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Vortex dynamics and statistics in thin elliptic ferromagnetic nanodisks

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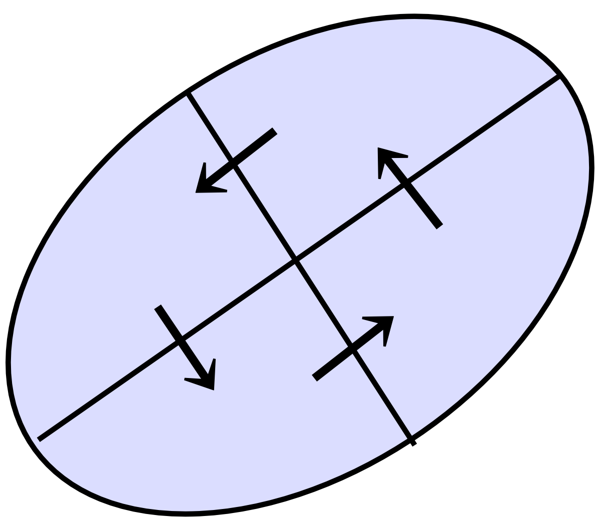

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Magnetic Vortex Core Observation in Circular Dots of Permalloy

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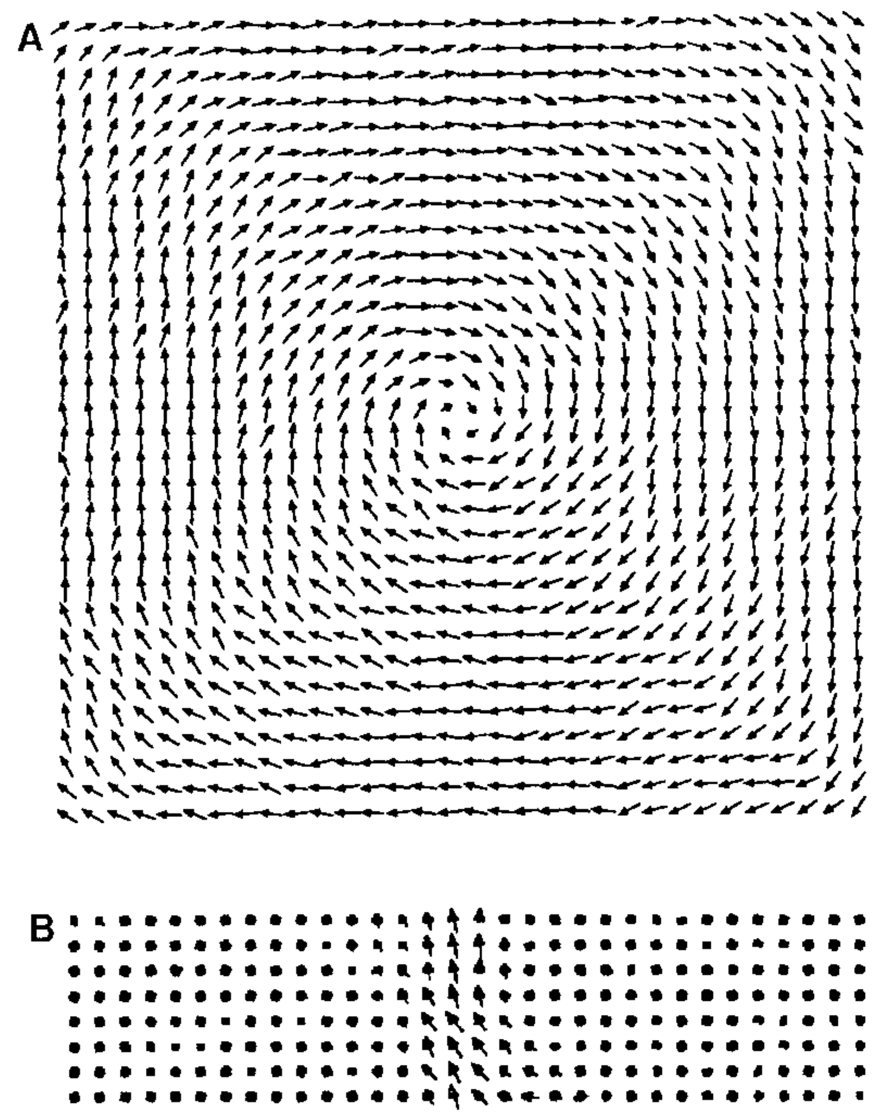
Fig. 1. Monte Carlo simulation for a ferromagnetic Heisenberg spin structure comprising 32 × 32 × 8 spins [courtesy of Ohshima *et al.* (*2*)]. (**A**) Top surface layer. (**B**) Cross-section view through the center. Beside the center, the spins are oriented almost perpendicular to the drawing plane, jutting out of the plane to the right and into the plane to the left, respectively. These figures represent snapshots of the fluctuating spin structure and are therefore not symmetric with respect to the center. The structure should become symmetric by time averaging.



M(r)

$M_z \approx \pm M_s$ in the core

The vortex cores are visible.

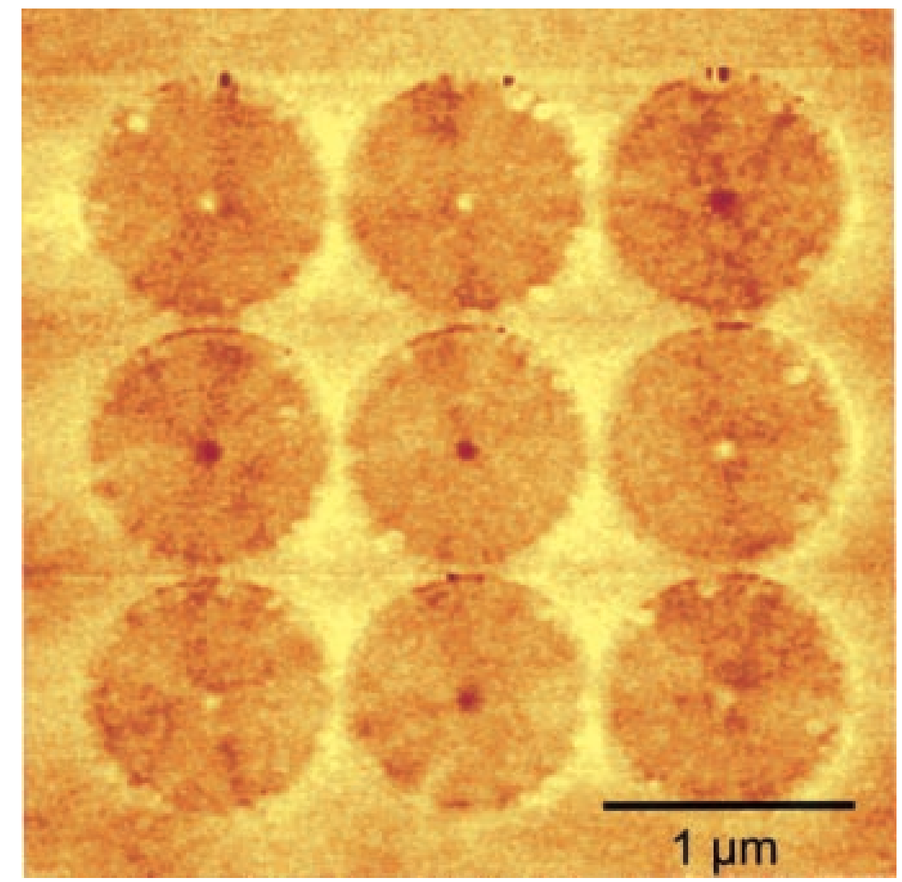


Fig. 2. MFM image of an array of permalloy dots 1 µm in diameter and 50 nm thick.

We can see up/down Mz = polarity of the core!

Why elliptically shaped nanodots?

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

- Consider effects due to an obvious breaking of circular symmetry. Vortex magnetization field will lose its perfect circular shape, due to demagnetization effects.

- Ellipticity introduces important changes in many classical problems $\Rightarrow$ Bohr orbits, 2D harmonic oscillators, elliptically polarized light & Faraday rotation, etc.

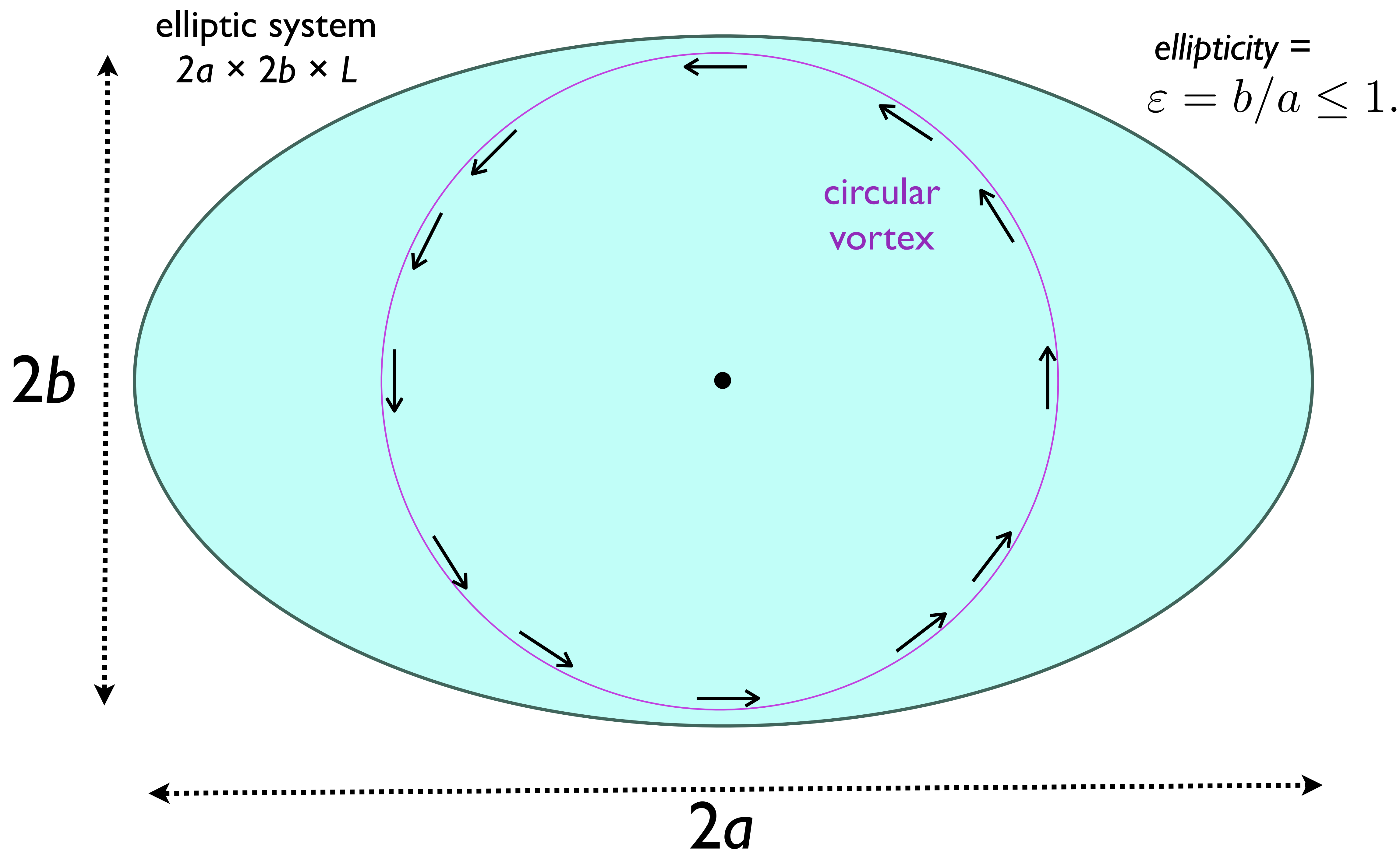
- Highly elliptic shape will destabilize a vortex in favor of a quasi-single-domain (QSD) state.

Consider vortex motion in a magnetic nanodot
in the shape of an *elliptical cylinder*.

- System: sat. magn. M_s, semi-major axes *a*, *b*, height *L*.
- Energetics for a stable vortex vs. QSD state.
- Find an effective potential of elliptic shape, by using a Lagrange constraint on vortex core position.
- Dynamics: 4th order Runge-Kutta for T=0,
2nd order Heun for magnetic Langevin equation at T>0.
- Theory: Compare micromagnetics with Thiele equation.

Vortex must deform in elliptical system, due to demagnetization effects.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$



Thiele equation analysis.

$$\mathbf{F} + \mathbf{G} \times \mathbf{V} = 0$$

vortex core position = $\mathbf{R} = (X, Y)$

core velocity = $\mathbf{V} = \dot{\mathbf{R}}$

Force on core:

$$\mathbf{F} = -\vec{\nabla} U(\mathbf{R})$$

Gyrovector due to polarization $p=\pm 1$, $q=1$:

$$\mathbf{G} = G\hat{z} = 2\pi pq \frac{m_0}{\gamma}\hat{z}. \qquad m_0 = LM_s$$

$$\boxed{\text{For a circular system:}}$$

symmetric potential:

$$U(\mathbf{R}) = \frac{1}{2}k_F \mathbf{R}^2 = \frac{1}{2}k_F(X^2 + Y^2).$$

force on vortex:

$$\mathbf{F} = -k_F \mathbf{R}$$

circular motion results:

$$\mathbf{V} = \frac{\mathbf{G} \times \mathbf{F}}{G^2} = \vec{\omega}_G \times \mathbf{R}.$$

gyrotropic frequency:

$$\boxed{\vec{\omega}_G = -\frac{k_F}{G}\hat{z}}$$

Thiele equation analysis.

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Gyrovector due to polarization $p=\pm 1$, $q=1$:

$$\mathbf{G} = G\hat{z} = 2\pi pq \frac{m_0}{\gamma}\hat{z}. \qquad m_0 = LM_s$$

For an elliptic system:

asymmetric potential:

$$U(\mathbf{R}) = U_0 + \frac{1}{2}\left(k_x X^2 + k_y Y^2\right).$$

force on vortex:

$$\begin{aligned} F_x &= -k_x X = -(\mathbf{G} \times \mathbf{V})_x = G\dot{Y} \\ F_y &= -k_y Y = -(\mathbf{G} \times \mathbf{V})_y = -G\dot{X} \end{aligned}$$

elliptic motion results:

$$\begin{aligned} X(t) &= X_0 \cos\omega_G t + Y_0 \frac{k_y}{G\omega_G} \sin\omega_G t \\ Y(t) &= Y_0 \cos\omega_G t - X_0 \frac{k_x}{G\omega_G} \sin\omega_G t \end{aligned}$$

gyrotropic frequency:

$$\boxed{\omega_G = -\frac{\sqrt{k_x k_y}}{G}.}$$

$$\vec{\omega}_G = \omega_G \hat{z}$$

Equivalent circular coordinates for the elliptic system.

$$U(\mathbf{R}) = \frac{1}{2}\left[\left(\sqrt{k_x}X\right)^2 + \left(\sqrt{k_y}Y\right)^2\right] = \frac{1}{2}\sqrt{k_x k_y}\left(\sqrt{\frac{k_x}{k_y}}X^2 + \underbrace{\sqrt{\frac{k_y}{k_x}}}_{1/e}Y^2\right)$$

$\boxed{\text{ave. force constant = } \bar{k} \equiv \sqrt{k_x k_y}}$ $\boxed{\text{energetic ellipticity = } e \equiv \sqrt{\frac{k_x}{k_y}}}$

From the vortex elliptic motion: $$\frac{Y_{\max}}{X_{\max}} = \frac{k_x}{G|\omega_G|} = \sqrt{\frac{k_x}{k_y}} = e$$

Define a new coordinate that symmetrizes the potential:

$$\vec{\rho} \equiv \left(\sqrt{e}X, \frac{1}{\sqrt{e}}Y\right) \quad \Rightarrow \quad U(\vec{\rho}) = \frac{1}{2}\bar{k}\vec{\rho}^{\,2}$$

Get an equivalent circular motion:

$$\dot{\rho}_x = \sqrt{e}\dot{X} = -\omega_G \rho_y$$
$$\dot{\rho}_y = \frac{1}{\sqrt{e}}\dot{Y} = \omega_G \rho_x$$

$$\Rightarrow \quad \boxed{\dot{\vec{\rho}} = (\dot{\rho}_x, \dot{\rho}_y) = \vec{\omega}_G \times \vec{\rho}.}$$

$$\vec{\omega}_G = \omega_G \hat{z}$$

Example force constants from numerics (vortex relaxation with a Lagrange-multipliers position constraint)

Hamiltonian

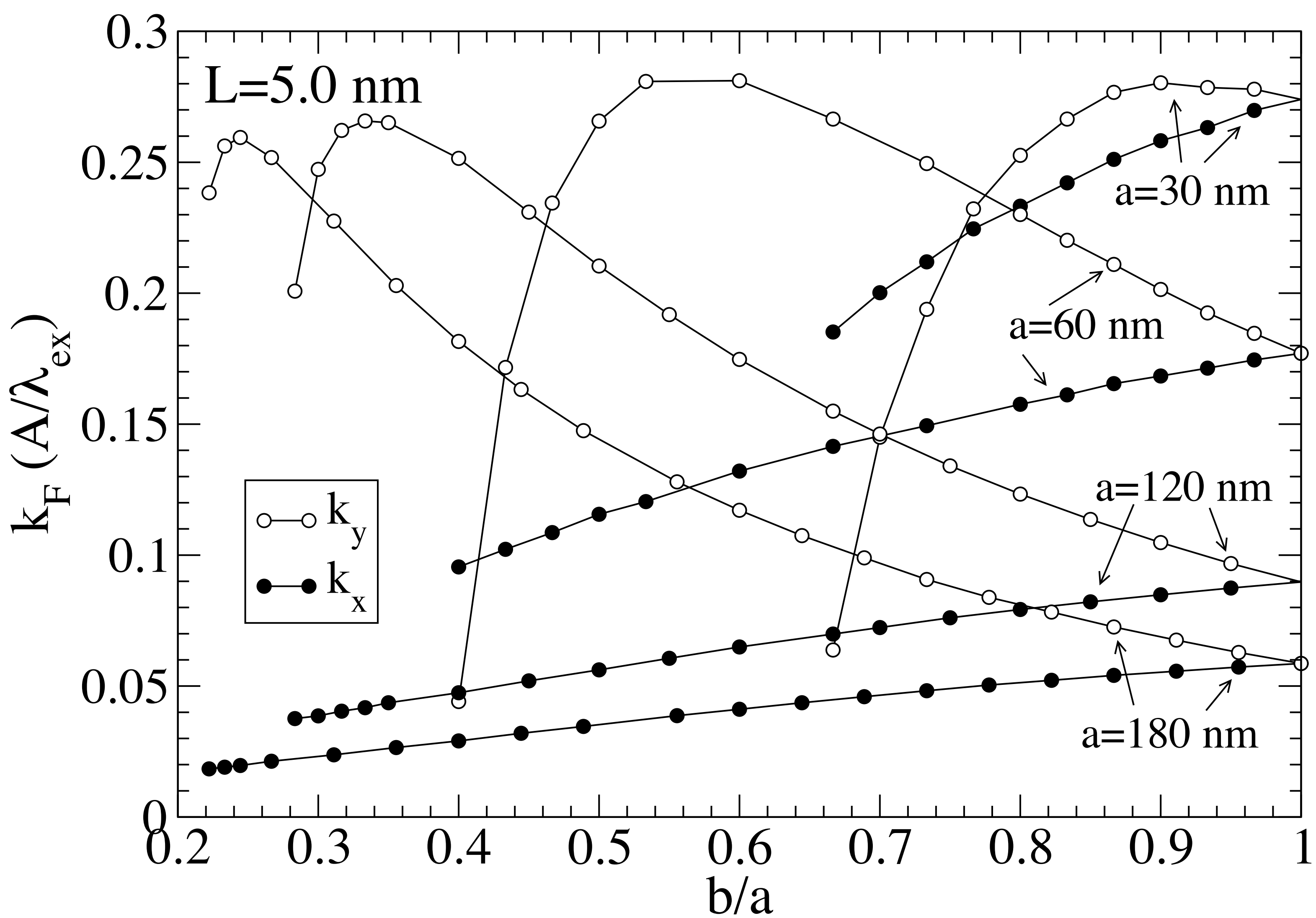
$$\mathcal{H} = \int dV \left\{ A\nabla\vec{m}\cdot\nabla\vec{m} - \frac{1}{2}\mu_0 \vec{H}^M \cdot \vec{M} \right\}$$

scaled magnetization

$$\vec{m} = \vec{M}/M_s$$

exchange length

$$\lambda_{\rm ex} = \sqrt{\frac{2A}{\mu_0 M_s^2}}$$



force constants vs system ellipticity

From system energies obtained by aligning spins to their local effective fields.

The potential is softer along the longer axis: $k_x < k_y$

Static force constants from numerics (vortex relaxation with a Lagrange-multipliers position constraint)

Hamiltonian

$$\mathcal{H} = \int dV \left\{ A\nabla\vec{m}\cdot\nabla\vec{m} - \frac{1}{2}\mu_0 \vec{H}^M \cdot \vec{M} \right\}$$

scaled magnetization

$$\vec{m} = \vec{M}/M_s$$

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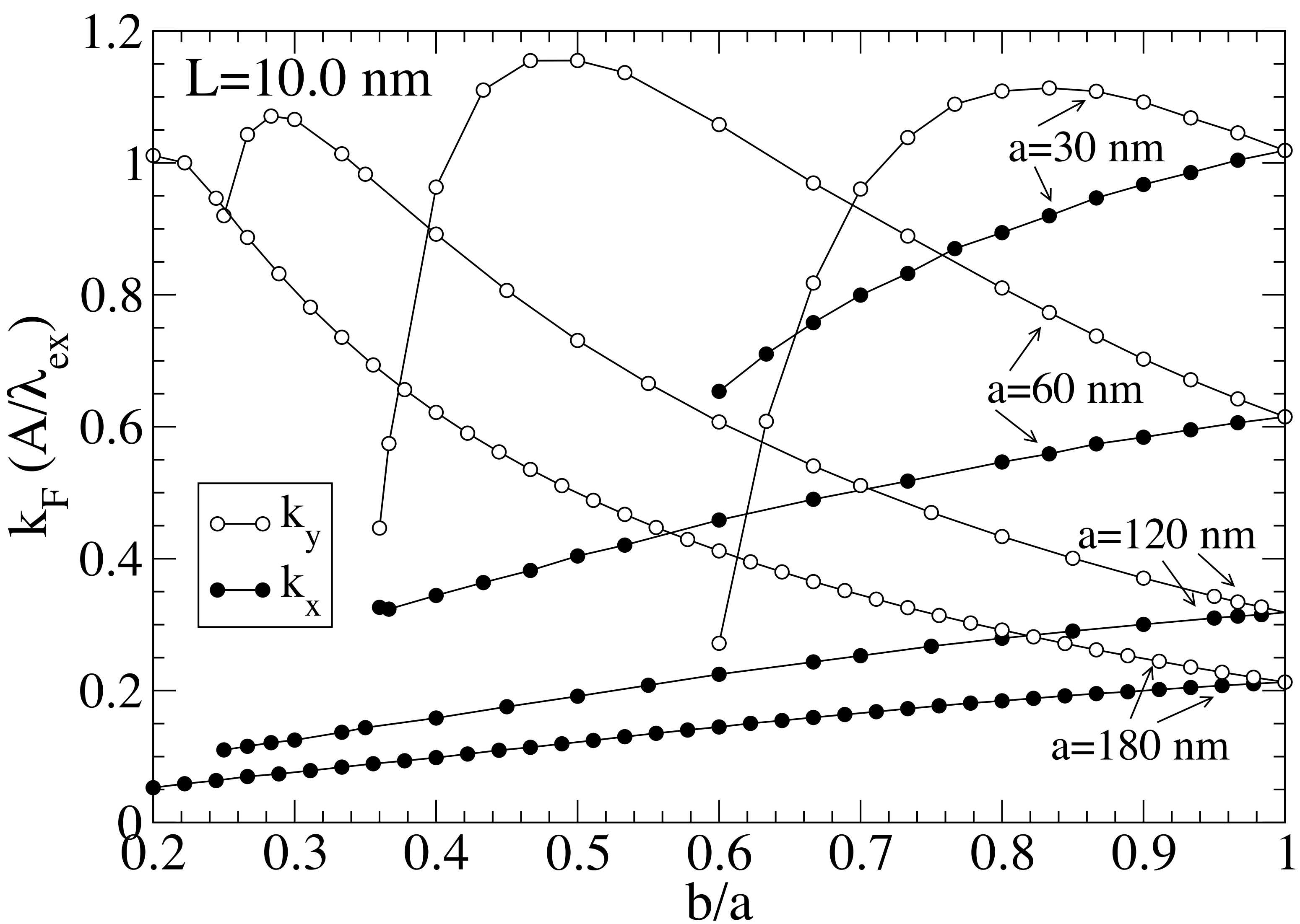


force constants vs system ellipticity

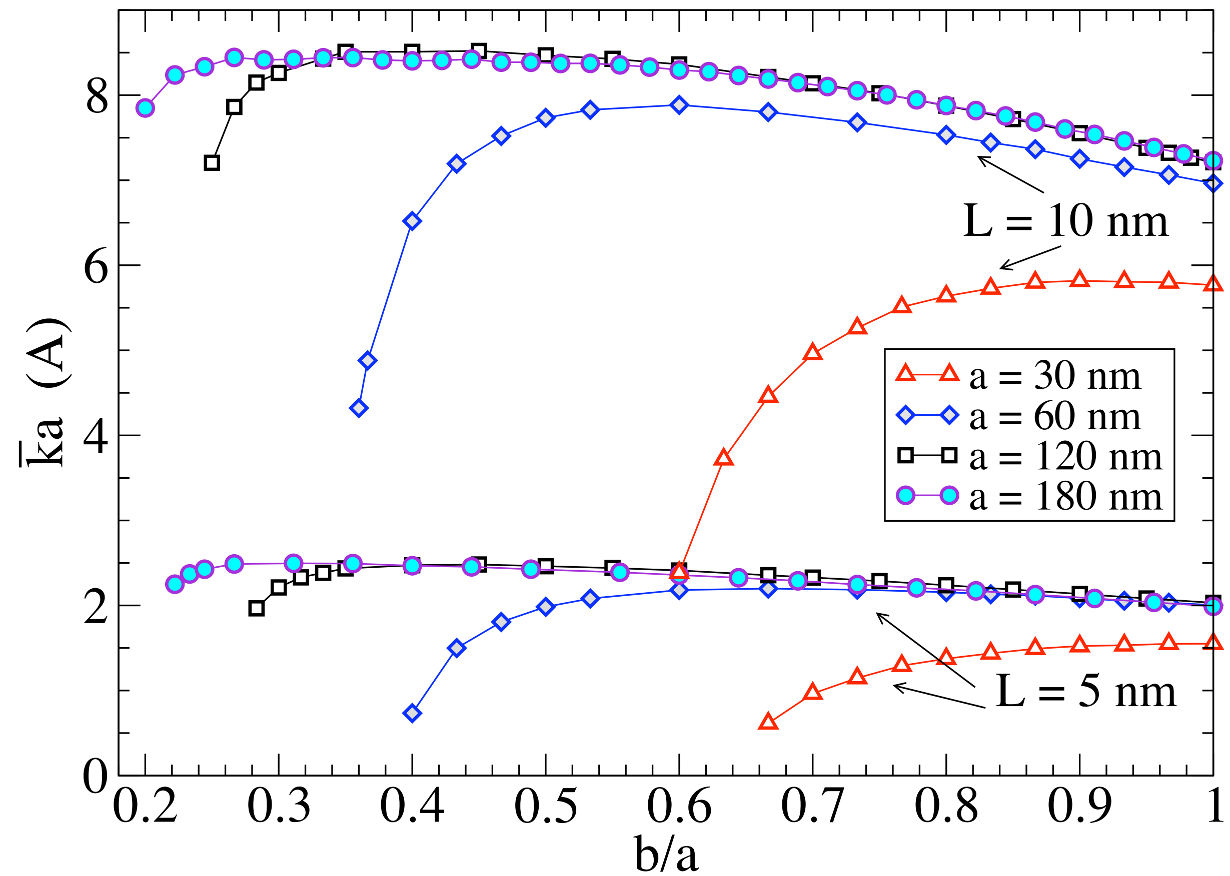
From system energies obtained by aligning spins to their local effective fields.

The potential and force constants scale approximately with L^2

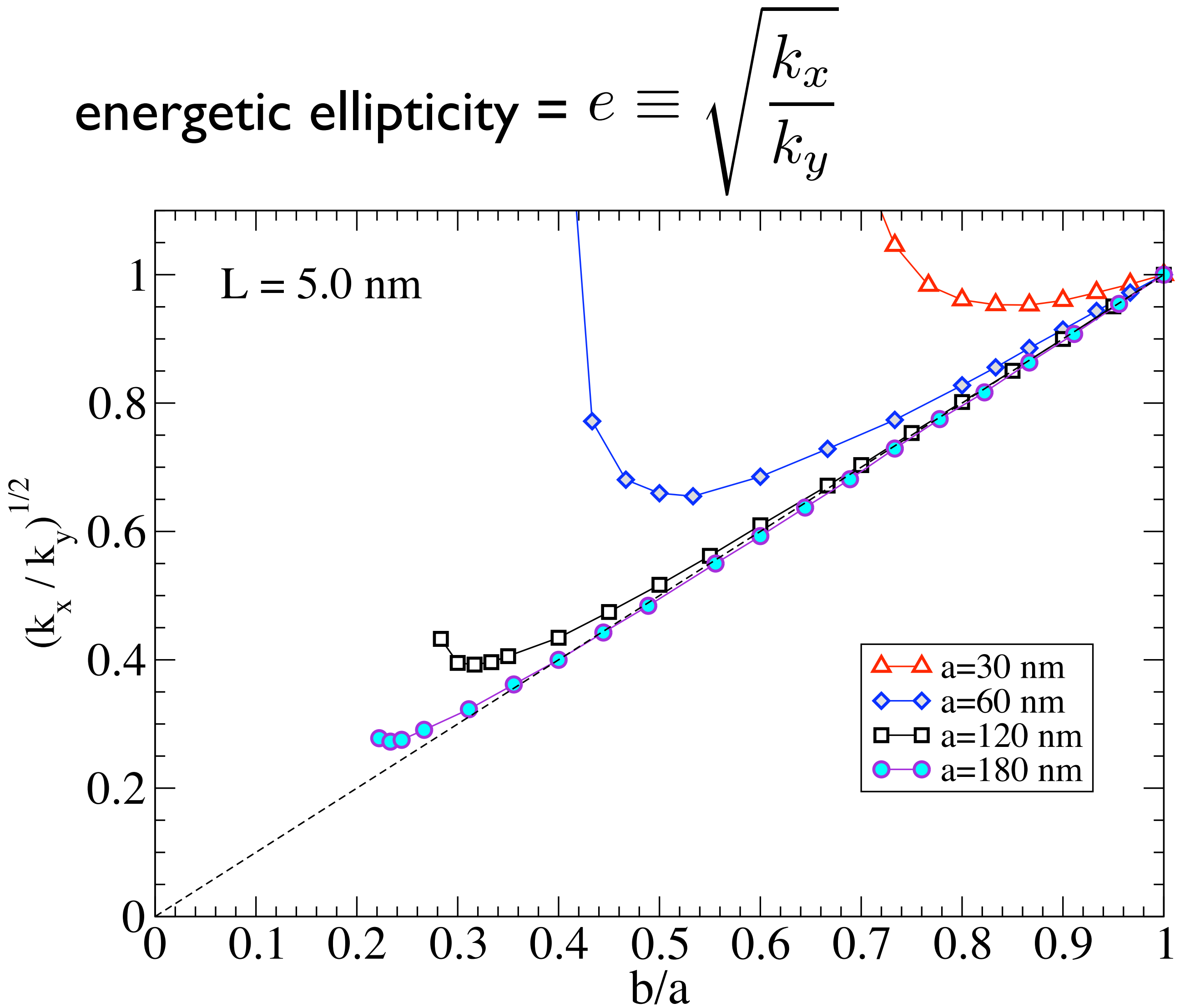
Derived static force parameters from vortex relaxation with Lagrange position constraint

ave. force constant = $\bar{k} \equiv \sqrt{k_x k_y}$

energetic ellipticity = $e \equiv \sqrt{\dfrac{k_x}{k_y}}$



system ellipticity ϵ=*b*/*a* →

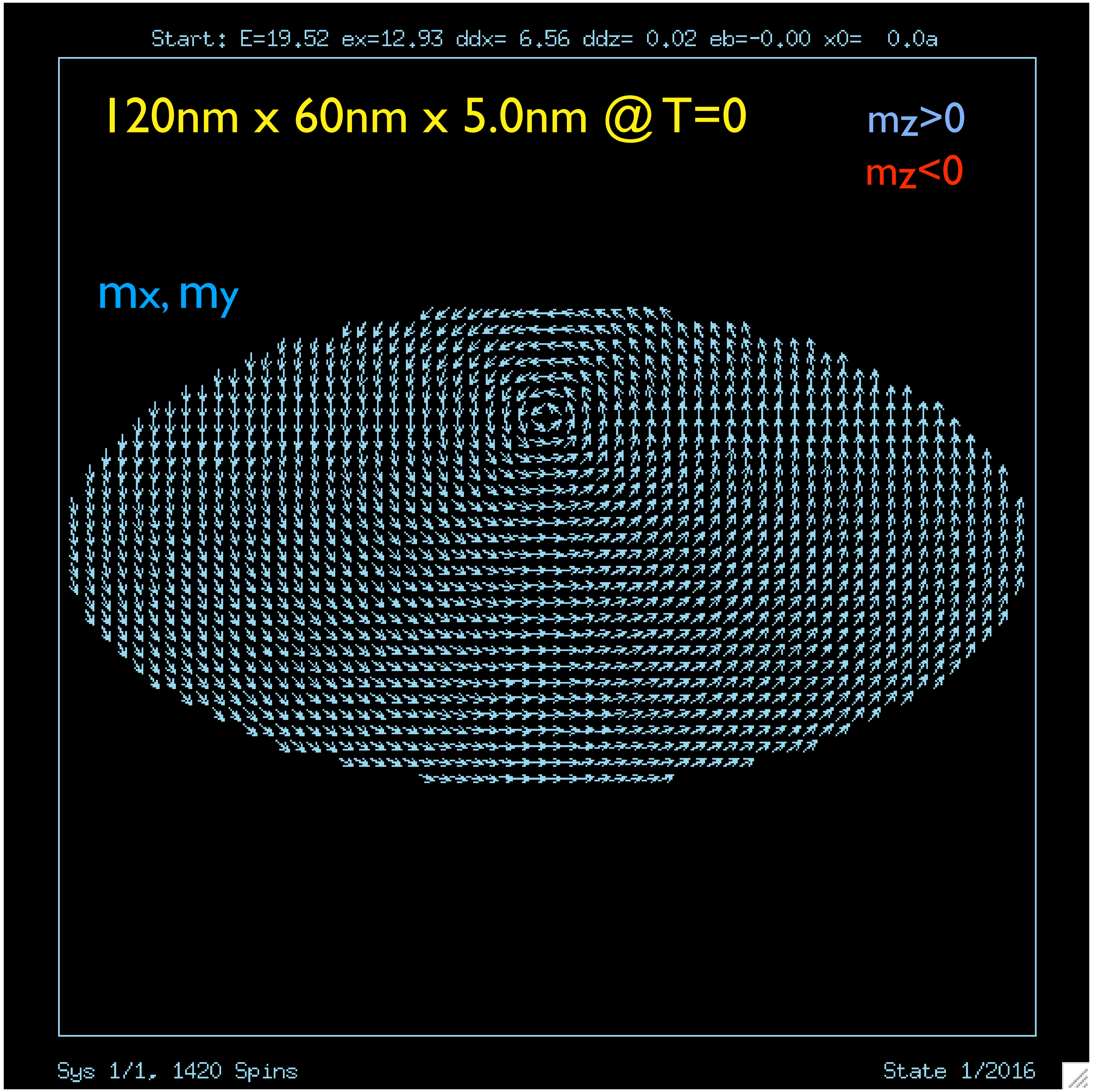


system ellipticity ϵ=*b*/*a* →

Result: e ≈ ϵ for larger disks

(also holds for *L*=10 nm)

Simulations by Runge-Kutta 4th order solutions of the Landau-Lifshitz-Gilbert equations (zero temperature)



simulation cell size is 2.0 nm

exchange length ≈ 5.3 nm for Py

simulation field unit is

$$B_0 = \frac{\lambda_{\rm ex}^2}{a_{\rm cell}^2}\mu_0 M_s$$

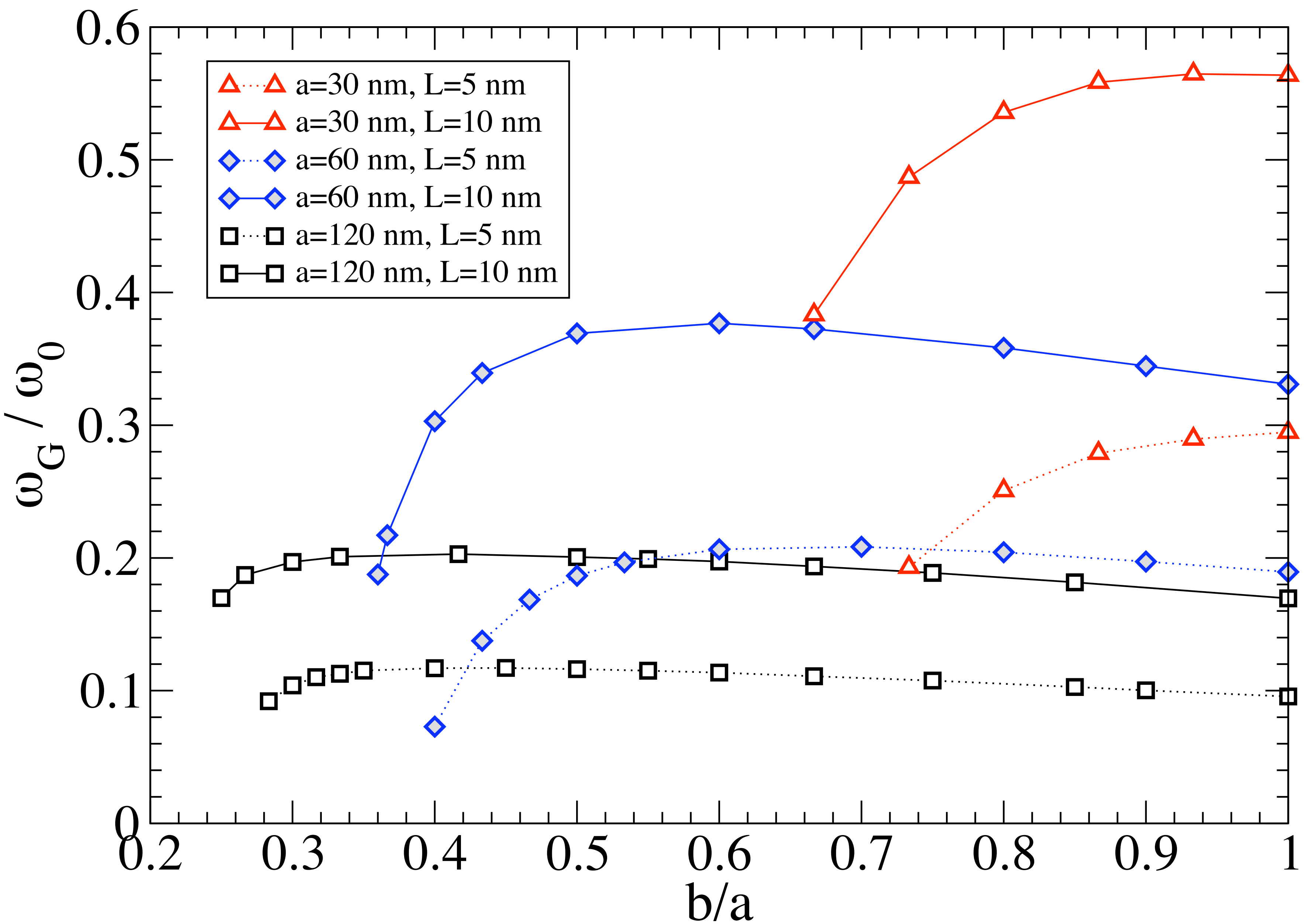
(≈ 7.6 T for Py)

simulation time unit is

$$t_0 \equiv (\gamma B_0)^{-1}$$

(≈ 0.75 ps for Py)

Dynamic gyrotropic frequencies from RK4 simulations of the Landau-Lifshitz-Gilbert equations (zero temperature)



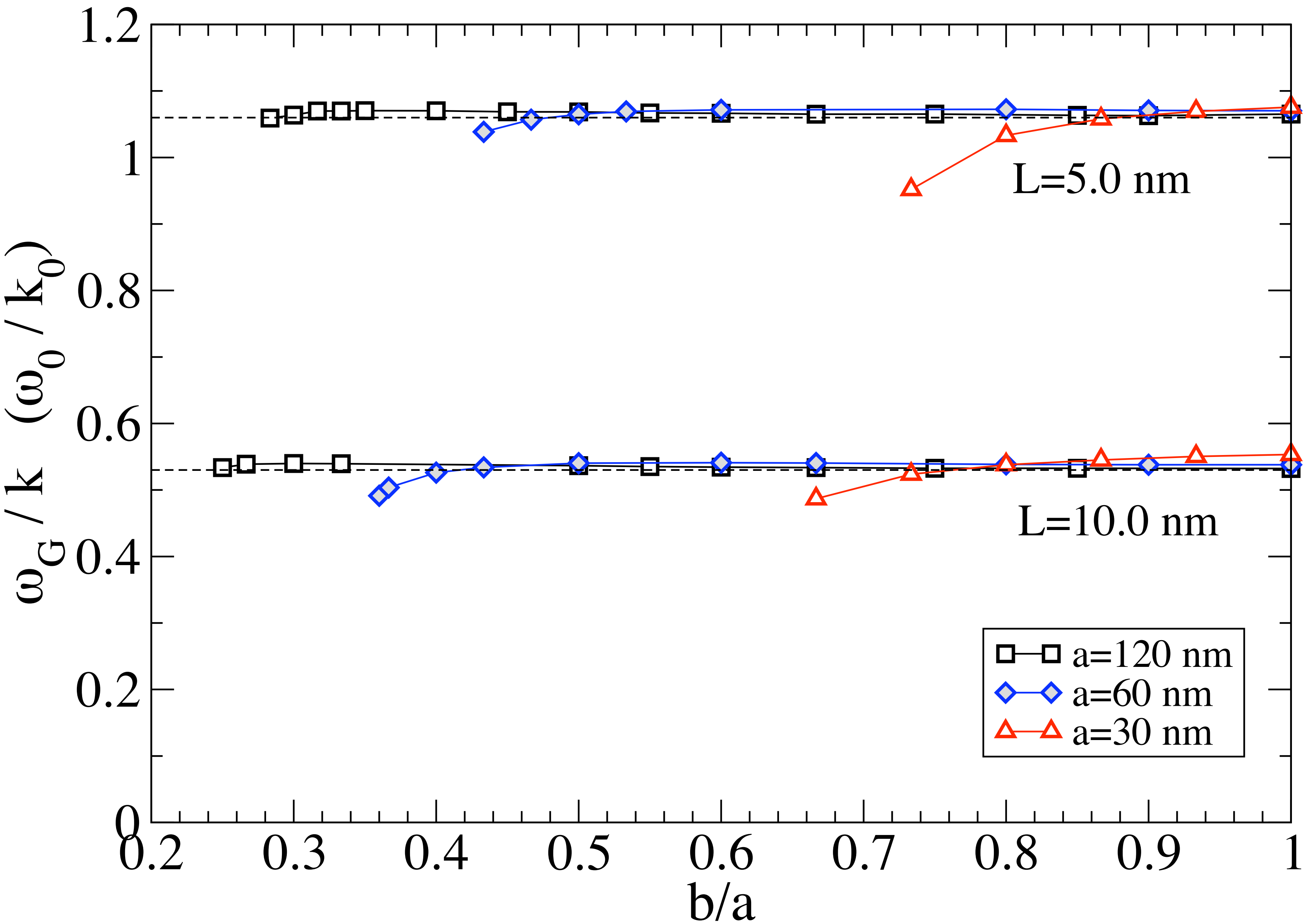
system ellipticity ϵ=*b*/*a* →

frequency unit is

$$\omega_0 \equiv \frac{\mu_0}{4\pi}\gamma M_s$$

(about 15 GHz for permalloy)

Dynamic gyrotropic frequencies scaled by average force constants (zero temperature)



frequency unit is

$$\omega_0 \equiv \frac{\mu_0}{4\pi}\gamma M_s$$

force constant unit is

$$k_0 \equiv A/\lambda_{\rm ex}$$

Dashed lines are the Thiele equation predictions:

$$\boxed{|\omega_G| = \frac{\bar{k}}{G} = \left(\frac{\mu_0}{4\pi}\gamma M_s\right)\left(\bar{k}\frac{\lambda_{\rm ex}}{A}\right)\left(\frac{\lambda_{\rm ex}}{L}\right)}$$

Vortex statistics in thermal equilibrium — Theory

Lagrangian for vortex core motion:

$$L = -\frac{1}{2}G(X\dot{Y} - Y\dot{X}) - \frac{1}{2}\left(k_x X^2 + k_y Y^2\right) \qquad \Rightarrow \quad \text{Thiele equation}$$

Canonical momentum:

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{V}} = -\mathbf{A} = \frac{1}{2}(GY, -GX). \qquad \leftarrow \quad \text{a constraint.}$$

Hamiltonian:

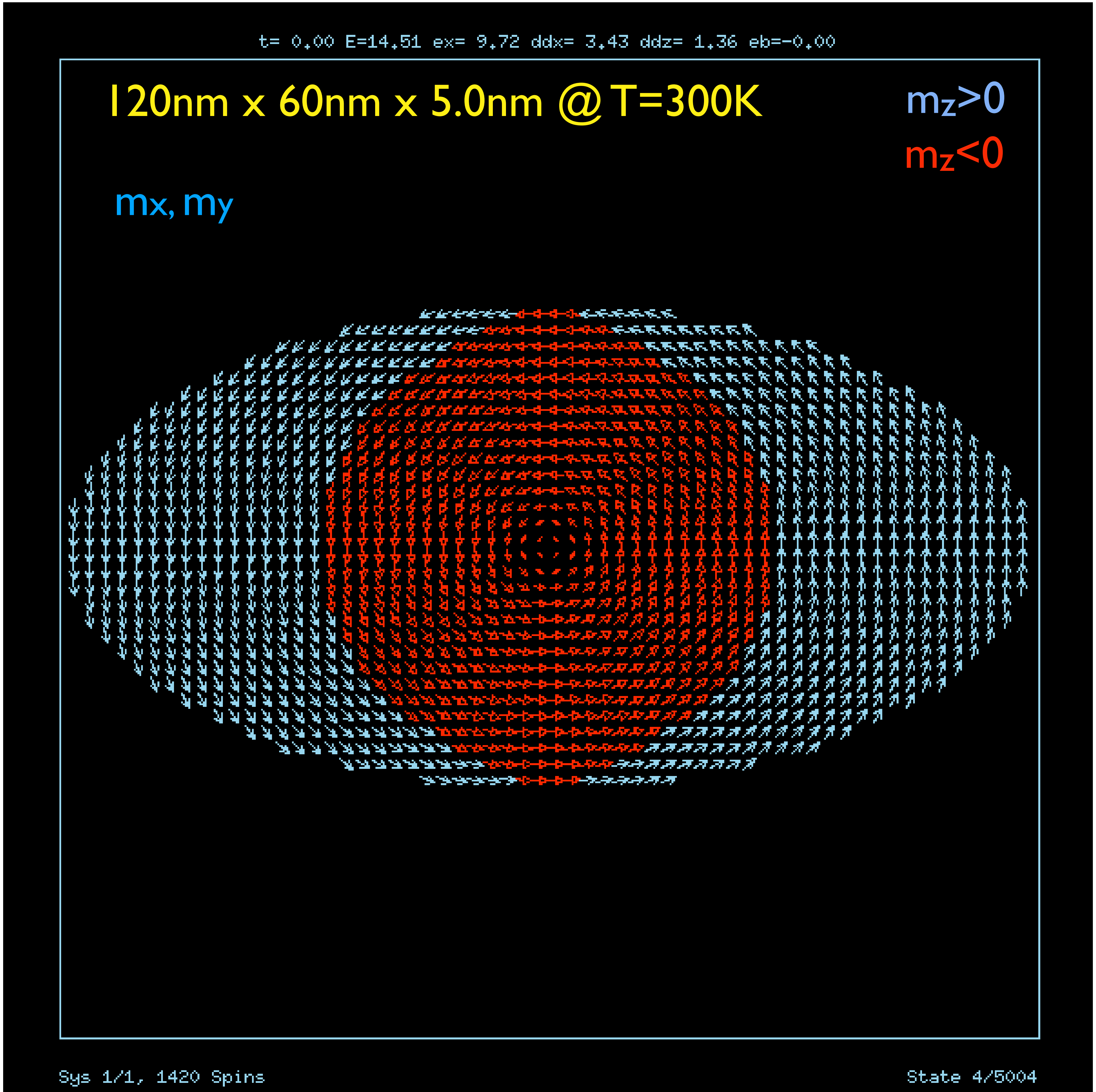
$$H = \mathbf{P}\cdot\mathbf{V} - L = U = \frac{1}{2}\left(k_x X^2 + k_y Y^2\right) = \frac{1}{4}\left(k_x X^2 + k_y Y^2\right) + \frac{1}{4}\left(\frac{2}{G}\right)^2\left(k_x P_y^2 + k_y P_x^2\right)$$

The phase space has only two independent quadratic degrees of freedom.

$$\langle H \rangle = k_B T$$

Expected vortex position distribution is a Boltzmann distribution:

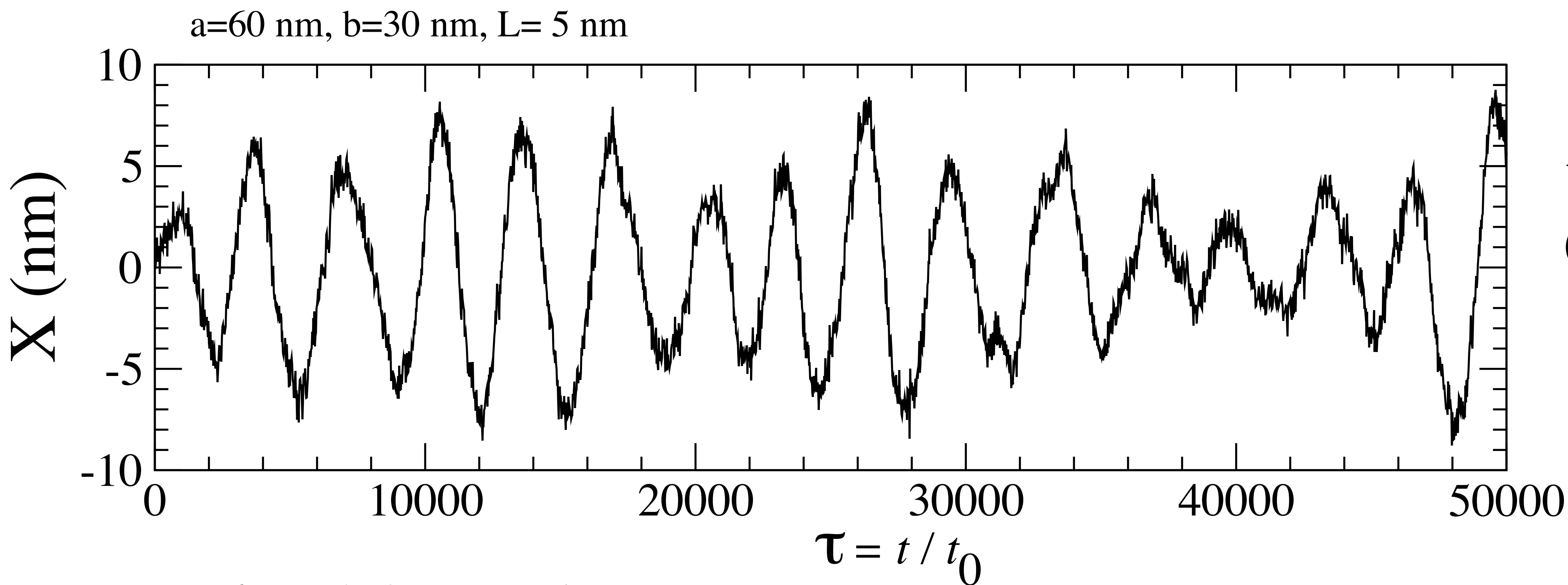
$$\boxed{p(X,Y) = \sqrt{\frac{\beta k_x}{2\pi}}e^{-\frac{1}{2}\beta k_x X^2}\sqrt{\frac{\beta k_y}{2\pi}}e^{-\frac{1}{2}\beta k_y Y^2}}$$



Dynamics from 2nd order Heun simulations of the Langevin-Landau-Lifshitz-Gilbert equations (finite temperature T>0)

Spontaneous motion from initial position $X_0=Y_0=0$.

Dynamics from 2nd order Heun simulations of the Langevin-Landau-Lifshitz-Gilbert equations (finite temperature T>0)

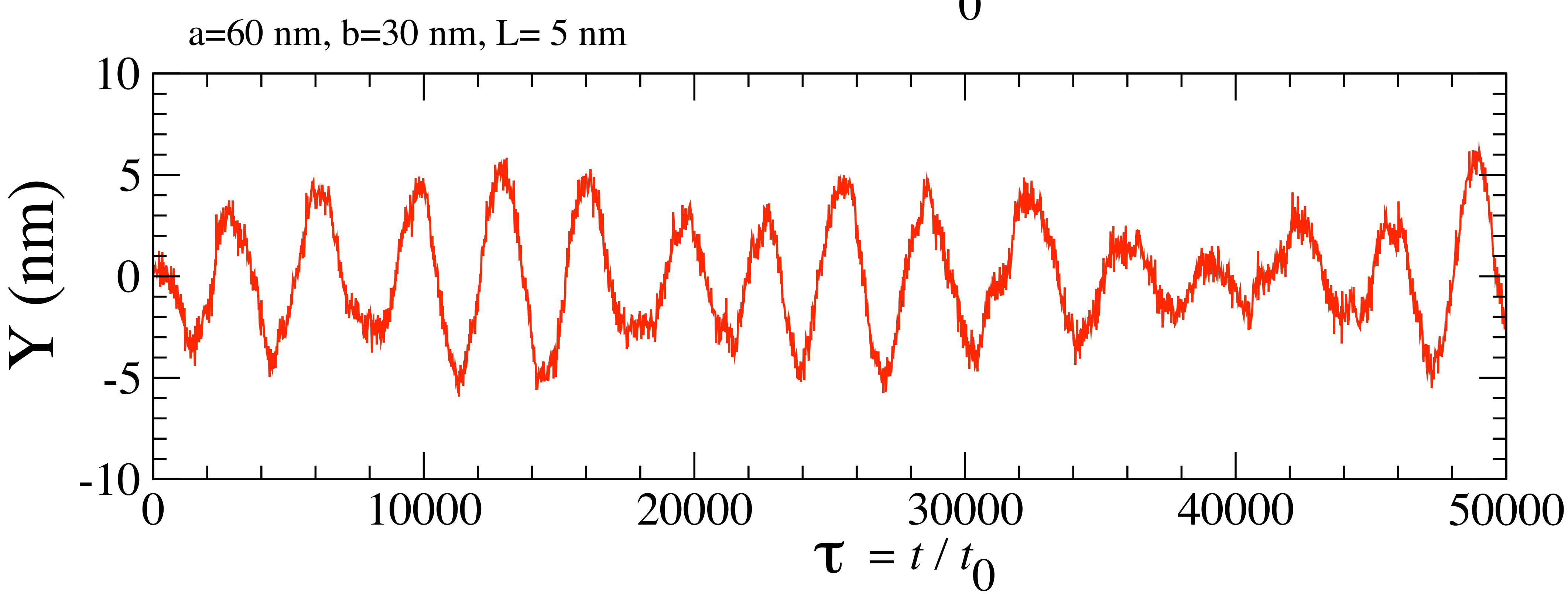


time unit

$t_0 \equiv (\gamma B_0)^{-1}$

(≈ 0.75 ps for Py)

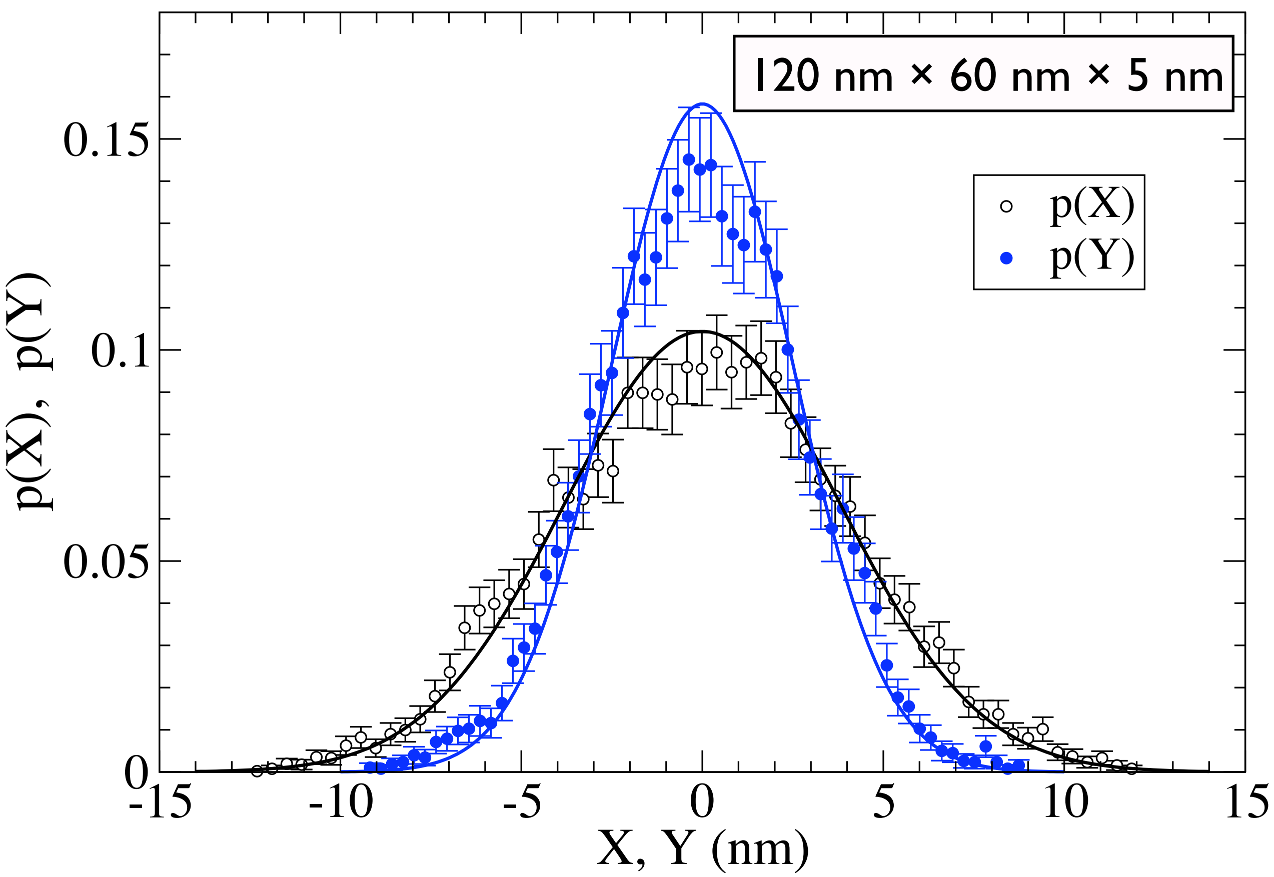
to τ = 2.5 × 10^5

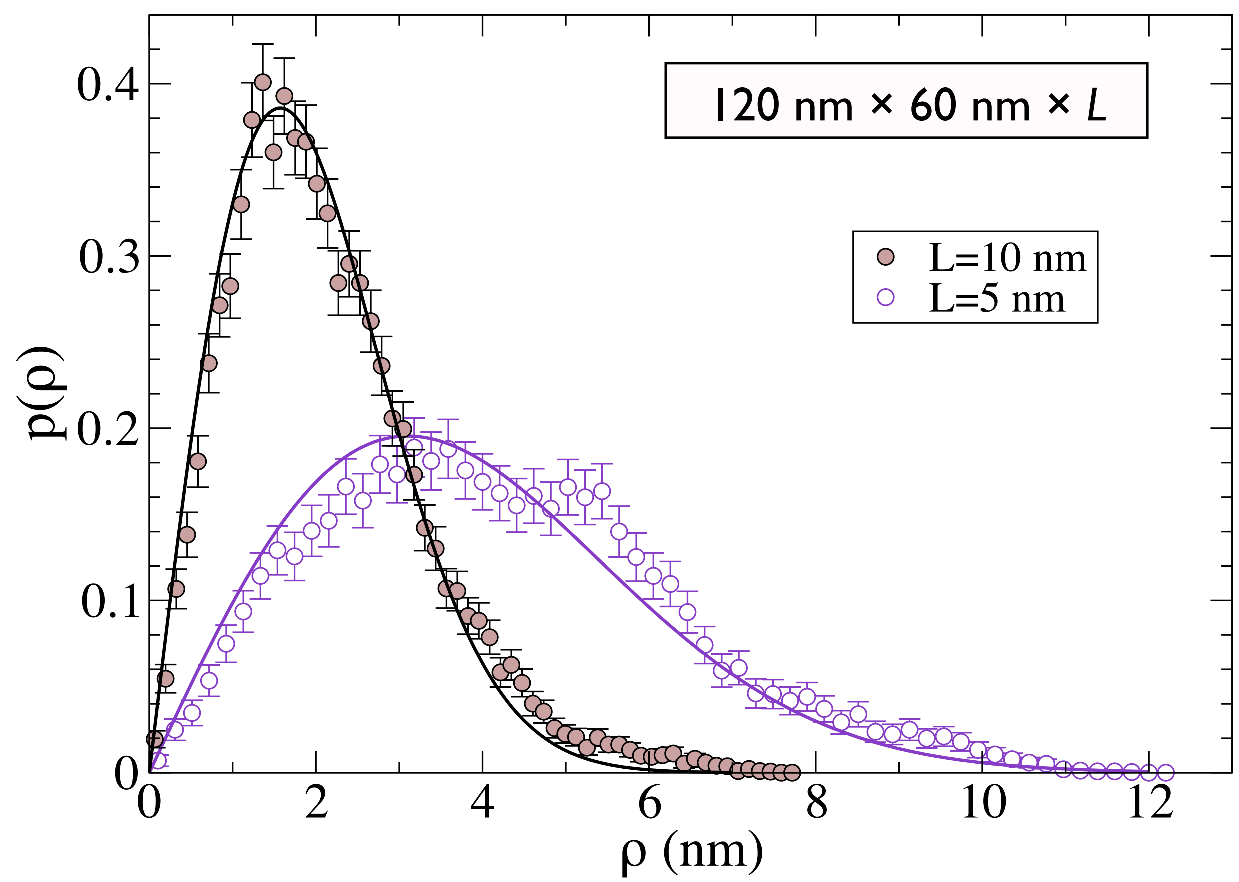


to τ = 2.5 × 10^5

Dynamics from 2nd order Heun simulations of the Langevin-Landau-Lifshitz-Gilbert equations (finite temperature T>0)

from averages over ~100 orbital periods (to $\tau = 2.5 \times 10^5$)
solid curves are the theoretical Boltzmann distribution.





radial coordinate $\vec{\rho} \equiv \left(\sqrt{e}X, \frac{1}{\sqrt{e}}Y \right)$

Summary, vortex gyrotropic dynamics in elliptic nanodisks *

Without an external magnetic field, elliptic gyrotropic movement begins naturally if the vortex is not in the center of the nanodot. The frequency ω_G is proportional to $\bar{k}$ / L .

The dynamics is well-described by a Thiele equation, provided the different force constants k_x and k_y along the principal axes are taken into account.

For large enough nanodisks, $\sqrt{\frac{k_x}{k_y}} = \frac{b}{a}$.

Thermal fluctuations can initiate the movement spontaneously. The amplitude along each axis is determined by the principle of equipartition of energy equally among degrees of freedom.

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