

CLASSICAL KINK DYNAMICS AND QUANTUM THERMODYNAMICS IN EASY-PLANE
MAGNETIC CHAINS WITH AN APPLIED MAGNETIC FIELD

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The classical dynamics of soliton-like kink excitations in one-dimensional (1-D) easy-plane ferromagnets (EPFs) and antiferromagnets (EPAs) with an applied magnetic field in the easy plane is studied. For EPFs, (e.g. CsNiF_3), numerical simulation has demonstrated an unusual energy vs. velocity dispersion, in contrast to the frequently assumed sine-Gordon (sG) theory. In addition to perturbed sG kinks, there are also stable kinks with negative effective mass. For fields greater than a critical field, only negative effective mass kinks exist. Simulation of kink-antikink ($\bar{K}K$) collisions has shown a variety of outgoing states, as a function of the field and incoming $\bar{K}K$ velocity. Generally, as a function of increasing field, the outgoing states are sG-like transmission, oscillatory bound states and reflection of the pair. Finally, transitions to chaos in the EPF with damping are demonstrated for specific applied AC plus DC fields.

For EPAs, (e.g. TMMC) improvements are made over previous work for the yz (out-of-plane) kinks by using spherical coordinates where the polar axis is parallel to the applied field (x-axis). Using these coordinates, linear stability analysis shows that there is a velocity-dependent critical field necessary for yz kink stability. The geometric similarity of yz and xy (in-plane) kinks has motivated a general kink Ansatz that includes the two as specific limits, and shows they can be considered as belonging to one continuously connected energy dispersion curve. The Ansatz and yz stability results are verified by a numerical simulation which also shows that xy kinks are stable both below and above the field

b_c at which static xy and yz kinks have equal energy. The xy kinks have a negative effective mass at fields greater than b_c , similar to the EPF kinks.

A quantum Monte Carlo method is used to obtain the thermodynamics of quantum EPF chains. For spin- $\frac{1}{2}$, vertex weights for the equivalent 2-D lattice are calculated analytically for 8 and 16 vertex models, and computations are applied to $(C_6H_{11}NH_3)CuBr_3$. The spin- $\frac{1}{2}$ effective Hamiltonian for the 2-D lattice is shown to consist of different nearest neighbor exchange in the two directions, plus a next nearest neighbor diagonal exchange and a 4-spin coupling. For spin-1, vertex weights are calculated analytically for 29 and 41 vertex models, and computations are applied to $CsNiF_3$. Comparison is made with experiment and sG soliton theory.

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by

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Biographical Sketch

Gary Wysin was born on October 21, 1956 in Port Clinton, Ohio, a small town on the shore of Lake Erie, where he also grew up. He was the fifth of the six children of Norbert and Rosemary Wysin. He graduated from Port Clinton High School in June of 1974, and in September of that year began studying electrical engineering at the University of Toledo, Ohio. After obtaining his Bachelor of Science degree in June of 1978, there was still a great deal more he wanted to know about science, so he entered the graduate school at Toledo to study in the Physics and Astronomy Department. Working with Professors Robert T. Deck and John Simon, he characterized some aspects of optical bistability at a linear dielectric-metal-nonlinear dielectric interface for his Master's thesis. After completing this work in August of 1980, he began further graduate work at Cornell University, with advisor Professor Jim A. Krumhansl. Upon looking for a summer research position in 1981, he found from JAK that there was an opening in the newly formed Center for Nonlinear Studies at Los Alamos National Laboratory, and so began the first of his visits to New Mexico. He started the present work on 1-D magnets with Dr. Alan R. Bishop that summer, and returned again in the summer of 1982, spring and summer of 1983, and all of 1984 for longer visits. On the return trips to Ithaca he also participated in work on nonlinear dynamics in phenomenological models of DNA, with JAK and fellow graduate students Denise Alexander and Angel Garcia.

In addition to physics he has many other intense interests including cycling, classical and folk guitar, running, fishing, skiing and hiking in the southwest.

After completing his Doctorate in August of 1985, he has taken a postdoctoral appointment at the University of Florida, working with Pradeep Kumar.

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