approach.

➡ The magnetostatics problem has no free currents:

$$-\tilde{\nabla}^2\tilde{\Phi} = \tilde{\rho} \qquad \tilde{\rho} \equiv -\tilde{\nabla}\cdot\hat{m} \qquad \tilde{H}_M = -\tilde{\nabla}\tilde{\Phi}$$

use Green's function solution:

$$\tilde{\Phi}(\vec{r}) = \int d^3r'\ G(\vec{r},\vec{r}\,')\,\tilde{\rho}(\vec{r}\,') \qquad G(\vec{r},\vec{r}\,') = \frac{1}{4\pi|\vec{r}-\vec{r}\,'|}$$

specialize to a thin cylinder (2D) geometry: $\tilde{r} \equiv (x,y)$

$$\tilde{H}_z(\tilde{r}) = \int d^2\tilde{r}\,'\ G_z(\tilde{r}-\tilde{r}\,')\ m_z(\tilde{r}\,')$$

$$\tilde{H}_{xy}(\tilde{r}) = \int d^2\tilde{r}\,'\ \vec{G}_{xy}(\tilde{r}-\tilde{r}\,')\ \tilde{\rho}(\tilde{r}\,')$$

$$G_z(\tilde{r}) = \frac{1}{2\pi L}\left[\frac{1}{\sqrt{\tilde{r}^2+L^2}} - \frac{1}{|\tilde{r}|}\right]$$

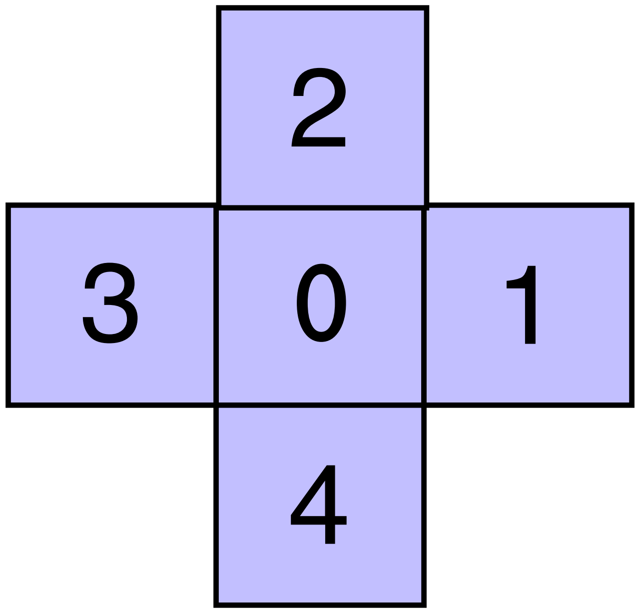
$$\vec{G}_{xy}(\tilde{r}) = \frac{1}{2\pi L}\left[\sqrt{1+\left(\frac{L}{\tilde{r}}\right)^2} - 1\right]\hat{e}_{\tilde{r}}$$

Some details.

The magnetic charge densities depend on the present magnetic configuration, such as:

$$\tilde{\rho}_0^{\mathrm{vol}} = \frac{q_M^{\mathrm{vol}}}{La^2} = -\frac{1}{2a}\left[m_1^x - m_3^x + m_2^y - m_4^y\right]$$

$$\tilde{\rho}_0^{\mathrm{sur}} = \frac{q_M^{\mathrm{sur}}}{La^2} = \sum_{\text{cell edges}} \frac{1}{2a}\hat{m}_0 \cdot \hat{n}_{\mathrm{edge}}$$



Convolutions are evaluated using fast fourier transforms.

Use zero padding to avoid the wrap-around problem:
FFT grid is 2X larger than original system to avoid false copies.

The solution for demagnetization field is that for a disk isolated from others.

How to minimize the energy for a vortex in a desired location? → Use Lagrange undetermined multipliers technique.

Energy functional:

$$\Lambda[\vec{m}_i] = E[\vec{m}_i] + \sum_i \alpha_i(\vec{m}_i^2 - m^2) - \vec{\lambda}\cdot\sum_n^{\text{core}} \vec{m}_n$$

hamiltonian

length constraints

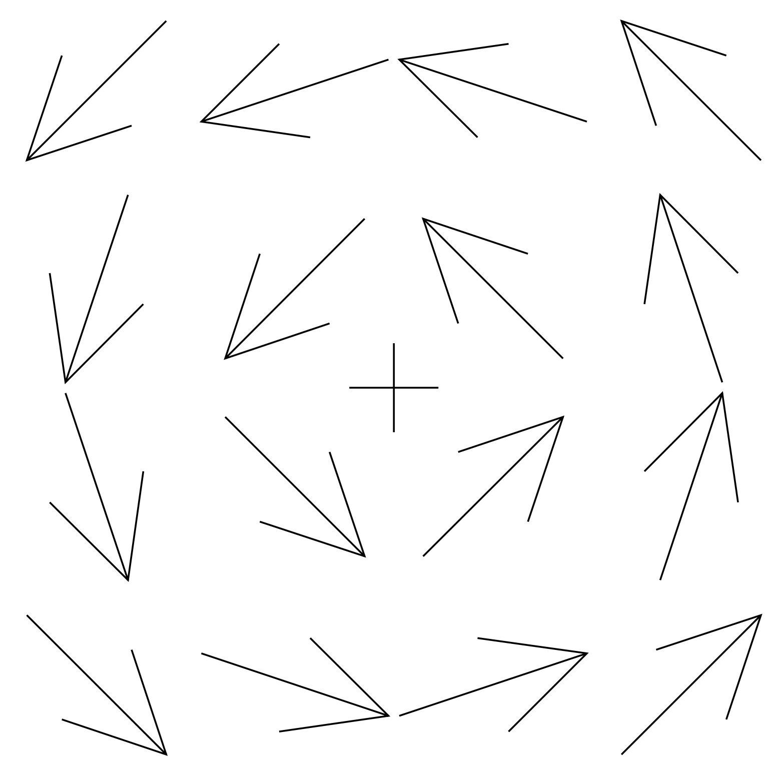
vortex position constraint

$$\frac{\partial \Lambda}{\partial m_n^x} = \frac{\partial E}{\partial m_n^x} + 2\alpha_n m_n^x - \lambda_x = 0$$

$$-F_n^x + 2\alpha_n m_n^x - \lambda_x = 0$$

in core:

$$m_n^x = \frac{1}{2\alpha_n}(F_n^x + \lambda_x)$$



But need to get α and λ by applying the constraints.

Iterations . . .

$$\vec{m}_n^2 = \frac{1}{4\alpha_n^2}\left[(F_n^x+\lambda_x)^2+(F_n^y+\lambda_y)^2+(F_n^z)^2\right] = m^2$$

A. $$\frac{1}{\alpha_n} = \frac{2m}{\sqrt{(F_n^x+\lambda_x)^2+(F_n^y+\lambda_y)^2+(F_n^z)^2}}$$

(length constraints)

B. $$\sum_{\text{core}} m_n^x = \sum_{\text{core}} \frac{1}{2\alpha_n}(F_n^x+\lambda_x) = 0 \longrightarrow \lambda_x = -\frac{\sum_{\text{core}} F_n^x/\alpha_n}{\sum_{\text{core}} 1/\alpha_n}$$

(vortex position constraint)

Iterate, placing each dipole along its effective field:

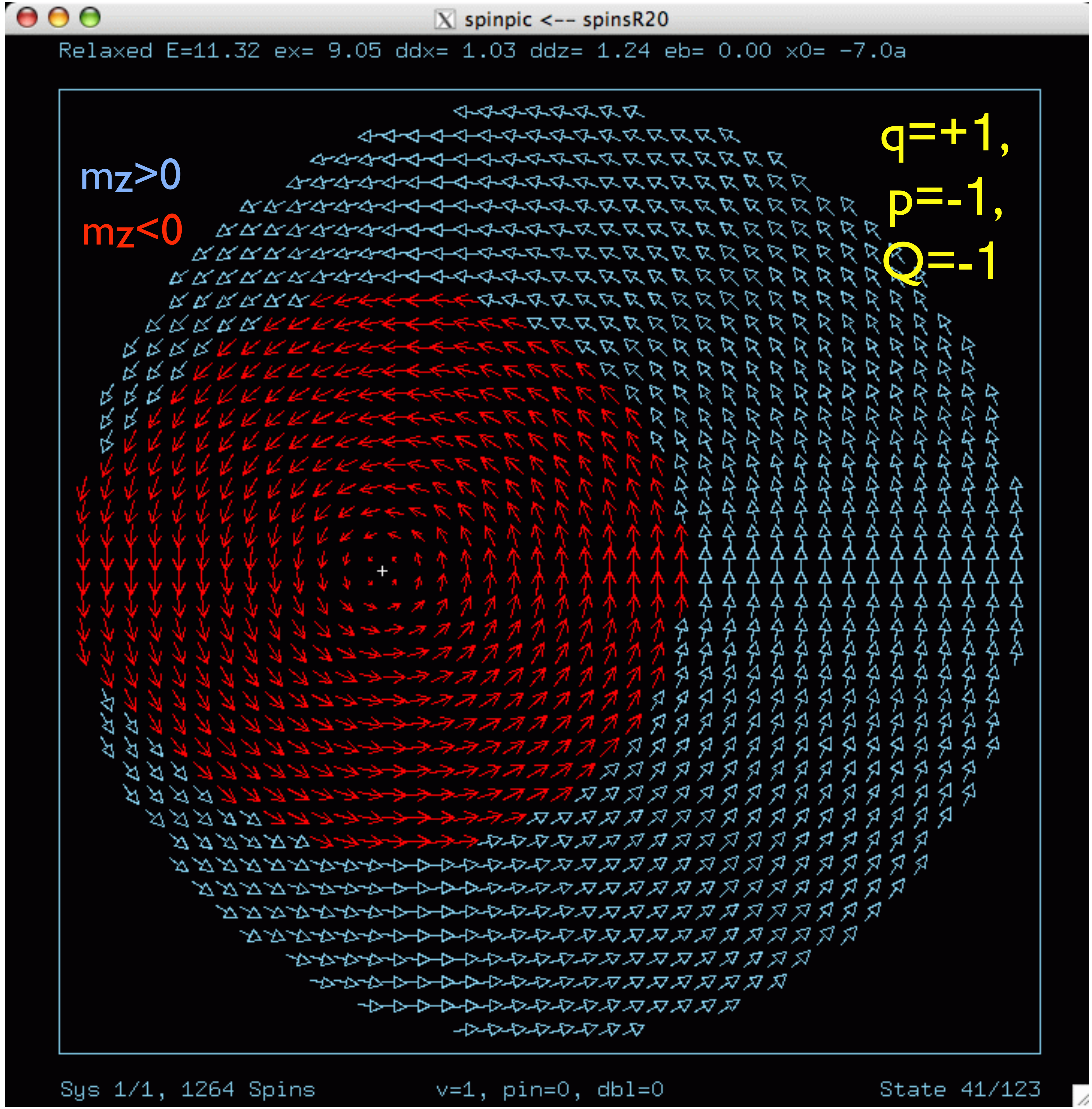
C. $$\vec{m}_n = m\frac{(F_n^x+\lambda_x)\hat{x}+(F_n^y+\lambda_y)\hat{y}+F_n^z\hat{z}}{\sqrt{(F_n^x+\lambda_x)^2+(F_n^y+\lambda_y)^2+(F_n^z)^2}}$$

(not using Landau-Lifshitz dynamic equations)

Example.
Typical vortex configuration.

$a=2.0$ nm,
$\lambda_{ex}=5.3$ nm,
$L=12$ nm,
$R=40$ nm,

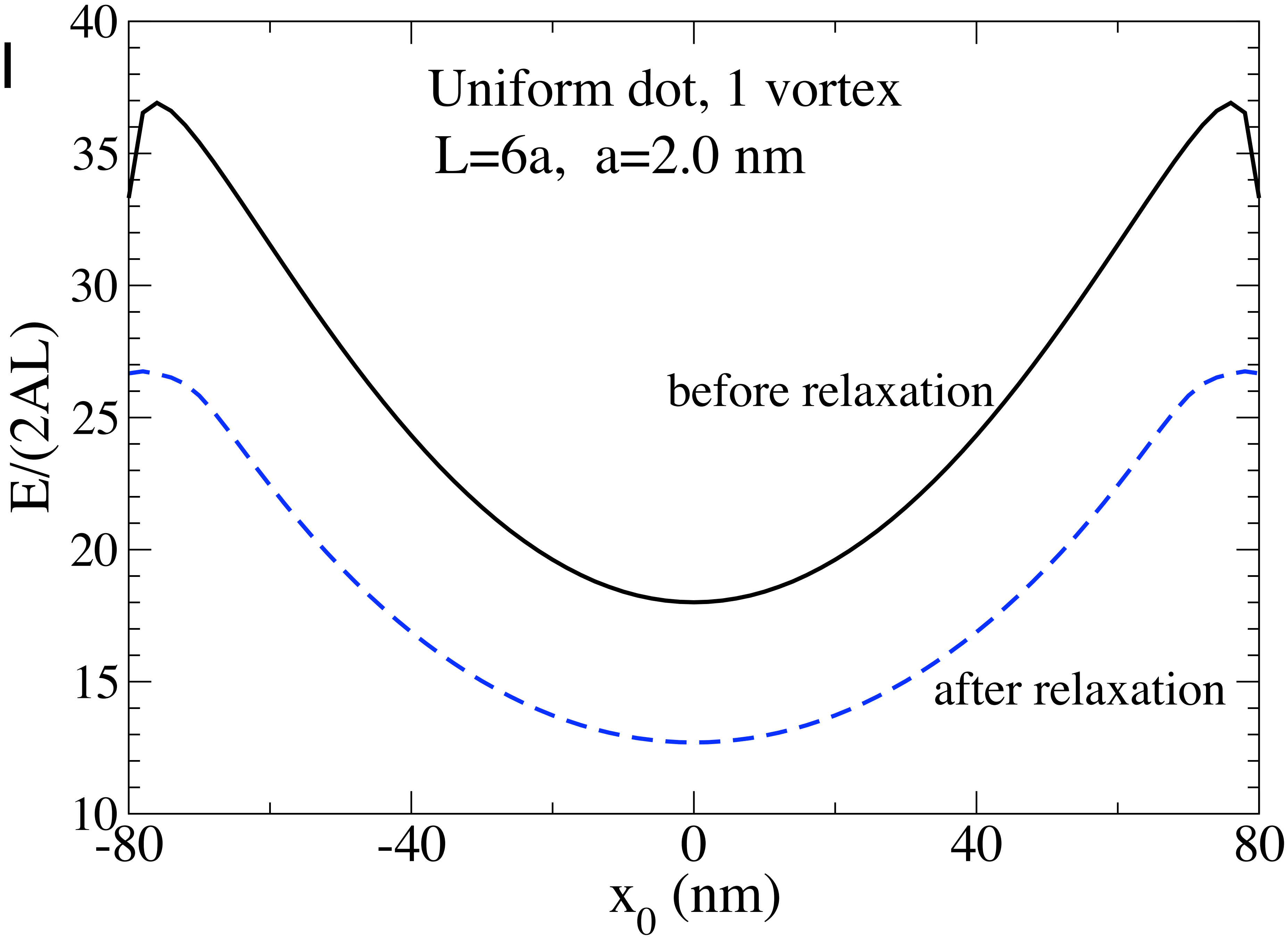
$x_0=-7a$



Example. Total energy of a vortex, $E(x_0) \approx \frac{1}{2} k_F x_0^2$

a=2.0 nm, λ_{ex}=5.3 nm, L=12 nm, R=80 nm

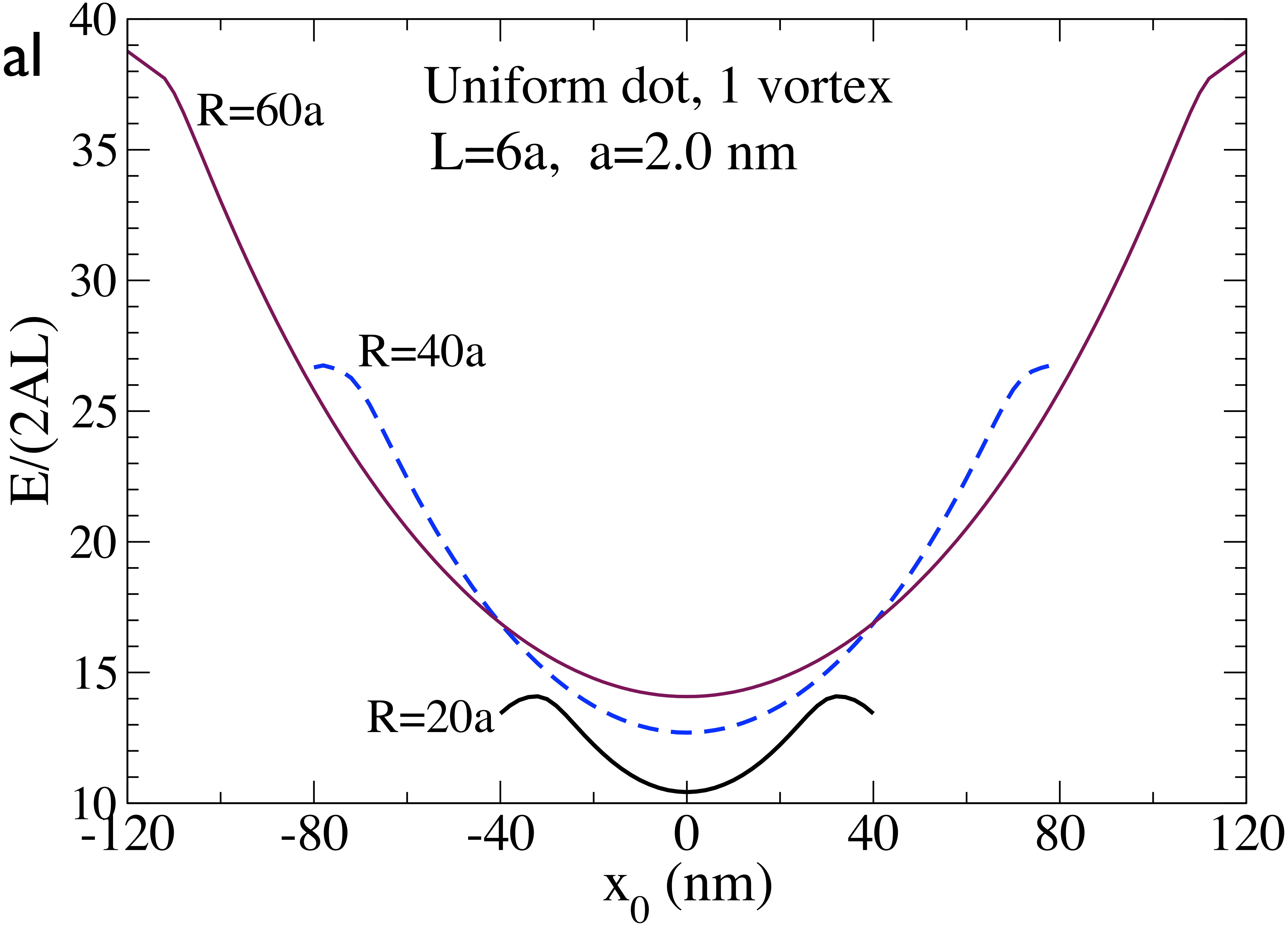
Potential $E(x_0)$



Example. Total energy of a vortex, $E(x_0) \approx \frac{1}{2} k_F x_0^2$

a=2.0 nm, λ_{ex}=5.3 nm, L=12 nm, R=40, 80, 120 nm

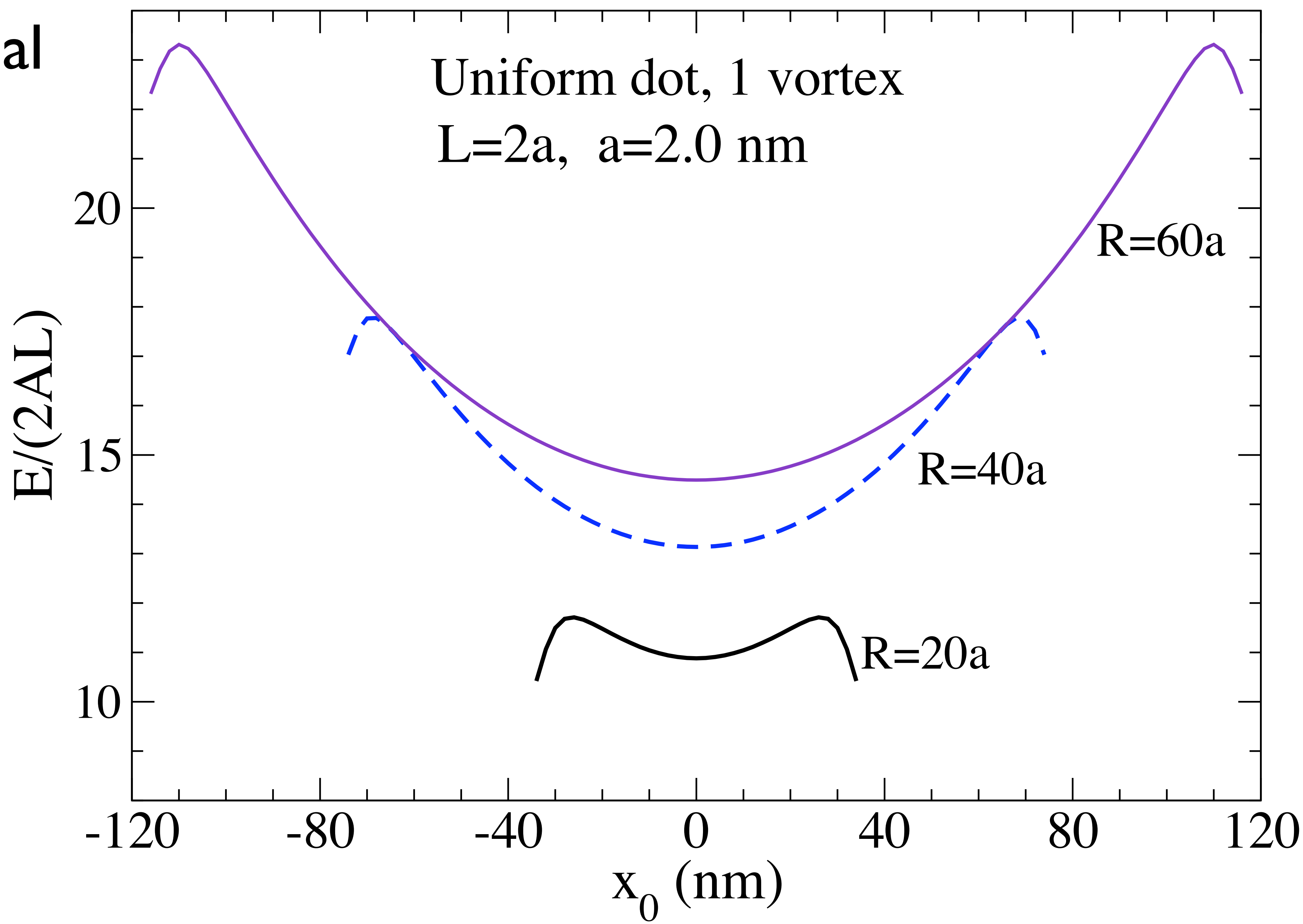
Potential $E(x_0)$



Example. Total energy of a vortex, $E(x_0) \approx \frac{1}{2} k_F x_0^2$

$a=2.0$ nm, $\lambda_{ex}=5.3$ nm, $L=4.0$ nm, $R=40, 80, 120$ nm

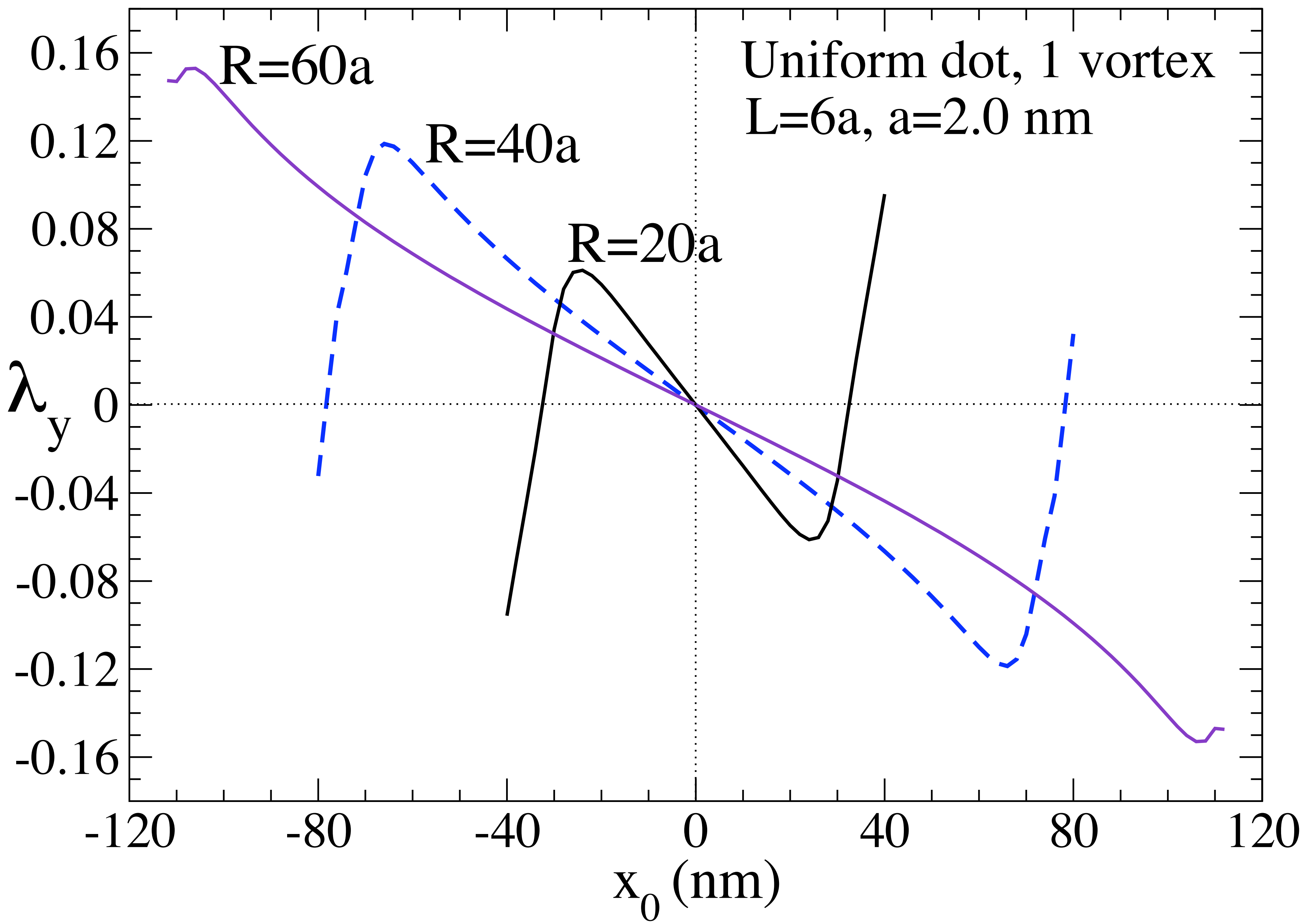
Potential $E(x_0)$



softer potentials

Example. Vortex constraint field $\lambda=(0,\lambda_y)$

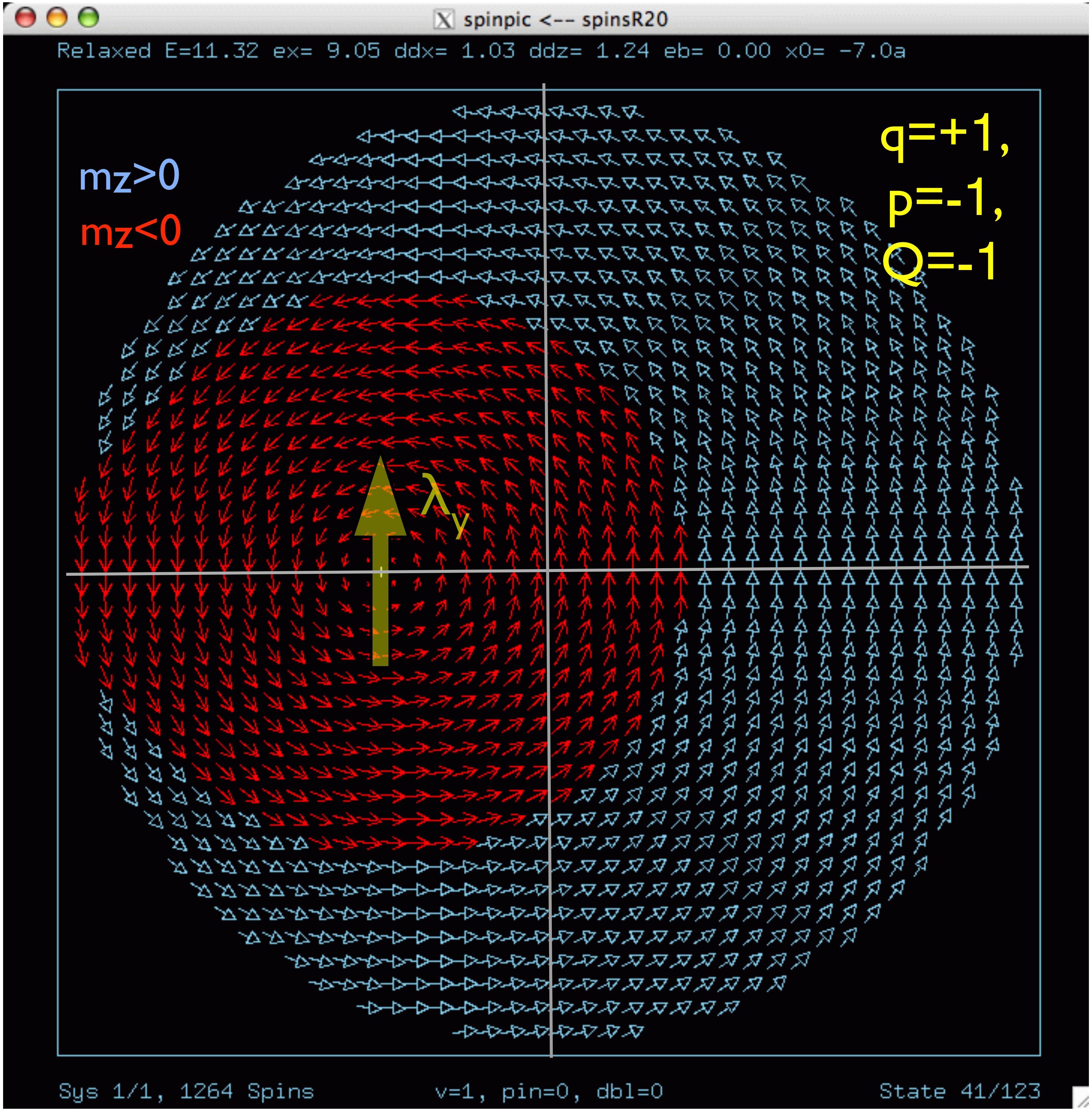
a=2.0 nm, λ_{ex}=5.3 nm, L=12 nm, R=40, 80, 120 nm



Example.
Typical vortex configuation.

$a=2.0$ nm,
$\lambda_{ex}=5.3$ nm,
$L=12$ nm,
$R=40$ nm,

$x_0=-7a$



Vortex Potentials E(X)

Using a modified micromagnetics for thin systems, the demagnetization field H_M is found using FFTs to evaluate the convolutions of M with the Green's functions for quasi-2D nanodots.

A magnetic constraint field (λ_x,λ_y) in the vortex core is used in Lagrange's method of undetermined multipliers, to enforce a desired vortex position X.

In this way it is possible to determine the effective potential E(X) for a vortex inside a nanodot, which could be useful in the study of their dynamics.

Next, we shall see what this has to do with the dynamics ⇒⇒⇒

Lagoinha
Brava
Canasvieiras
Ingleses
Jurerê
Daniela
Santinho
Santo Antonio de Lisboa
Cacupé
Moçambique
Barra da Lagoa
Lagoa da Conceição
Galheta
Praia Mole
Joaquina
Campeche
Ilha do Campeche
Morro das Pedras
Ribeirão da Ilha
Armação
Matadeiro
Lagoinha do Leste
Pântano do Sul
Naufragados
Florianópolis
Ilha de Santa Catarina
OCEANO ATLÂNTICO

About Dynamics:

a cell has a magnetic dipole = $\vec{\mu}_i = La^2 M_s \hat{m}_i$

$$\frac{d\vec{\mu}_i}{dt} = \gamma \vec{\mu}_i \times \vec{B}_i.$$

$$\vec{B}_i = -\frac{\delta \mathcal{H}}{\delta \vec{\mu}_i} = \frac{J_{\text{cell}}}{La^2 M_s} \vec{b}_i$$

$$\frac{d\hat{m}_i}{d\tau} = \hat{m}_i \times \vec{b}_i,$$

$$\tau = \gamma B_0 t.$$

$$\vec{b}_i \equiv \sum_{\text{nbrs}} \hat{m}_j + \frac{a^2}{\lambda_{\text{ex}}^2} \left(\tilde{H}^{\text{ext}} + \tilde{H}^M \right)$$

$$B_0 \equiv \frac{2A}{a^2 M_s}$$

$t_0 = (\gamma B_0)^{-1}$ ≈ 1.5 ps for Permalloy

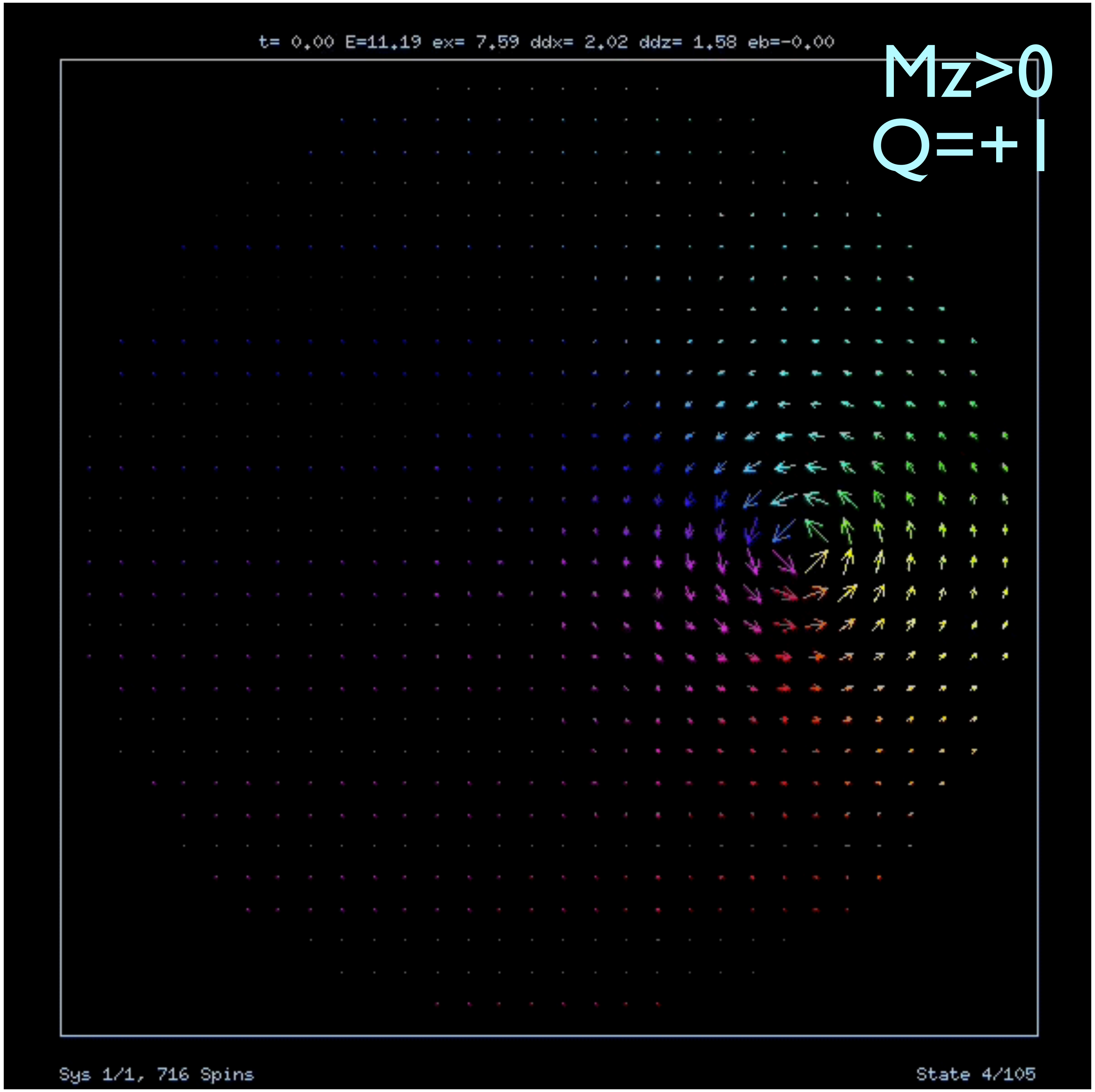
This defines the dynamics at temperature T=0.

We can integrate with fourth order Runge-Kutta.

$$\frac{d\hat{m}_i}{d\tau} = \hat{m} \times \vec{b}_i - \alpha \hat{m} \times \left(\hat{m} \times \vec{b}_i \right)$$

← (if damping is present)

Gyrotropic movement



R=30 nm,
L=5 nm,
cells a=2.0 nm
α=0.02

gyrovector:

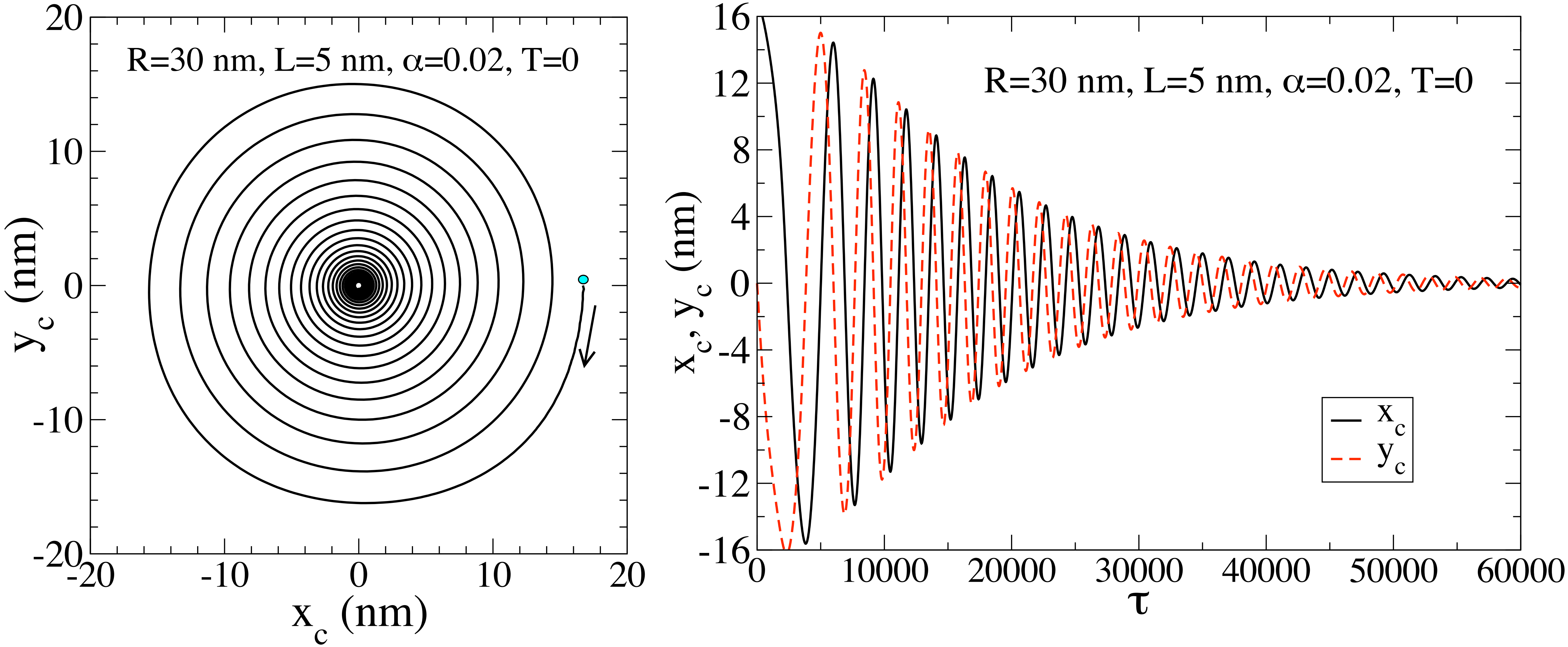
$$\mathbf{G} = 2\pi Q \hat{z}$$

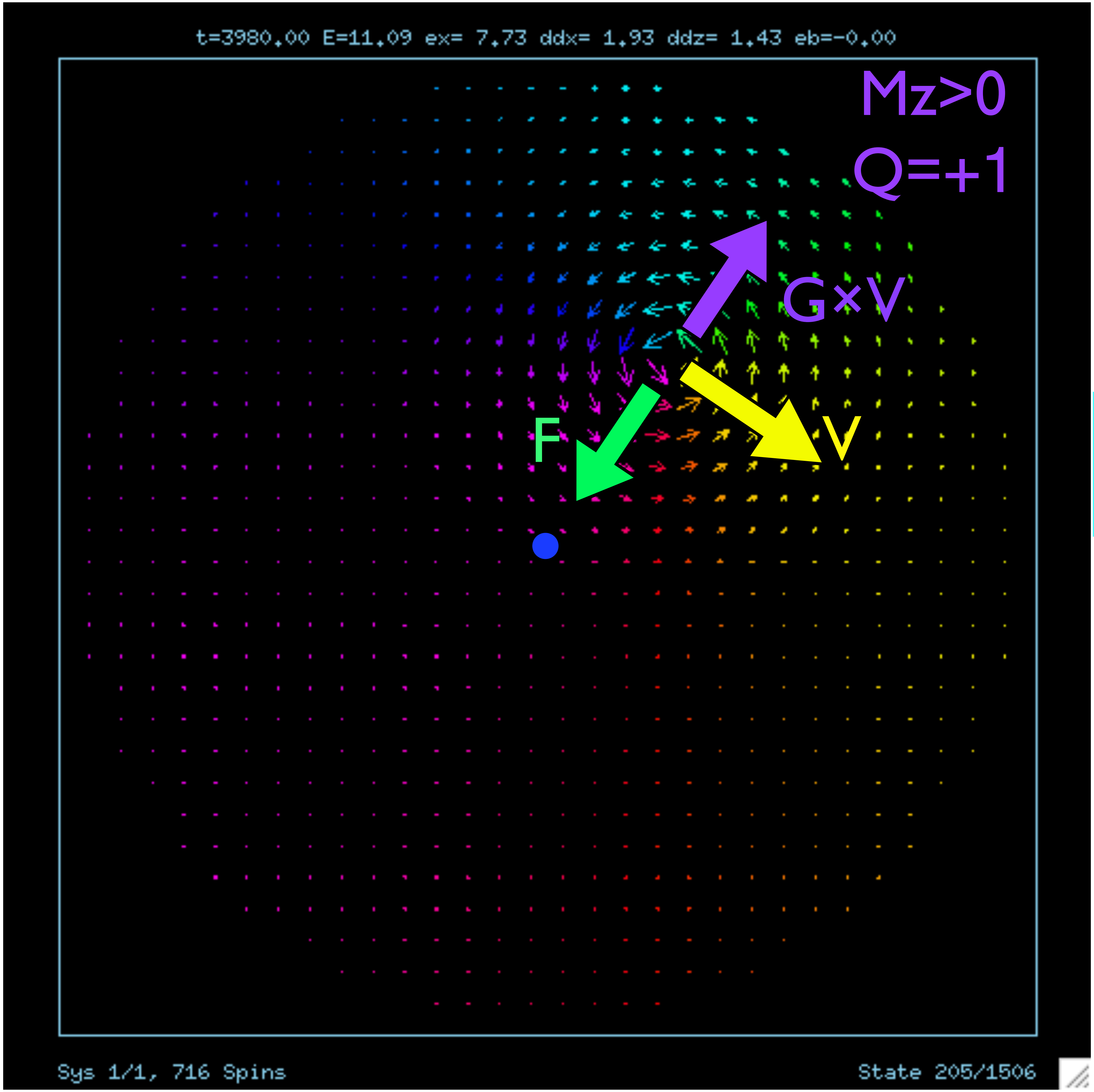
$$Q = \pm 1$$

$$\frac{\gamma}{m_0}\mathbf{F} + \mathbf{G} \times \mathbf{V} = 0.$$

$$m_0 = \mu/a^2 = L M_s$$

Position of the vortex core:





gyrovector:

$$\mathbf{G} = 2\pi Q \hat{z}$$

Q=qp=+1

$$\frac{\gamma}{m_0}\mathbf{F} + \mathbf{G} \times \mathbf{V} = 0.$$

$$m_0 = \mu/a^2 = LM_s$$

Thiele equation:

$$\frac{\gamma}{m_0}\mathbf{F} + \mathbf{G} \times \mathbf{V} = 0.$$

A central force:

$$\mathbf{F} = -k_F r \hat{r} = -k_F X$$

Solution (for α=0): circular movement of the core:

$$\mathbf{V} = \frac{\gamma}{GLM_s}\hat{z} \times \mathbf{F} = -\frac{\gamma k_F r}{2\pi QLM_s}\hat{\phi}$$

Frequency of the gyrotropic movement:

$$\omega_G = \frac{V}{r} = -\frac{\gamma k_F}{2\pi QLM_s}$$

$$f_G = \omega_G / 2\pi$$

The important part is k_F/L.

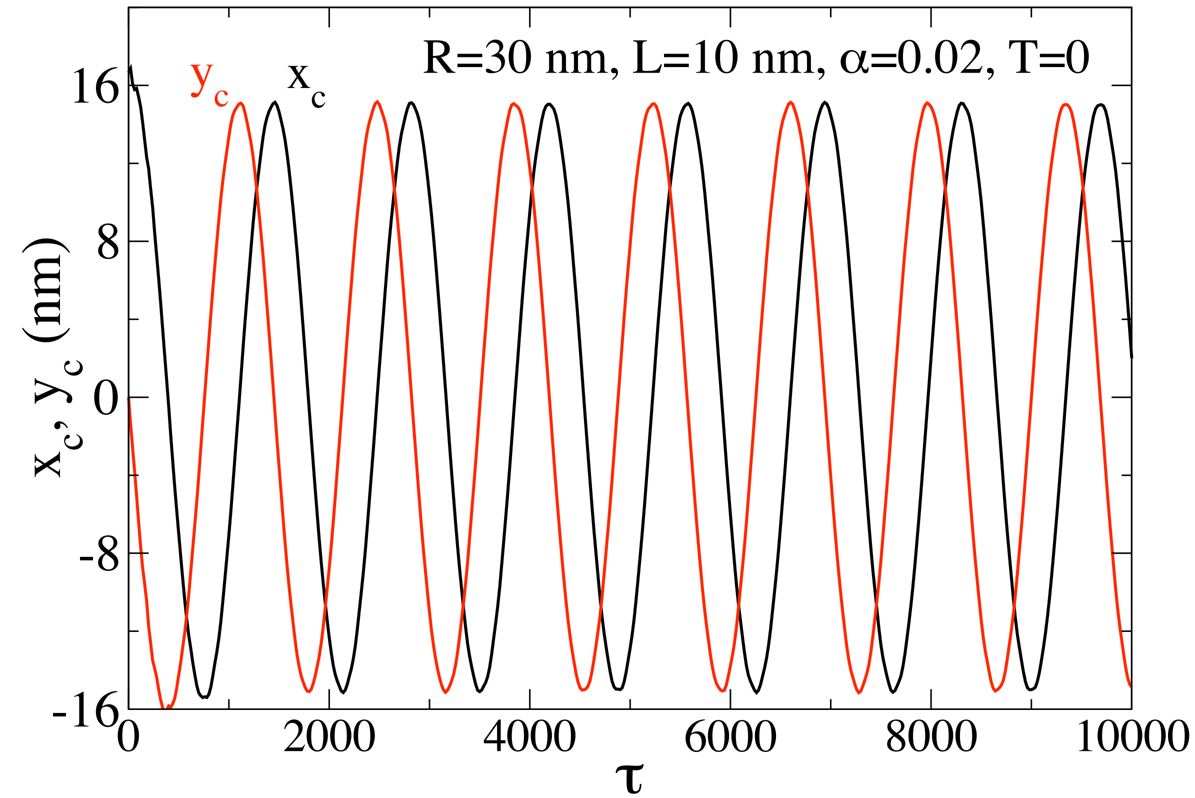
Simulations:

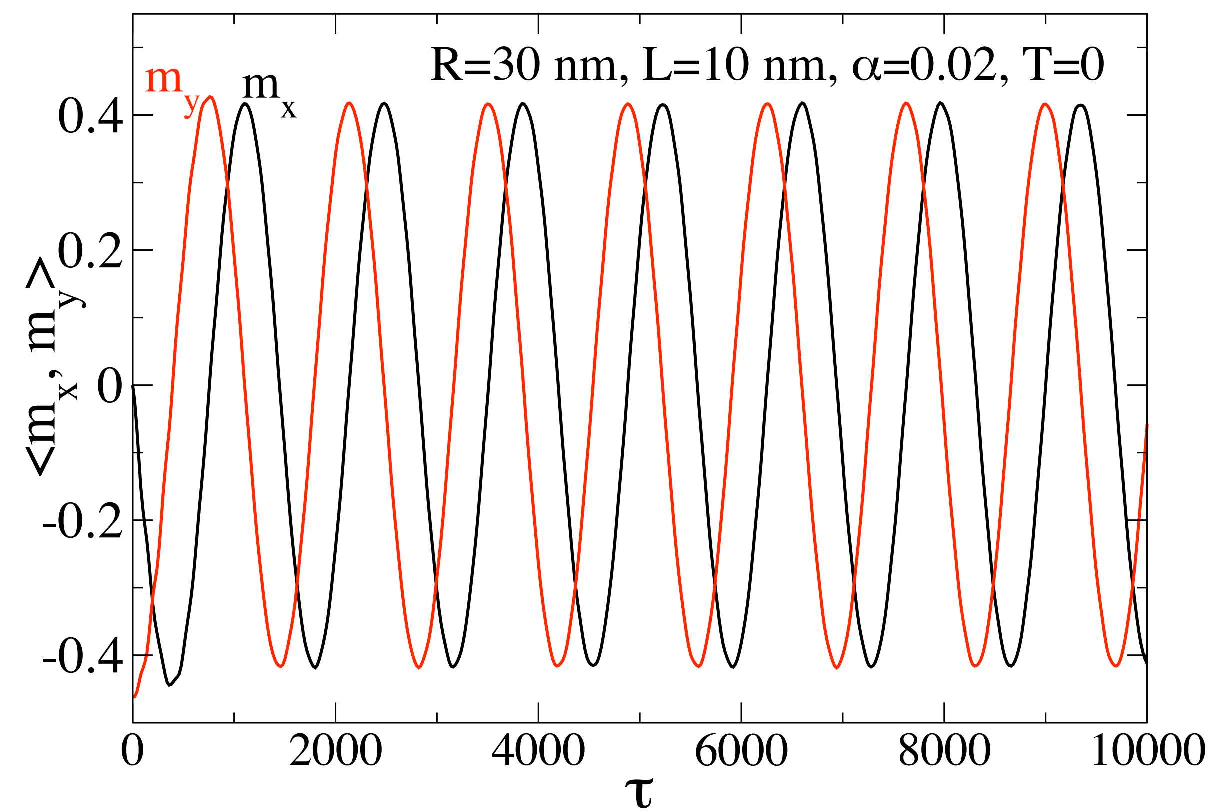
1. Constrained relaxation ⇒ initial position (x_0, y_0)

2. Evolution with Runge-Kutta-4 with $\alpha=0.02$ to $\tau=1000$.

3. Evolution with Runge-Kutta-4 with $\alpha=0.0$ for several periods

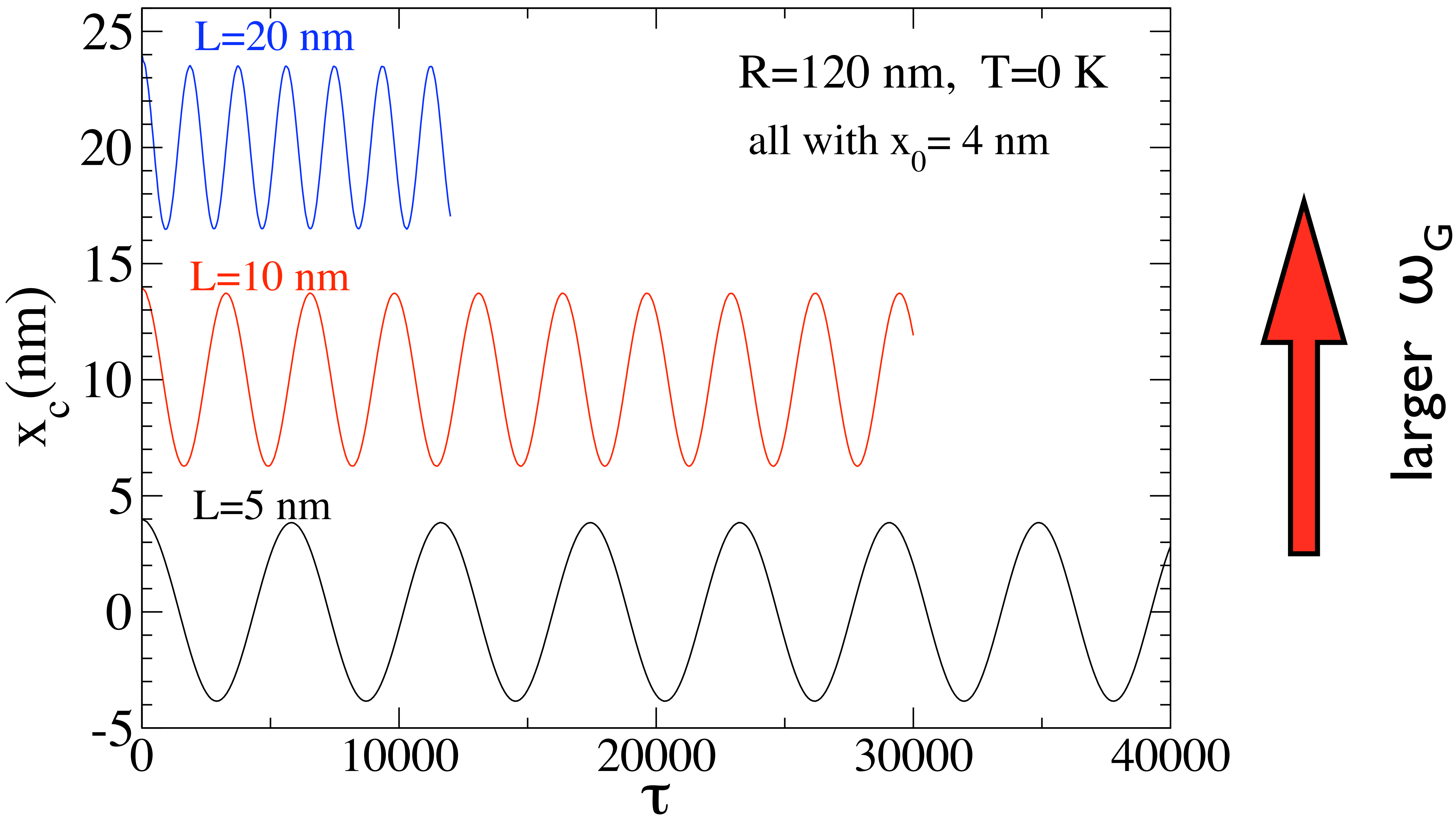
4. Measure the rotation frequency: $\nu_G = 1/\tau_G$.





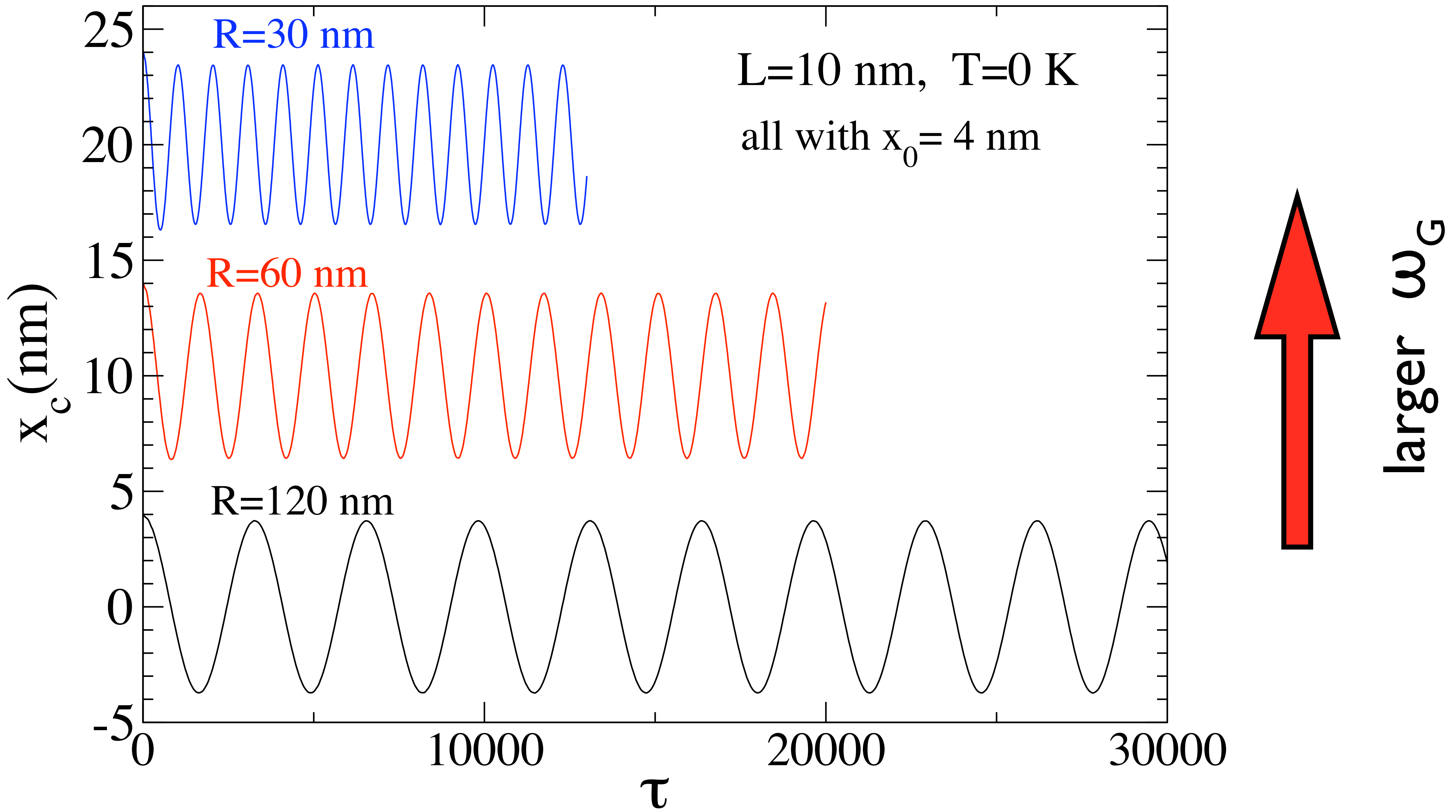
Differences with nanodot thickness L .

x_c= vortex core coordinate



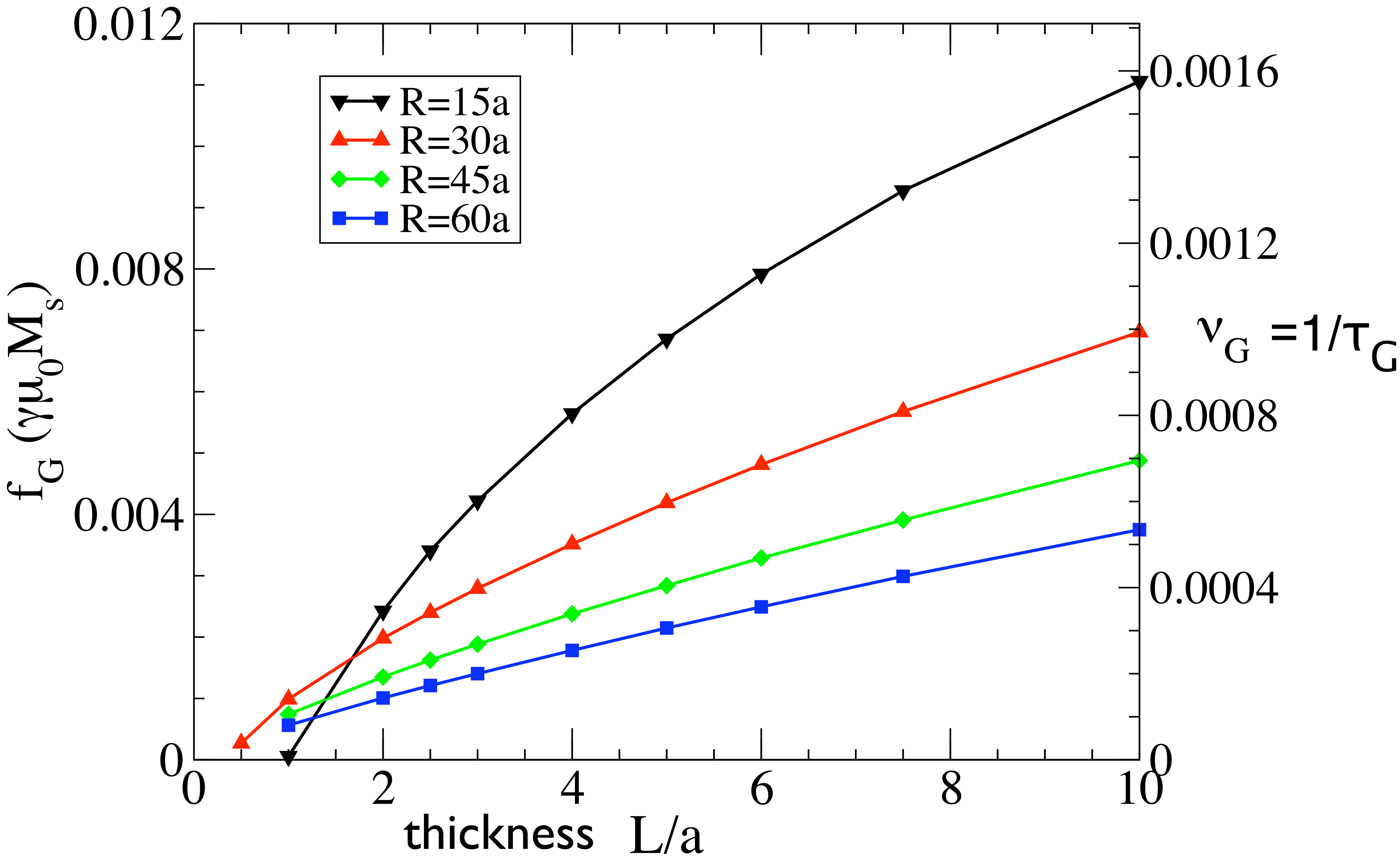
Differences with nanodot radius R.

x_c= vortex core coordinate



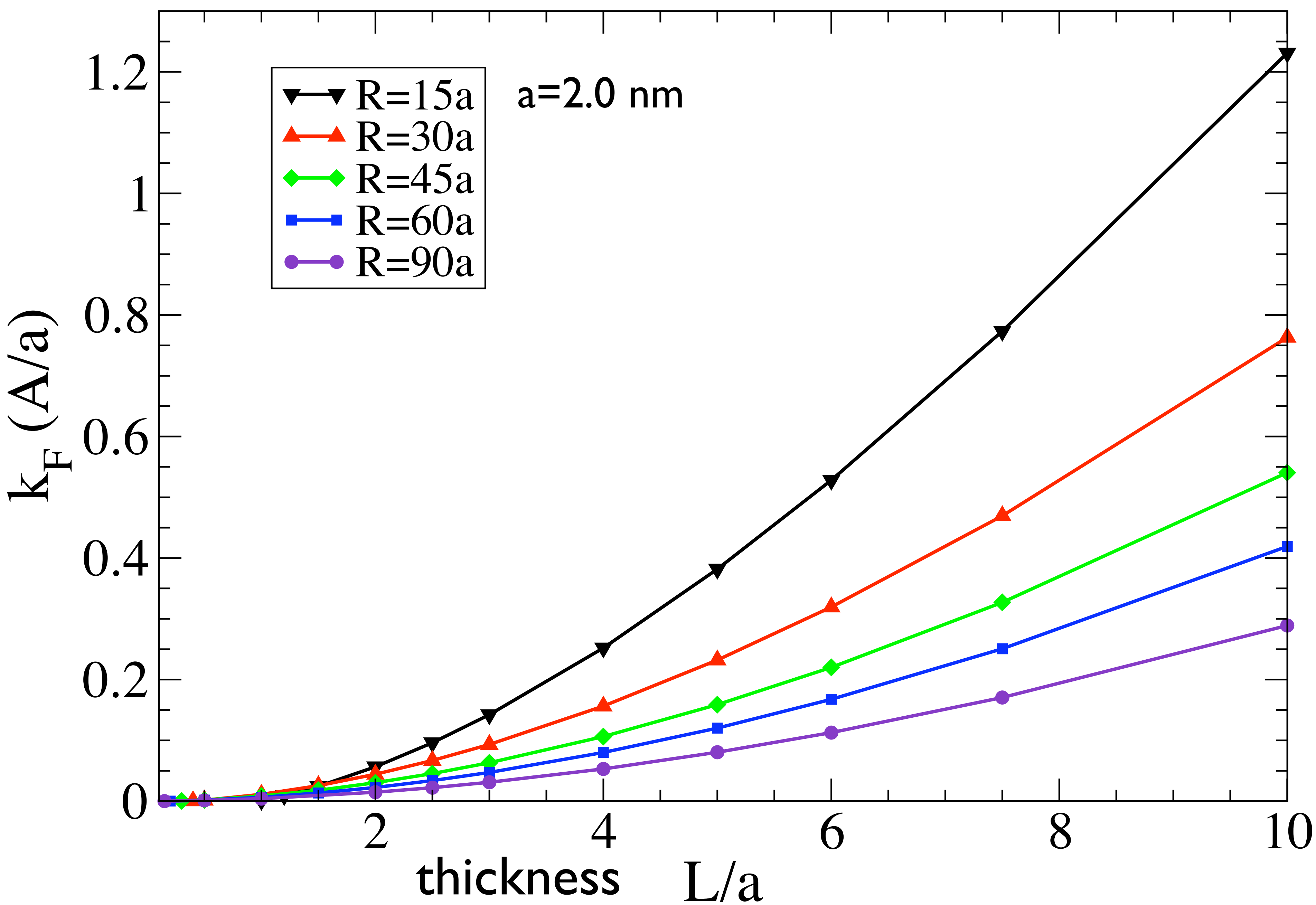
Gyrotropic frequencies:

(Calculated from the dynamics)



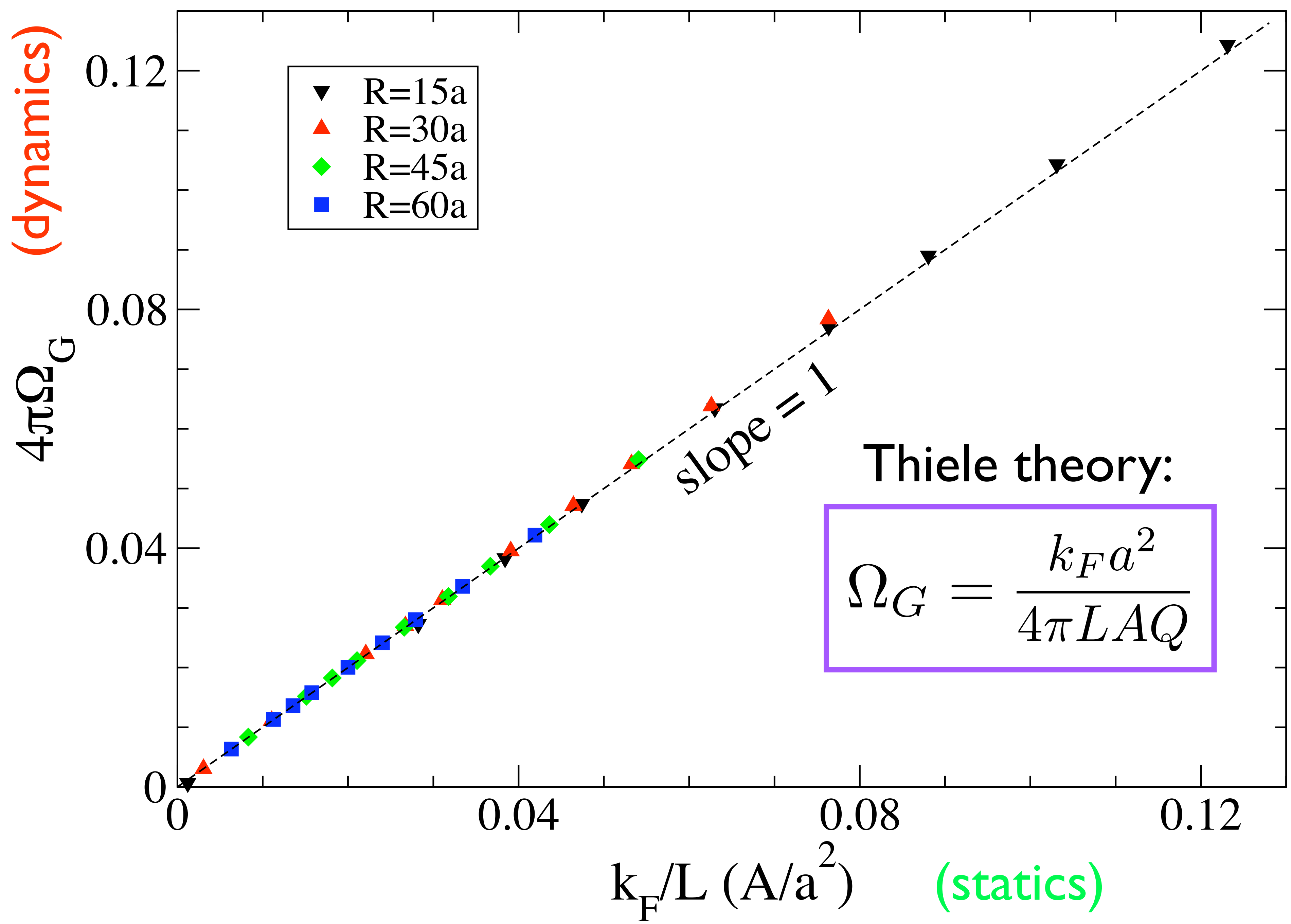
Force constants: $E(x) \approx E(0) + \frac{1}{2} k_F x^2$

(Calculated only from the static energy, not the dynamics)



Dynamic frequency vs. force constant:

$$\omega_G = \gamma B_0 \Omega_G \qquad \Omega_G = 2\pi\nu_G = 2\pi/\tau_G$$



Thiele theory:

$$\Omega_G = \frac{k_F a^2}{4\pi L A Q}$$

With temperature T>0. For the movement in one cell:

$$\frac{d\hat{m}}{d\tau} = \hat{m} \times \left(\vec{b} + \vec{b}_s\right) - \alpha \hat{m} \times \left[\hat{m} \times \left(\vec{b} + \vec{b}_s\right)\right]$$

stochastic fields

fluctuation-dissipation theorem:

$$\langle b_s^{\alpha}(\tau)\, b_s^{\beta}(\tau')\rangle = 2\alpha\, \mathcal{T}\, \delta_{\alpha\beta}\, \delta(\tau - \tau') \qquad \mathcal{T} \equiv \frac{kT}{J_{\text{cell}}} = \frac{kT}{2AL}$$

(the stochastic fields carry thermal energy & power)

We can integrate with Heun's 2nd order algorithm:

A. Euler predictor step.

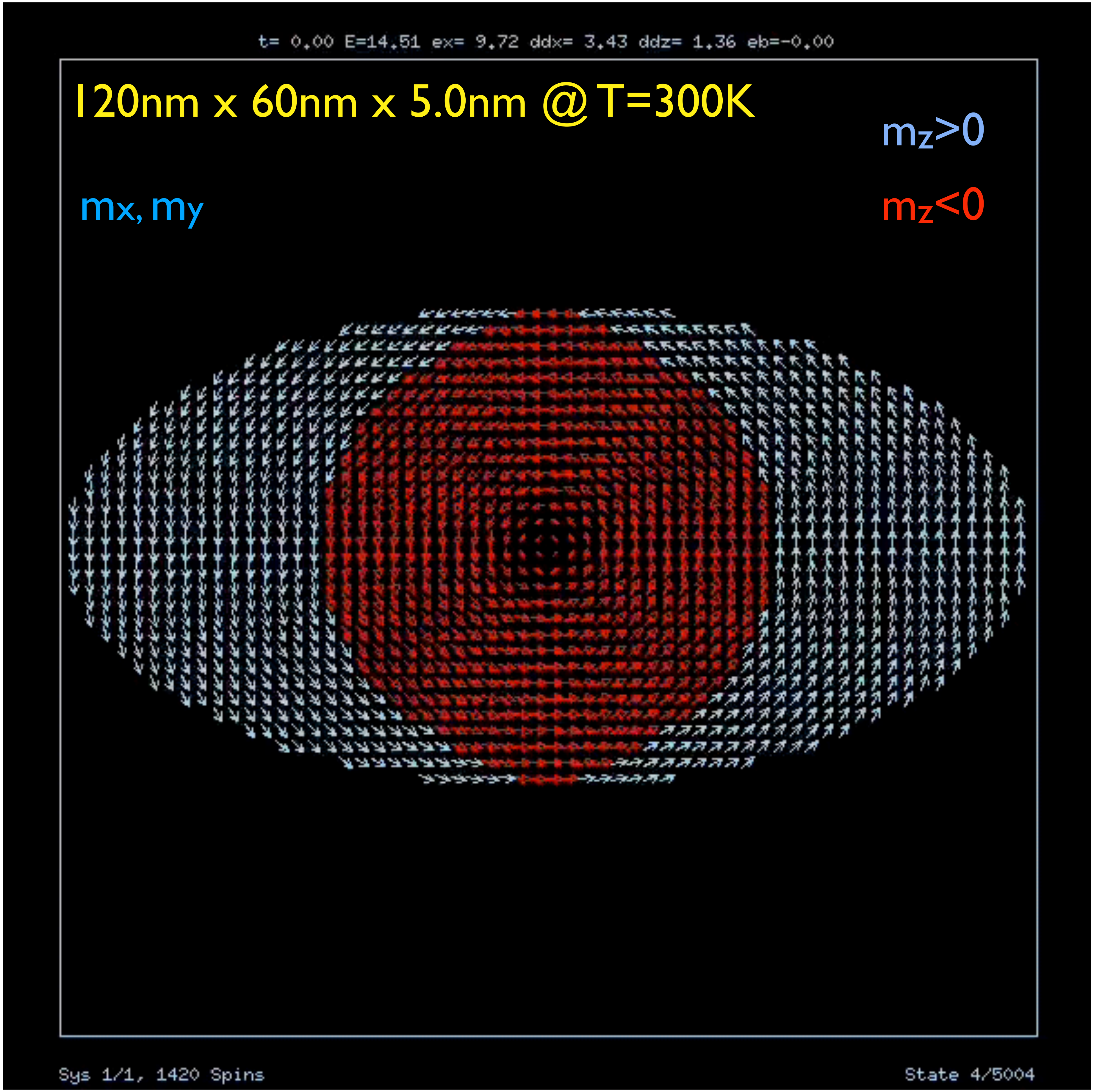
B. Trapezoid corrector step.

$$\int_{\tau_n}^{\tau_n + \Delta\tau} d\tau\ b_s^x(\tau) \longrightarrow \sigma_s w_n^x$$

ran( )

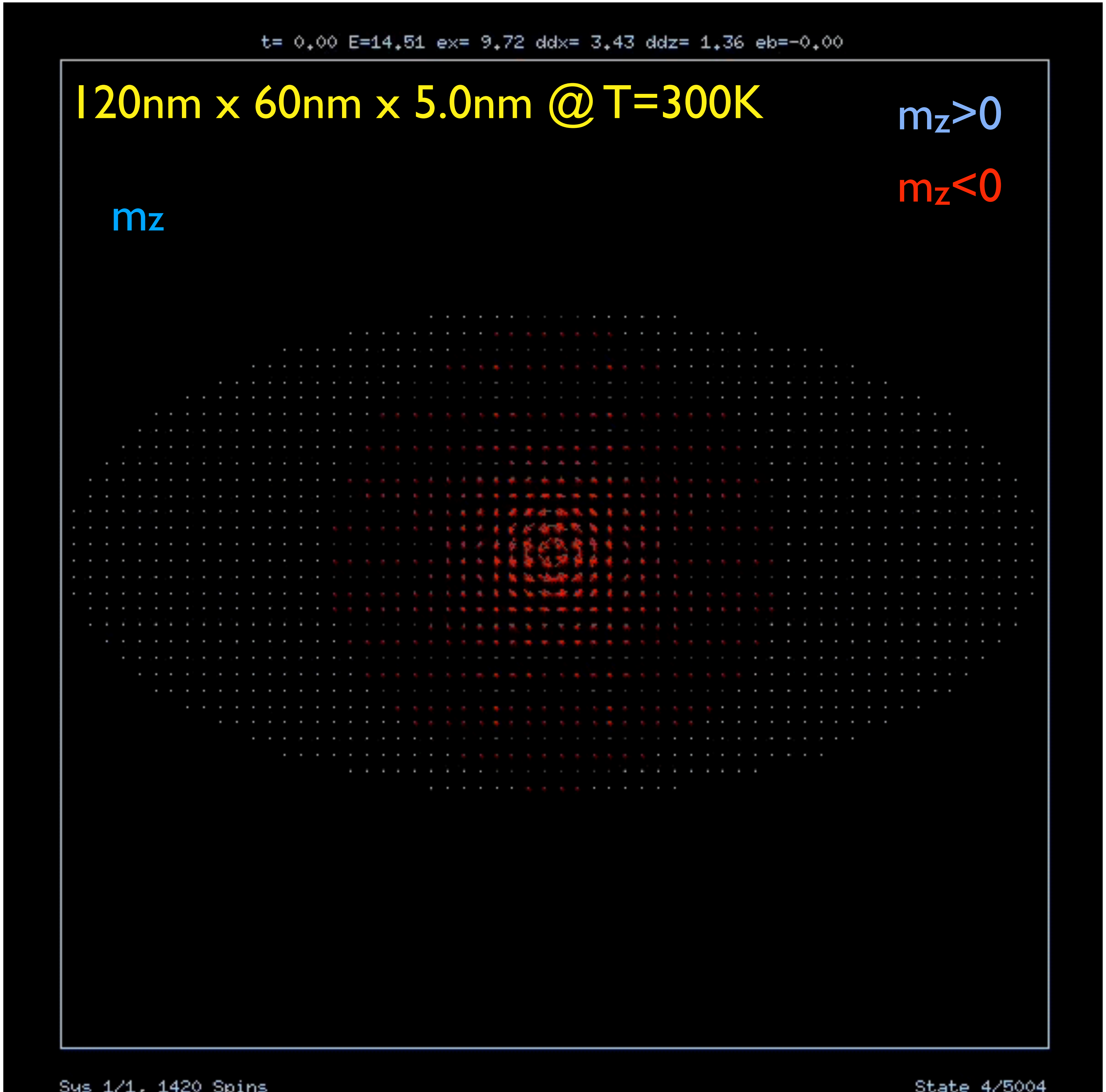
$$\sigma_s = \sqrt{2\alpha\mathcal{T}\,\Delta\tau}$$

Spontaneous gyrotropic movement for T>0 (ellipse)



initial position
$x_0=y_0=0$

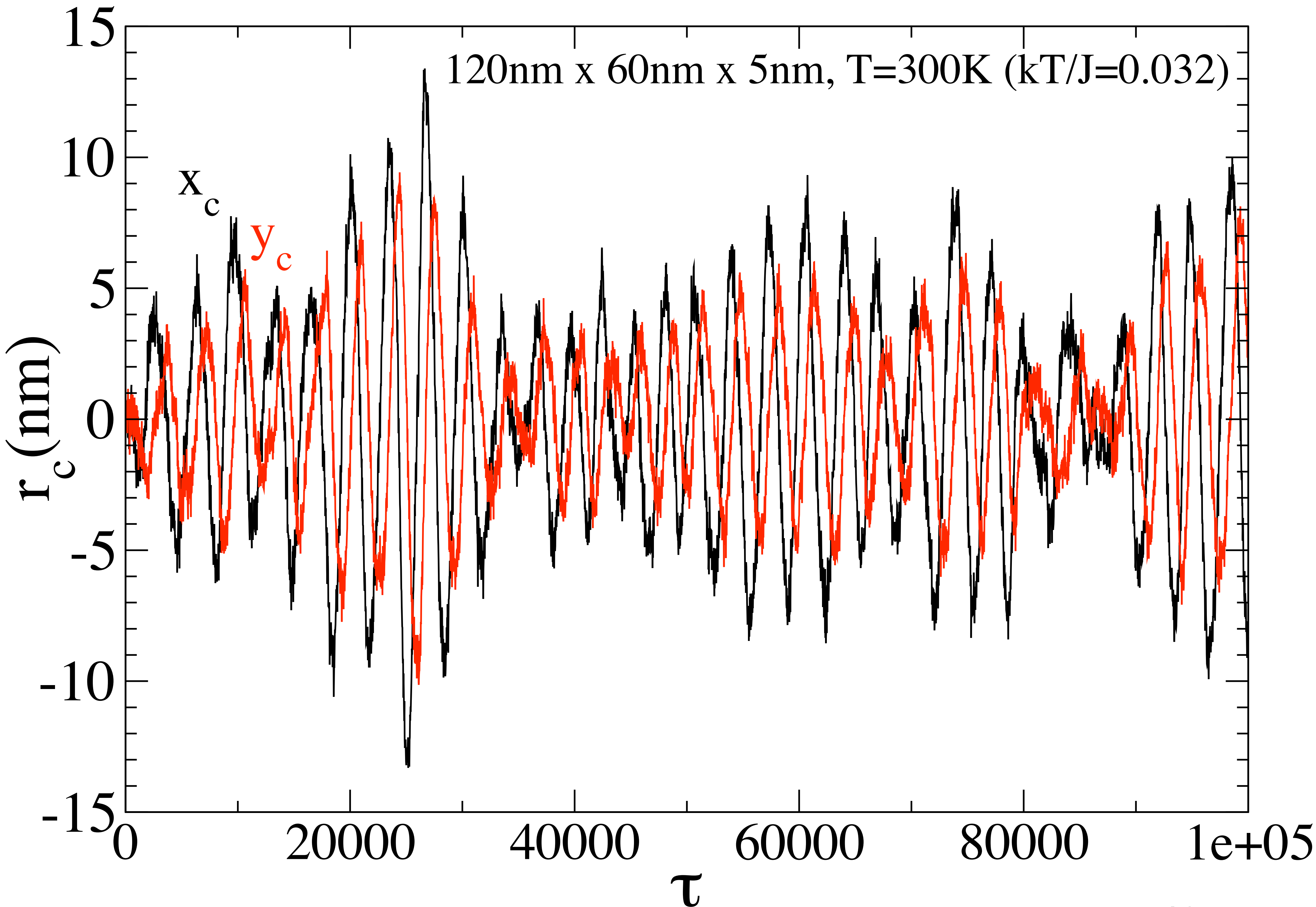
Spontaneous gyrotropic movement for T>0 (ellipse)



initial position

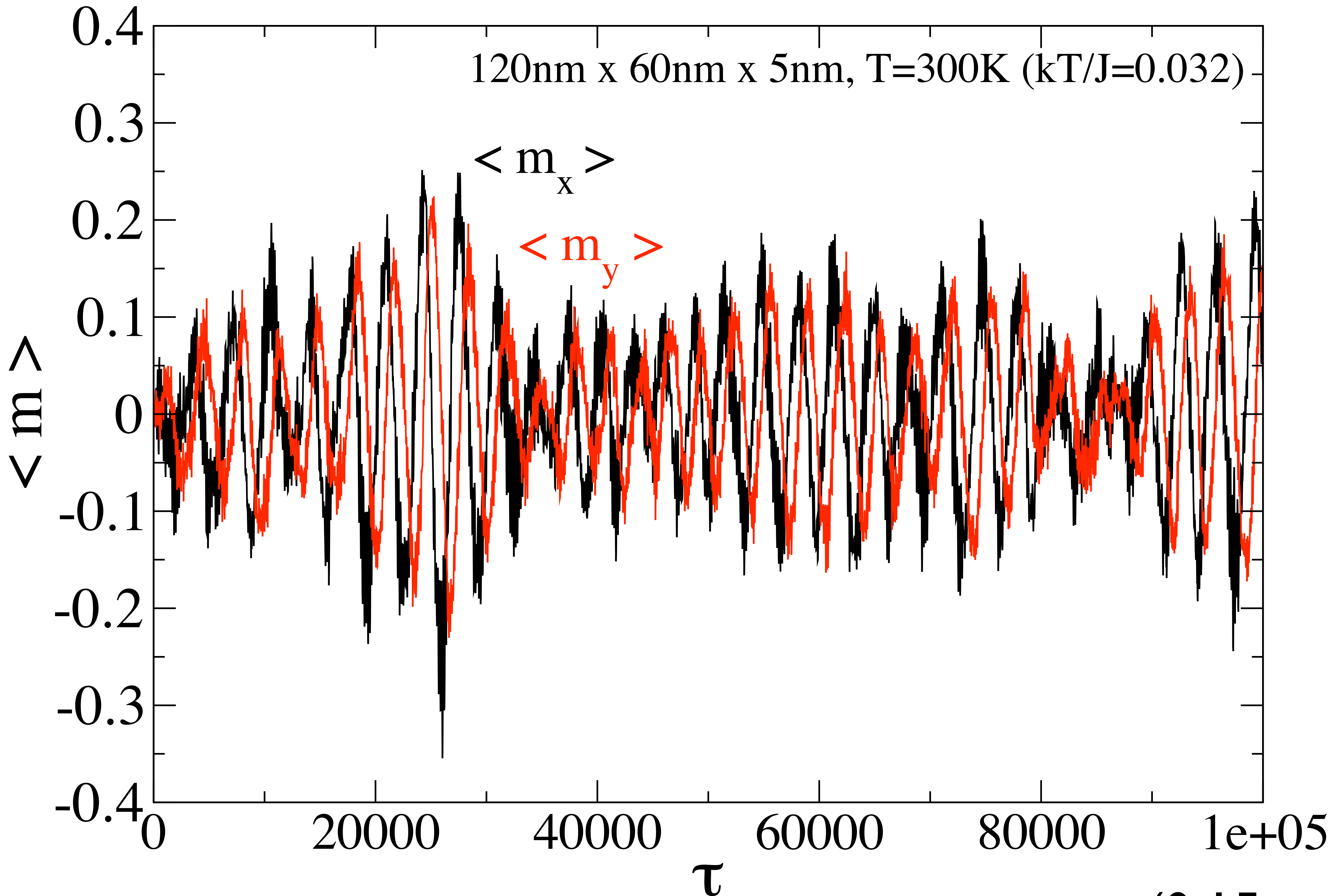
$x_0=y_0=0$

vortex core position: (ellipse)



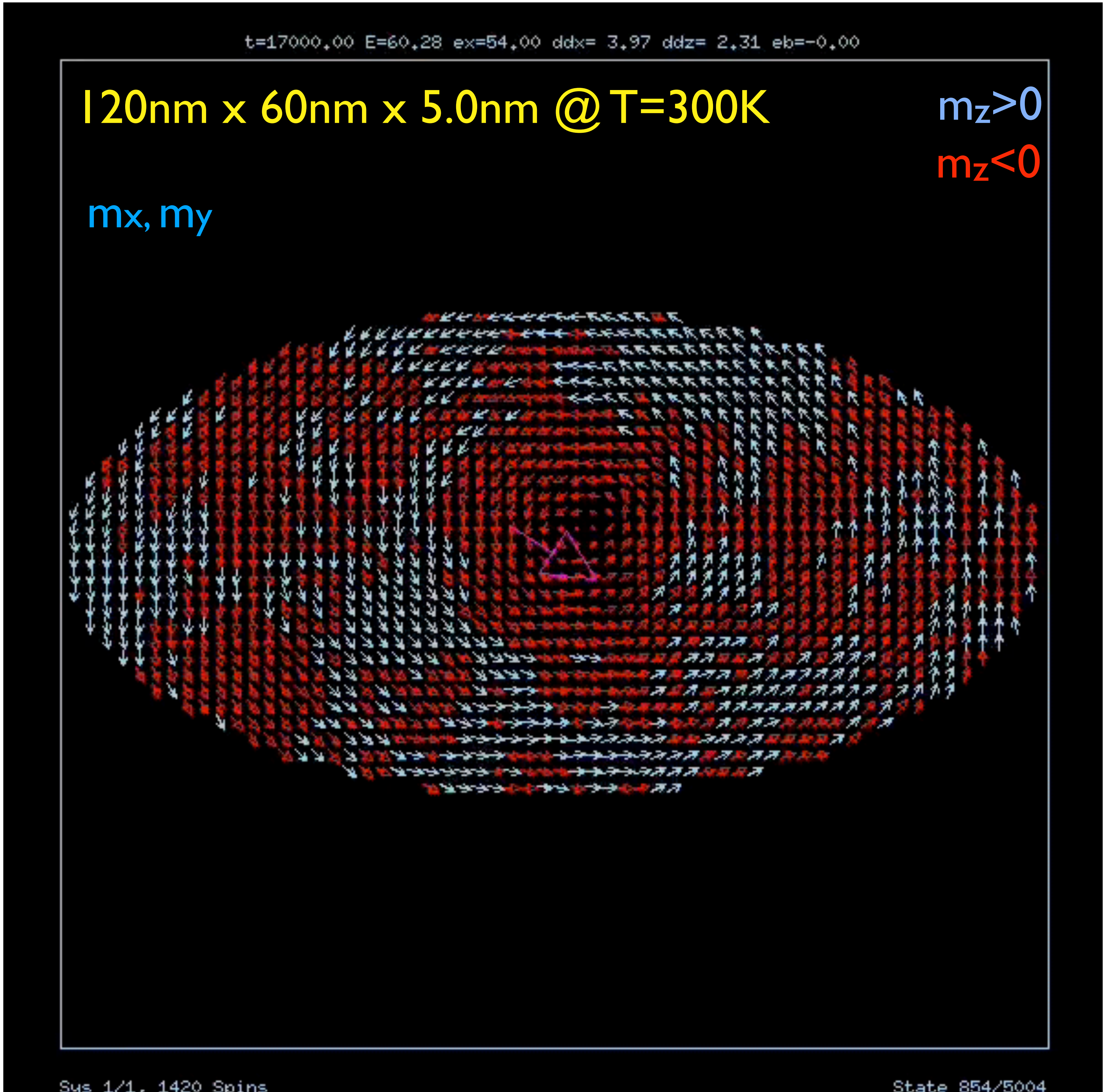
(0.15 μs)

total magnetization: (ellipse)



(0.15 μs)

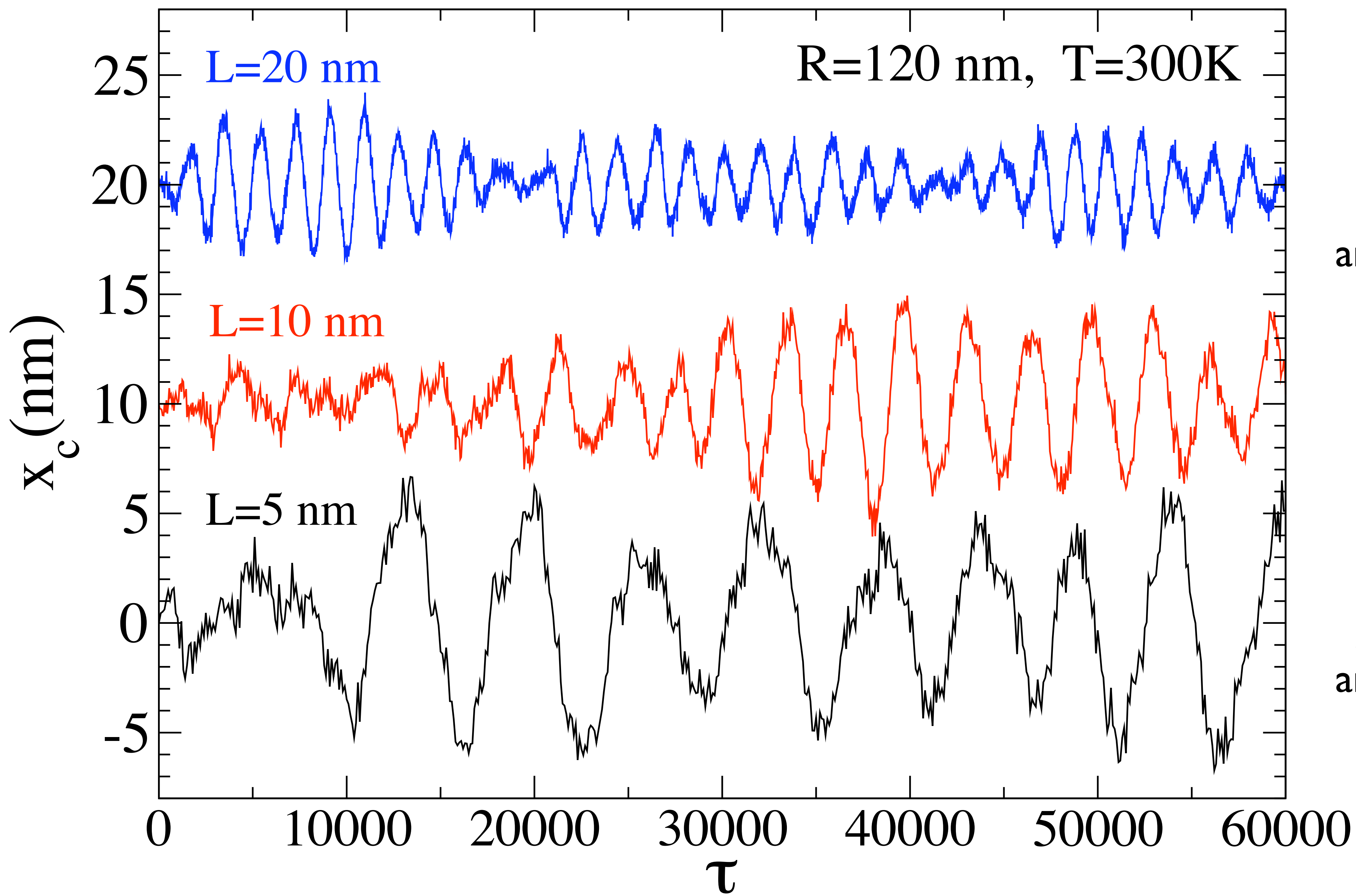
Spontaneous gyrotropic movement for T>0 (ellipse)



initial position
$x_0=y_0=0$

The large arrow = <M>.
Note its faster oscillations.

Spontaneous vortex movement (circular nanodots):



lesser amplitude

greater amplitude

Summary, vortex gyrotropic dynamics:

Without an external magnetic field, the vortex gyrotropic movement begins naturally, when the vortex is not in the center of the nanodot.

The frequency ω_G of gyrotropic movement is proportional to k_F/L for circular nanodots.

Even thermal fluctuations can initiate the movement spontaneously, which should have an amplitude determined by the principle of equipartition of energy equally among degrees of freedom.

The dynamics in ellipses should be even more interesting, due to the presence of two inequivalent axes.

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