Influence of a perpendicular current on the circulation of a pinned magnetic vortex

G. M. Wysin*
Department of Physics, Kansas State University, Manhattan, KS 66506-2601
(Dated: Mar. 7, 2008)

The influence of a current's magnetic field on a vortex pinned in a thin magnetic nanodot is considered. Pinning due to a non-magnetic region or hole in the center of the nanodot is assumed. Then the vortex ground state is planar and has vorticity $q = +1$, with a double degeneracy due to the two opposite directions (curling or *circulation*) in which the spins can align around the hole. Dipole interactions lead to a finite energy barrier between the two states. Monte Carlo relaxation is used to study the current-induced reversal of the circulation. At least two different processes can take place during reversal: formation of an outward moving circular domain wall, or, nucleation of two outward moving vortices of opposite vorticity ($q = +1$ followed by $q = -1$).

VORTICITY IN A THIN CIRCULAR NANOMAGNET

In a thin circular nanomagnet of sufficient size, it is known that the lowest energy stable configuration of magnetic moments forms a vortex [1, 2], rather than a state of a single domain [3]. Such states have been observed, for example, in nanodots of permalloy [4, 5], Fe [6, 7] and Co [8, 9], and offer great possibilities for high-density and high-speed magnetic storage [10].

It is usual to expect that the spins interact via isotropic short-range ferromagnetic exchange interactions, together with long-range but weaker dipole-dipole interactions. The spins become mostly confined within the (xy) plane of the material, due to dipole-dipole interactions which act equivalent to an easy-plane anisotropy [11] which even varies with position in the nanomagnet [12]. Additionally, the dipole interactions cause the spins to lie parallel to the circular boundary at the edge, leading to a vortex state with either a clockwise (CW, negative) or counterclockwise (CCW, positive) curling or *circulation* of the spins around the circle. At the center, however, to reduce their exchange energy, the vortex core spins must tilt out of the xy-plane (out-of-plane vortex), acquiring either a positive or negative out-of-plane magnetization, M_z (positive/negative vortex *polarity*). This concentrated region of nonzero M_z has been used to locate the vortex [13]. The core out-of-plane tilting is similar to that found in easy-plane magnetic vortices [14–16], which exhibit a critical anisotropy strength above which the spins become confined in the easy plane [16–18].

Thus, in a uniform circular system, there are actually four different types of *out-of-plane* magnetic vortices that could be the ground state (+/− circulation, with +/− polarity). In all four of these cases, the usual quantized vortex *charge* or *vorticity*, is $q = +1$, being the charge that refers to a line integral of the gradient of in-plane spin angle, taken around any path that encloses the vortex core:

$$q = \frac{1}{2\pi} \oint \vec{\nabla}\phi \cdot d\vec{r}. \tag{1}$$

A vortex with a negative vorticity (i.e., antivortex, $q = -1$) would not have its spins follow the contour of the boundary, and hence, would possess considerably higher dipolar energy, but the same exchange energy.

The presence of multiple degenerate discrete ground states, separated by energy barriers, suggests using various tactics for switching between them. For example, an out-of-plane applied magnetic field removes the polarity degeneracy [19] and results in light and heavy vortices [20]. Vortex polarity switching due to a magnetic field pulse has been observed experimentally [21]. It is also expected that application of a spin-polarized current [22] or an in-plane magnetic field pulse [23] should switch the vortex polarity.

The above examples were concerned with changing the out-of-plane spin configuration. It is our intention here, rather, to concentrate on the switching of the *circulation* of a vortex pinned around a "hole" within a nanodot, whose effect is to minimize the out-of-plane spin tilting and eliminate the polarity. Changes in the circulation might be detected using a nonlocal spin-valve measurement [24]. We concentrate mainly on the effects caused by the magnetic field of the switching current (Oersted field) flowing perpendicular to the xy-plane. Miltat *et al.* [25] found using micromagnetics for rectangular permalloy platelets, that the Oersted field can have a significant effect on the switching of S and Leaf states. The current's inhomogeneous field was found to cause vortex nucleation, propagation, and interaction during switching. Here we analyze a simpler problem with higher symmetry, and focus mostly on the effects of the Oersted field.

Switching a vortex formed around a hole. In the study here, we consider some aspects of how an *unpolarized* central current could affect the vortex in a nanodot. To avoid the discussion of electron-magnetic ion scattering effects, we consider a current applied through the center of the dot, in a small region or "hole" that is separate from the magnetic ions. It is supposed that the current itself does not flow through the magnetic lattice. This may be difficult to accomplish in the laboratory, but nevertheless it is interesting to consider.

The magnetic model employed here is that for a thin nanomagnet: an array of spins on a two-dimensional (2D) square lattice, in a circular system with a hole (i.e., missing magnetic ions) cut out of the center. The spins are assumed to interact via isotropic exchange between nearest neighbors, and long-range dipole interactions, as well as with the circular magnetic field due to the applied current. The interest here is in two primary effects: (i) how the magnetic field due to the current through the vortex center can cause reversal of vortex circulation, and (ii) any fundamental effects initially produced on the vortex due to the hole.

The hole effects. For two-dimensional magnets with easy-plane anisotropy, analytical calculations and Monte Carlo simulations [26–29] predicted that vortices would be attracted and pinned by nonmagnetic impurities, which has been confirmed by experiment [30, 31]. The vortex energy is lowered when pinned at a nonmagnetic site, resulting in greater preference of single vortex [32] and vortex-pair [33] formation there. Monte Carlo calculations show that vortices preferentially form in equilibrium around nonmagnetic sites [34], causing the Berezinskii-Kosterlitz-Thouless transition to move to zero temperature when the impurity density reaches the percolation limit [35, 36].

The motion of a vortex pinned by a defect has been measured using time-resolved Kerr microscopy; high enough excitation amplitude can free the vortex [13]. Pinned vortices in nanoparticles are expected to affect the hysteresis curves [37] and can be manipulated by applied magnetic fields [13, 38].

An additional effect is that even on a single missing magnetic site, the strength of easy-plane anisotropy needed to stabilize a vortex in the planar form ($M_z = 0$ everywhere, resulting in zero polarity) is reduced compared to the pure system [39]. Taking this a step further, vortex stability calculations for a continuum system show that as the nonmagnetic region (or hole) is increased in radius, eventually any vortex pinned around that region will be forced to be planar [40]. This means that even for a model with isotropic exchange, such as in a nanodot, there is no stable out-of-plane vortex around a hole. Once dipolar interactions are included together with (isotropic) exchange, the tendency is even stronger for spins to remain within the plane of the nanomagnet. Because the singular core of the vortex is now removed by the presence of the hole, there is no need for spins to tilt out of the xy-plane. Thus, in this situation (and with low enough temperature), the spin dynamics of such a pinned vortex will be predominantly confined within the xy-plane, with only smaller dynamic out-of-plane motions. That only leaves the vortex circulation as a candidate for reversal.

Energy barrier and current effects. Clearly the presence of a current through the hole on which a vortex is centered will give an energetic preference for the vortex circulation having the same sense as the current's magnetic field. Then we are primarily interested in how the vortex circulation can be switched from its current state by a current whose field is in the opposite sense. The goal of the calculations here is to explore different possible paths that the spin configurations can follow when reversing the circulation.

Naturally, during the reversal, the spin system must have at least some of its members point against the magnetic field. There must be an energy barrier Δ over which the spin configuration's internal energy must pass, to get to the preferred state. This barrier would vanish in the limit of vanishing dipole coupling, because all spins could rotate together to their preferred direction, with no change in exchange energy. With dipolar interactions present, the barrier must increase. But if the spins reverse their alignment (and circulation) in well-defined groupings or processes, the time evolution of the system's internal energy will be subsequently affected. Therefore, the switching might possibly proceed along different paths in the configuration space, depending on the relative strengths of the dipole couplings compared to the applied current. Here we make some estimates of the possibilities, using a Monte Carlo (MC) approach that includes thermal fluctuations.

THE MODEL AND ENERGY BARRIERS

For a thin nanomagnet, rough estimates can be obtained by using a 2D model, taking classical spins of length S and magnetic moment $g\mu_B S$, located on sites of a square lattice in the xy-plane. The system is a circle of radius R, with a hole cut out of the center, of radius R_h, and the origin of coordinates is at the center of the hole. Any point in the system can be specified either by its Cartesian coordinates (x, y) or equivalently, by radius and azimuthal angle, (r, ϕ). We assume nearest neighbor isotropic ferromagnetic exchange coupling, J, together with long-range dipolar interactions, the interaction with the field of a central current I, and thermal fluctuations via Monte Carlo.

The exchange hamiltonian between spins $\vec{S}_i$ is

$$H_{\text{ex}} = -J \sum_{(i,j)} \vec{S}_i \cdot \vec{S}_j \tag{2}$$

where (i, j) indicates summing over all nearest neighbor pairs, with i and j denoting lattice sites. Any magnetic moment $\vec{m}_i = g\mu_B \vec{S}_i$ generates a dipole field at position $\vec{r}$ measured away from that spin's site, according to

$$\vec{B}_i(\vec{r}) = \left(\frac{\mu_0}{4\pi}\right) \frac{3(\vec{m}_i \cdot \hat{r})\hat{r} - \vec{m}_i}{r^3} \tag{3}$$

Any pair of spins $\vec{S}_i$ and $\vec{S}_j$ separated by a displacement $\vec{r}_{ij} = \vec{r}_j - \vec{r}_i$, contributes a dipolar interaction, $U_{i,j} = -\vec{B}_i \cdot \vec{m}_j$. Then the total dipolar interaction in the system is

$$H_{\rm dd} = -\left(\frac{\mu_0}{4\pi}\right) \sum_{i>j} \frac{[3(\vec{m}_i \cdot \hat{r}_{ij})(\vec{m}_j \cdot \hat{r}_{ij}) - \vec{m}_i \cdot \vec{m}_j]}{r_{ij}^3}, \quad (4)$$

where $\hat{r}_{ij}$ is the unit vector pointing from site i towards site j. The sum with $i > j$ avoids double counting the interactions.

It is convenient to work the calculations in terms of unit length spins, $\hat{\sigma}_i \equiv \vec{S}_i/S$, and additionally, scale out the lattice constant a to give dimensionless distances. Then the exchange terms are proportional to the energy unit JS^2, whereas, the dipole interaction is proportional to the energy parameter,

$$D = \left(\frac{\mu_0}{4\pi}\right) \frac{(g\mu_B S)^2}{a^3}. \quad (5)$$

In terms of D we have a form more convenient for numerical calculations,

$$H_{\rm dd} = -D \sum_{i>j} \frac{[3(\hat{\sigma}_i \cdot \hat{r}_{ij})(\hat{\sigma}_j \cdot \hat{r}_{ij}) - \hat{\sigma}_i \cdot \hat{\sigma}_j]}{(r_{ij}/a)^3}. \quad (6)$$

Taking the exchange as the basic energy unit, the ratio $\delta \equiv D/JS^2$ will indicate the relative dipole coupling strength. Exchange and dipole terms form the intrinsic or internal system energy,

$$H_{\rm int} = H_{\rm ex} + H_{\rm dd}. \quad (7)$$

One of our primary interests will be in the evolution of $H_{\rm int}$ over any barrier during a switching process.

Additionally, the central current I (along $\hat{z}$, positive when out of the xy-plane) will be the driving force for evolution over a barrier. It produces a magnetic field with only an azimuthal component, along the local azimuthal axis $\hat{\phi}_i$,

$$\vec{B}(\vec{r}_i) = \frac{\mu_0 I}{2\pi r_i} \hat{\phi}_i, \quad (8)$$

leading to the interaction,

$$H_{\rm B} = -\sum_i \vec{m}_i \cdot \vec{B}(r_i). \quad (9)$$

Writing this in scaled form,

$$H_{\rm B} = -(g\mu_B S) \frac{\mu_0 I}{2\pi a} \sum_i \frac{\hat{\sigma}_i \cdot \hat{\phi}_i}{(r_i/a)} \quad (10)$$

Then we define the effective energy scale of the current,

$$K = (g\mu_B S) \frac{\mu_0 I}{2\pi a}, \quad (11)$$

and the dimensionless ratio, $\kappa \equiv K/JS^2$ gives its relative importance compared to the exchange forces. In general,

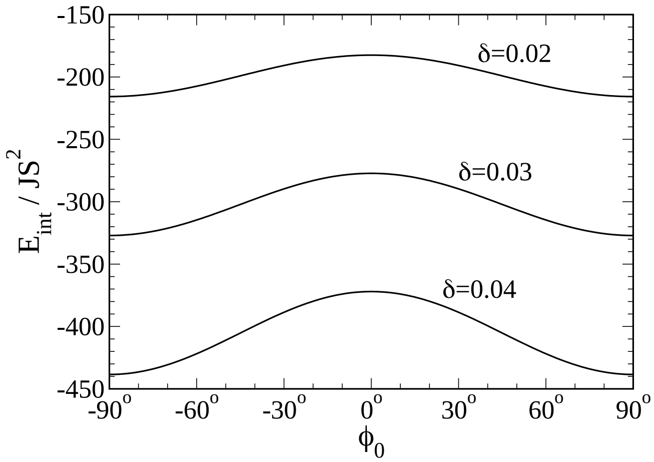

FIG. 1: Planar vortex internal energy $E_{\rm int}(\phi_0)$ for a system of radius $R = 40a$ with a hole of radius $R_h = 4a$, at the scaled dipole strengths $\delta = D/JS^2$ indicated. The vortex circulation is $C = \sin\phi_0$, where ϕ_0 is the global rotation of the spins away from the radial direction.

then, we have the parameters δ, κ and scaled temperature $\tau = k_B T/JS^2$ to describe the couplings.

Planar vortex, circulation, and energy barriers. Next we do a simple evaluation of the changes that could be expected in the intrinsic system energy when a planar vortex reverses its circulation. We can consider only in-plane spin components, because we assume an adequate sized hole or sufficient dipole strength such that the preferred alignments are within the xy-plane.

A $q = +1$ in=plane vortex structure can be defined by giving the in-plane spin angles, $\Phi_i \equiv \tan^{-1}(S_i^y/S_i^x)$ (with all $S^z = 0$),

$$\Phi_i = \phi_i + \phi_0, \quad (12)$$

where $\phi_i = \tan^{-1}(y_i/x_i)$ is the usual azimuthal position of a site i (angle measured from the $+x$-axis). The adjustable parameter ϕ_0 gives a global rotation of all the spins starting from an initial radial direction. Thus, $\phi_0 = 0$ simply gives all spins pointing radially outward from the system center, whereas, $\phi_0 = 90°$ produces a vortex with all spins directed counterclockwise around its center, following the system boundary. Similarly, $\phi_0 = -90°$ directs all spins in the opposite sense, clockwise around the system, but again following the system boundary. The choices $\phi_0 = \pm 90°$ give two degenerate vortices, for which we say the *circulation* is $C = \pm 1$, respectively. Of course, letting $\phi_0 = 0$ still produces a $q = +1$ vortex, but we could say its circulation is $C = 0$. In the presence of a current through the system's center, it is clear that the vortex with $C = +1$ will be preferred if the current is along $\hat{z}$, and the vortex with $C = -1$ is preferred when the current is along $-\hat{z}$.

More generally, in the presence of thermal fluctuations and even for states not containing a vortex, the circula-

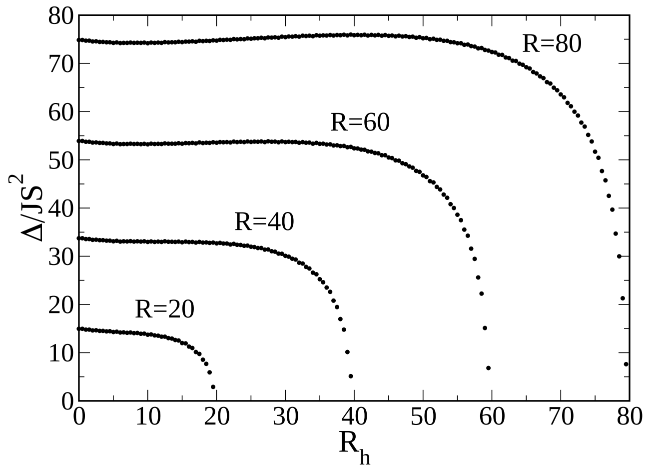

FIG. 2: Estimated energy barriers $\Delta = E_{\rm int}(0^\circ) - E_{\rm int}(90^\circ)$, for reversal of circulation of a planar vortex, assuming a coherent rotation of spins, for relative dipole strength $D/JS^2 = 0.02$. The system radius R and hole radius R_h are given in units of the lattice constant a. The barrier is directly proportional to the dipole coupling D.

tion (or curling) of any spin configuration can be defined according to a general expression

$$C = \frac{1}{N}\sum_i \hat{\sigma}_i \cdot \hat{\phi}_i. \quad (13)$$

where again, $\hat{\phi}_i$ is the azimuthal unit vector at a site. Each term in the sum ranges from -1 to $+1$, which is then normalized by the total number of spins, N. Hence, the circulation falls in the continuous range $-1 \le C \le +1$, and how closely it approaches the limits gives a sense of the alignment of the spins around the circular boundary. For the planar vortex (12), it is obvious that the circulation is $C = \sin\phi_0$. Clearly, larger absolute values of C should be more greatly favored at stronger dipole coupling, δ.

Initially, it is interesting to observe the change in vortex internal energy $E_{\rm int}$ as a function of ϕ_0, or equivalently, as a function of C. The expression (12) will be close to the actual vortex structure on the square lattice because the dipolar and discreteness effects only make minor modifications. Typical results for $E_{\rm int}(\phi_0)$ are close to sinusoidal, as shown in Fig. 1.

Assuming a vortex could reverse its circulation via a coherent rotation of all spins, just by slowly changing ϕ_0, results in an obvious energy barrier. It is clear that the barrier, $\Delta = E_{\rm int}(0^\circ) - E_{\rm int}(90^\circ)$, is zero when $D = 0$ and must be proportional to D otherwise. Also, the barrier changes slowly as the hole size increases, but it increases with increasing system size R, as shown in Fig. 2.

When a current I is turned on, the magnetic energy effect [hamiltonian (9)] for this planar vortex can be estimated quickly by a continuum integral:

$$E_B = -K \int \frac{d^2r}{a^2}\, \frac{\hat{\sigma}\cdot\hat{\phi}}{(r/a)} = -2\pi K \frac{(R-R_h)}{a} \sin\phi_0. \quad (14)$$

If the current's magnetic field has the opposite sense as $C = \sin\phi_0$, then the energy shifts upward by $2\pi K(R - R_h)/a$ compared to the situation without a current. Then roughly one could expect that a reversal must become easy when the extra magnetic energy lifts the system over the barrier, or $2\pi K(R-R_h)/a \approx \Delta$. This last relation can be considered to define a critical current level for switching, which is tested in the MC simulations. Specifically, it suggests that the critical current could decrease as the effective "system radius" $R - R_h$ is increased (but only if Δ does not change with $R - R_h$). Of course, all of this is only an upper limit, because the barrier found assumes all spins rotate in unison. If the system reverses circulation by other paths (such as a circular domain wall around the system), then the barrier that is surpassed could be smaller. This possibility is tested by using a Monte Carlo scheme to watch the relaxation after turning on a current in the "wrong" direction (i.e., a reversing current whose field is opposite to C).

MONTE CARLO RELAXATION

A Monte Carlo approach is useful for investigating vortex relaxation and stability, because it realistically includes thermal fluctuations. It also will take into account the dynamically important out-of-plane motions.

To test these ideas, we applied a standard Metropolis algorithm using single spin flip moves, as developed in many references [41–45], and applied to easy-plane Heisenberg models with vacancies in Ref. 34. For a chosen temperature T, the total hamiltonian $E = H_{\rm ex} + H_{\rm dd} + H_B$ for a system of N spins is employed. A Monte Carlo step (MCS) is defined by making trial spin moves on all N spins, chosen in a random sequence. A chosen spin σ_i is changed by adding a small increment in a random three-dimensional direction, and then renormalizing the spin to unit length, accepting or rejecting each change according to the Metropolis algorithm: Changes that reduce the total system energy are always accepted, whereas, changes that increase the system energy are accepted only with a probability of $\exp(-\Delta E/k_BT)$. The spin increments are dynamically adjusted in length so that the acceptance rate falls between 30% and 60%. Tables of inter-spin distances (and their powers) were determined once and then re-used to speed the dipole energy evaluation. Although the sequence of MC states is not a real time evolution, it gives a good idea of what could happen in the presence of thermal fluctuations and is an interesting alternative to the usual micromagnetic simulations.

MC experiments: In a typical simulation, the planar vortex (12) with $C = +1$ is the initial state, but its perfect order can be present only at zero temperature. Therefore, before turning on a reversing current, a number of MC steps (about 1000–2000 MCS) are performed to equilibrate the system for the chosen temperature. The thermal fluctuations substantially increase the exchange energy, have a lesser effect on the dipolar energy, and cause the circulation to acquire a magnitude less than one. Then the desired current is switched on suddenly, causing an energy increase due to the magnetic field energy E_B, close to the value in equation (14). Then the subsequent evolution of spins is tracked. In particular, the internal energy is monitored to watch the size of barrier it must pass over. Full reversal of the vortex can be detected after C changes sign and acquires the magnitude it had before the current was turned on.

REVERSAL PROCESSES

The MC experiments just described were carried out for small systems at different relative dipole coupling $\delta = D/JS^2$ and current $\kappa = K/JS^2$. The dipole interactions relative to exchange define a physical length scale for the problem, the magnetic exchange length, $l = a\sqrt{\frac{JS^2}{4\pi D}}$. Caputo *et al.* [12] showed that in a thin system without a hole, the vortex state (an out-of-plane one) is the ground state provided the system is large enough, $2R > 30l$. Otherwise, the ground state would be of homogeneous in-plane magnetization. For instance, relative dipole strength $\delta = 0.08$ gives $l \approx a$, which is the limit at which discreteness of the lattice becomes important (i.e., at $\delta > 0.08$, or weak exchange). Here, with the hole present, the vortex state is more greatly preferred, even for smaller systems, although we have not carried out a full analysis of the necessary conditions on R, R_h and l. Based on the discreteness limit, we tested some δ both above and below 0.08, keeping in mind that the higher values of δ relate to larger energy barriers for reversal, whereas, the lower values give a more continuum model.

We used many simulations on a small system ($R = 20$, $R_{\rm hole} = 2.0$, $\tau = 0.1$), at $\delta = 0.02, 0.04, 0.08, 0.12$, and various other simulations on larger systems. We found general trends for reversal as follows.

At very low reversing current κ, there is no reversal. The system stays in the original vortex of positive circulation, as can be expected. Clearly, there is a minimum current needed to supersede the minimum energy barrier to arrive at the opposite circulation.

As κ is increased beyond that minimum, reversal begins to be possible, and it can take place by at least two different processes. The first is via formation of a circular domain wall (CDW). The spins start to rotate together only in the region just outside the hole, where the current's magnetic field is strongest. These central spins are the first to rotate sufficiently until they have aligned with the magnetic field. In the meantime, a circular 180° domain wall propagates outward toward the system boundary, reversing the spins at larger radius as it moves outward (see Fig. 3). Eventually the CDW exits the system, leaving behind the vortex still pinned around the hole, but with reversed circulation.

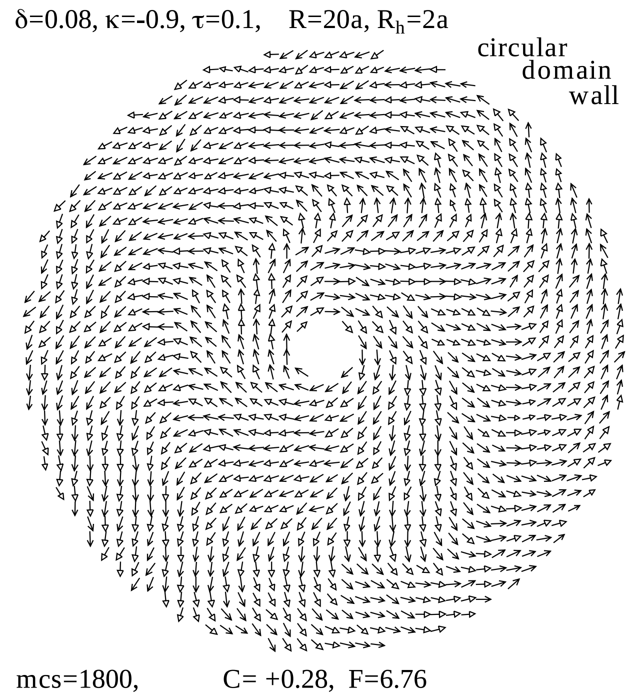

FIG. 3: An example of how a circular domain wall reverses the circulation, for a system with parameters indicated. The reversing current (into the page) was turned on after 1000 MCS. Projections of the spins on the xy-plane are shown.

We can define an additional quantity that we used to identify this kind of reversal. Presence of a circular domain wall forces some spins to point either radially inward or outward, hence it is helpful to compute the average of the radial spin components, weighted by the radius,

$$F = \frac{1}{N} \sum_i \hat{\sigma}_i \cdot \vec{r}_i. \tag{15}$$

This quantity is somewhat like a *flux* and we will refer to it by this name, because it acquires larger absolute values only when there is a net outward or inward organization in the spin configuration. In particular, it tends to be large when the circulation is changing rapidly, hence C and F carry complementary information.

An example of the behavior of C and F during reversal is shown in Fig. 4, for a system of radius $40a$, hole radius $4a$. A reversing current $\kappa = -0.3$ was used for dipole strength $\delta = 0.02$, corresponding to magnetic exchange

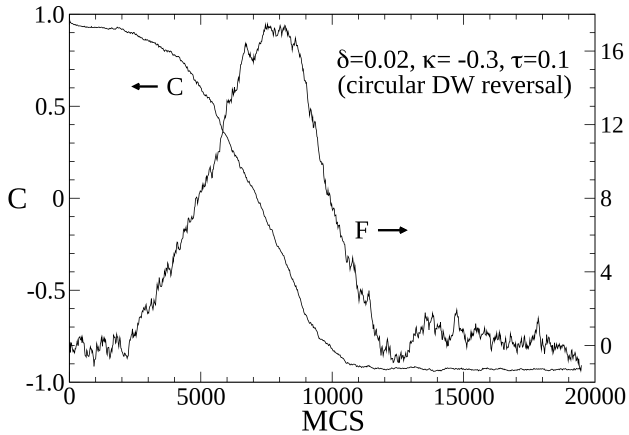

FIG. 4: For a system with $R = 40a$, $R_h = 4a$, $D = 0.02JS^2$, the circulation C and flux F during reversal via formation of a circular domain wall. The reversing current was turned on after 2005 MCS. The large values of F are due to large numbers of spins pointing radially outward as the CDW moves outward through the system.

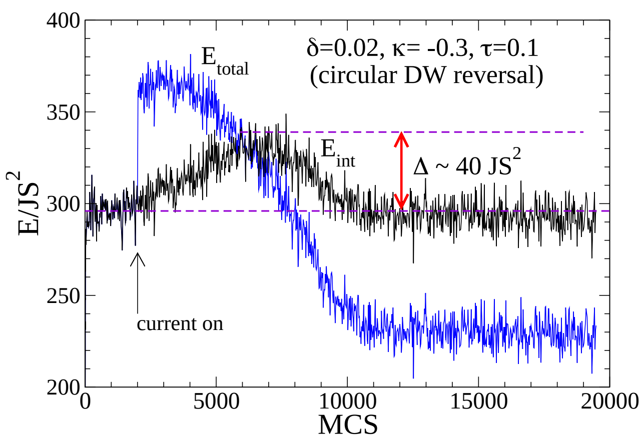

FIG. 5: For a system with $R = 40a$, $R_h = 4a$, $D = 0.02JS^2$, the internal energy and total energy ($E_{\rm int} + E_{\rm B}$) during the reversal via formation of a circular domain wall of Fig. 4. Δ is a rough estimate of the actual energy barrier surpassed during the process.

length $l \approx 2a$. The system was initially thermalized for 2005 MCS before turning on the current. It is seen that C makes a smooth transition from near +1 to near -1, while F also smoothly rises to large values, until falling back towards smaller values when the CDW leaves the system. The associated internal energy and total energy ($E_{\rm int} + E_{\rm B}$) are shown in Fig. 5. The internal energy passes over an obvious barrier, estimated around $40JS^2$ for this system. Somewhat surprisingly, this is slightly *larger* than the barrier estimated from a uniform global rotation of the vortex angle ϕ_0 of Eq. (12) [seen in Fig. 1 or Fig. 2, $\Delta = 33.2JS^2$ at these parameters]. This is probably due to the extra exchange energy of the CDW, that was not included in the global rotation calculation.

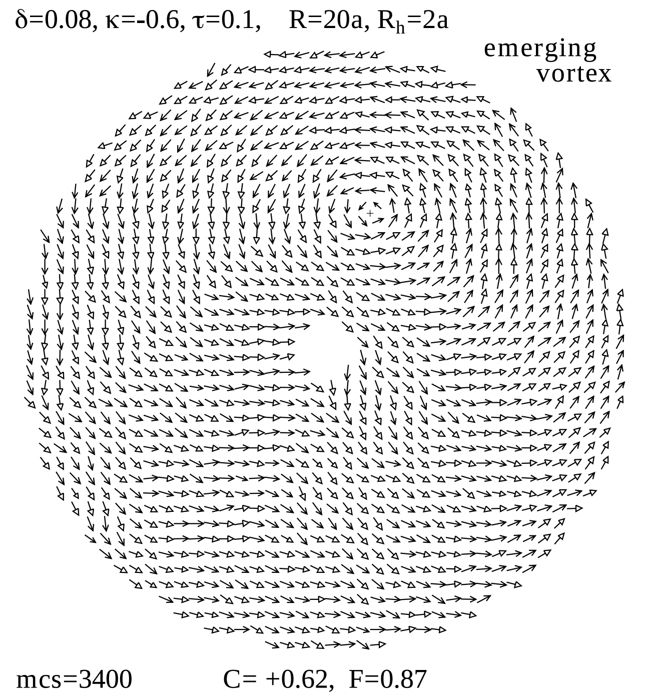

FIG. 6: The spin structure caused by a vortex that forms near the hole, moving outward and eventually reversing the circulation. The system parameters are indicated. The reversing current (into the page) was turned on after 1000 MCS.

A second type of reversal process is via formation and propagation of vortices out of the system. Initially, a $q = +1$ vortex can form within the spins, near the hole. The vortex finds it energetically favorable to move outward (see Fig. 6), until it leaves the system. This then results in there being no net vorticity [Eq. (1)] in the system, however, a radial 180° domain wall (RDW) can form, connecting the hole to the outer edge, as in Fig. 7. If the domain wall remains, the reversal of circulation is incomplete, which sometimes occurs if the reversing current is too weak. In that case, the circulation may not approach very close to -1, becoming "stuck" near a value around -2/3, although the net vorticity of the system would still be zero. To finish the reversal process, a $q = -1$ (anti) vortex must also form near the hole, and propagate outward through the system, typically moving along the domain wall. Once that has taken place, the final state is that of a $q = +1$ vortex pinned around the hole, with reversed circulation near -1.

Generally, but not universally, reversal via vortices seems to be favored as the reversing current is increased. It is interesting to contrast reversal via vortices with reversal via a CDW, changing only the current. For the same system parameters that gave CDW reversal above ($R = 40a$, $R_h = 4a$, $\delta = 0.02$, Figs. 4 & 5), and even using the same random seed and same thermalized state (at 2005 MCS), the reversing current was doubled to

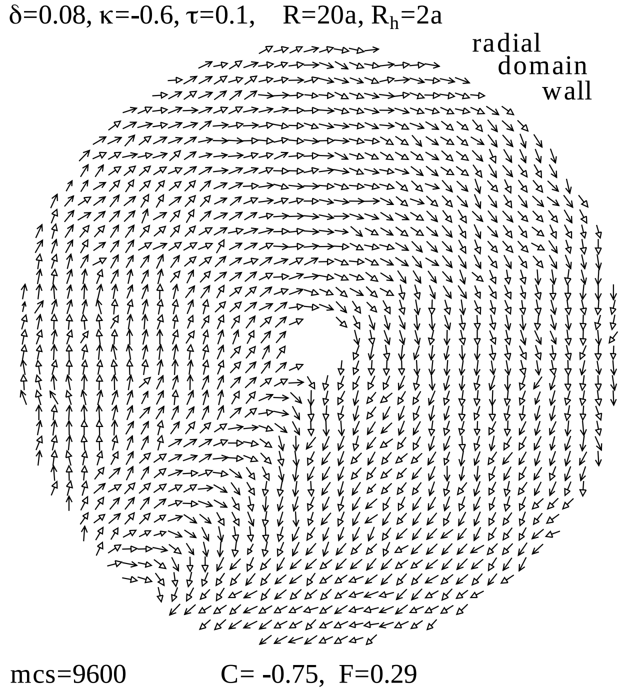

FIG. 7: A radial domain wall spin structure that results after a positive vortex (the one in Fig. 6) has left the system, for the parameters indicated. This structure is metastable. Complete reversal of the circulation from here is possible only if a $q = -1$ vortex can form near the hole and leave the system.

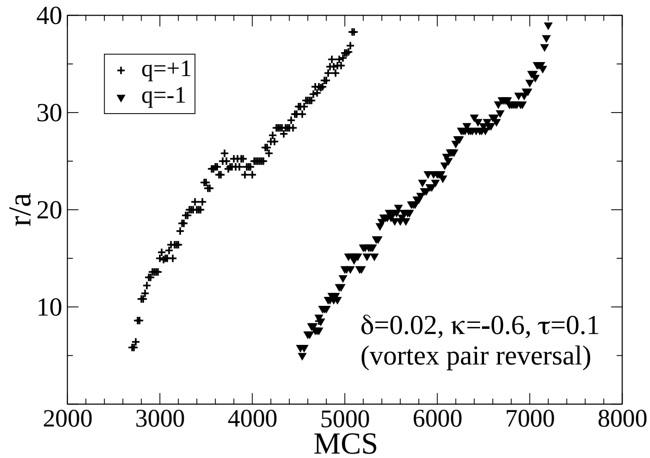

FIG. 8: The radial distances from the center of the hole for a vortex ($q = +1$) and antivortex ($q = -1$) that form during a reversal process, for the parameters indicated. The reversing current was switched on at 2005 MCS. Once their radial positions reach 40a, vortices have left the system. (See also Figs. 9 and 10)

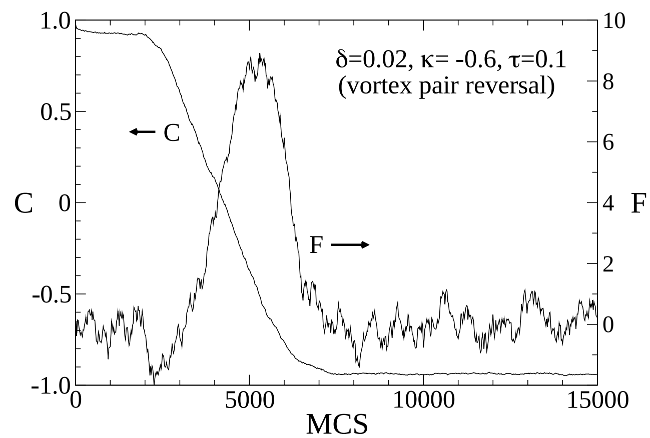

FIG. 9: The circulation and flux during a vortex pair reversal process (see Figs. 8 and 10), for the parameters indicated.

$\kappa = -0.6$. What resulted was an emerging vortex, which was joined shortly by an emerging antivortex ($q = -1$) on the opposite side of the system. The vortex positions were tracked. Their radial positions versus MC-steps are plotted in Fig. 8. Both eventually leave the system and facilitate the reversal. These movements result in the circulation and flux as shown in Fig. 9, and corresponding energy plot in Fig. 10. The presence of the vortices causes changes in F, but not as great as during reversal with a CDW. Even more interesting, is that the energy barrier now surpassed is somewhat larger, about $\Delta \approx 60JS^2$, although it is the same system that could give reversal by CDW at a lower current. The other difference is that the reversal now proceeds at a higher rate with the higher current.

Fig. 10 also displays another effect. It can be seen that the internal energy before and after the reversal do not match exactly. After the reversal, the internal energy is slightly lower, but this could be expected, because there was no current *before* reversal, whereas, after reversal, the reversing current is still being applied. This slight difference can be traced primarily to the exchange energy: the continued application of the current and its magnetic field keeps the spins more organized, reducing their fluctuations and exchange energy. The dipolar energy before and after reversal is nearly unchanged, hence the internal energy is reduced after the reversal in this simulation.

Near the weakest current that causes reversal, it seems that either CDW or vortex formation may take place. But when the reversing current is set to larger values, the reversal tends to take place more rapidly, and tends to do so via vortex formation. In the typical reversal via vortex formation, first a vortex emerges, followed by formation of a radial DW, and then followed by emergence of an antivortex to complete the reversal. At high enough current, as seen above, it is possible to skip the intermediate RDW step, with the system generating the $q = -1$ vortex *before* the $q = +1$ vortex has left the system. This type of reversal appears to be the fastest.

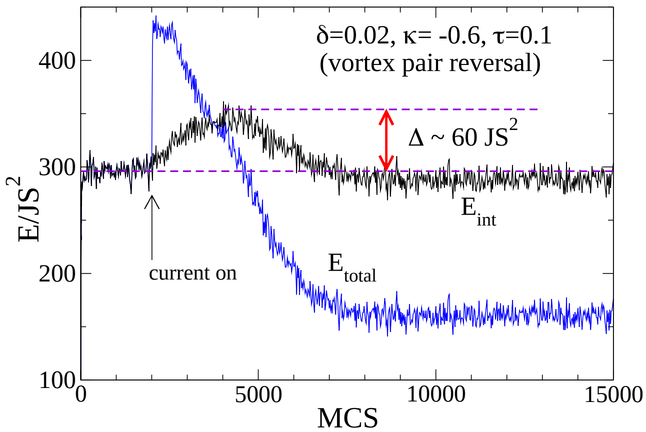

FIG. 10: The internal energy and total system energy (includes E_B) vs. Monte Carlo steps during the vortex pair reversal process of Fig. 8, for the parameters indicated. Δ is a rough estimate of the energy barrier surpassed.

DISCUSSION AND CONCLUSIONS

These results do not include a precise determination of the actual sizes of energy barriers that need to be crossed, in general, nor of the critical currents for reversal. Also, the temperature was fixed at $\tau = 0.1$, and used simply to include the typical thermodynamic fluctuations.

Roughly speaking, however, there seems to be no reversal at all unless the magnetic field energy (E_B, due to the current) is at least about 30% of the dipolar energy ($E_{\rm dd}$) in the system. Surprisingly, the surpassed barriers are somewhat larger than the barriers calculated for a uniform rotation of all spins, as appears in $E_{\rm int}(\phi_0)$ for the ideal vortex. Of course, that idealized calculation refers to a zero-temperature limit, and does not include any exchange energy. Furthermore, the global uniform rotation is thermodynamically unlikely, and the reversal must be dominated by other more probable (i.e., higher entropy) paths. The processes found here effectively rotate smaller groups of spins at any time (localized vortex or DWs), which requires extra exchange energy with the neighbors of those spins, hence the actual barriers are higher than expected from $E_{\rm int}(\phi_0)$ for an ideal vortex.

In summary, we found that a central reversing current passing through a hole in a thin circular nanomagnet can cause the vortex circulation to reverse by different processes. Near the minimum current that causes reversal, it appears to be more likely to see reversal proceed through formation of a circular domain wall that forms at the hole, and moves outward. Higher reversal current, however, leads to formation of an emergent $q = +1$ vortex, which leaves the system, followed by formation of a radial domain wall, with reversal finally completed if a $q = -1$ vortex can form and emerge from the system. In fact, at moderate to higher currents, the reversal almost always takes place via vortex formation; it was very difficult to see the reversal proceed due to a circular domain wall. At still higher current, the system might not form an intermediate state with a radial domain wall, because the reversal can even take place with both the $q = +1$ and $q = -1$ vortices emerging from the system together. Possibly, the different processes could be distinguished experimentally using methods such as a nonlocal spin-valve measurement [24].

In some instances, at moderate current, the reversal becomes "stuck" at the radial domain wall stage, after a vortex has already emerged. The system's final state has a circulation near -2/3, $F \approx 0$, and no net vorticity. The direction of the RDW was random, but once formed, it preserved its azimuthal position. This is also an interesting metastable state; control of the direction of the RDW could be a future challenge.

The formation of propagating vortices during reversal is consistent with micromagnetics simulations for rectangular permalloy platelets that included the Oersted field [25]. Certainly there are other more complex spin states present during reversal of the circular nanodot, that will deserve further study. Especially in larger systems, the dipole interactions begin to favor a complex set of domains, whose reversal must take even more complicated paths than discussed here.

* Electronic address: wysin@phys.ksu.edu; URL: http://www.phys.ksu.edu/personal/wysin

[1] N.A. Usov and S.E. Peschany, J. Magn. Magn. Mater. **118**, L290 (1993).
[2] A. Hubert and R. Schäfer, *Magnetic Domains* (springer-Verlag, Berlin, 1998).
[3] P.-O. Jubert and R. Allenspach, Phys. Rev. B **70**, 144402 (2004).
[4] R.P. Cowburn, D.K. Koltsov, A.O. Adeyeye, M.E. Welland and D.M. Tricker, Phys. Rev. Lett. **83**, 1042 (1999).
[5] M. Schneider, H. Hoffmann and J. Zweck, Appl. Phys. Lett. **77**, 2909 (2000).
[6] M. Hanson, O. Kazakova, P. Blomqvist, R. Wäppling and B. Nilsson, Phys. Rev. B **66**, 144419 (2002).
[7] R. Pulwey, M. Zölfl, G. Bayreuther and D. Weiss, J. Appl. Phys. **91**, 7995 (2002).
[8] M. Hehn, K. Ounadjela, J.-P. Boucher, F. Rousseaux, D. Decanini, B. Bartenlian and C. Chappert, Science **272**, 1782 (1996).
[9] O. Kazakova, M. Hanson, P. Blomqvist and R. Wäppling, J. Appl. Phys. **90**, 2440 (2001).
[10] R.P. Cowburn, J. Magn. Magn. Mater. **242-245**, 505 (2002).
[11] G. Gioia and R.D. James, Proc. R. Soc. London, Ser. A **453**, 213 (1997).
[12] J.-G. Caputo, Y. Gaididei, V.P. Kravchuk, F.G. Mertens and D.D. Sheka, Phys. Rev. B **76**, 174428 (2007).
[13] R.L. Compton and P.A. Crowell, Phys. Rev. Lett. **97**, 137202 (2006).

[14] S. Hikami and T. Tsuneto, Prog. Theor. Phys. **63**, 387 (1980).
[15] S. Takeno and S. Homma, Prog. Theor. Phys. **64**, 1193 (1980); **65**, 172 (1980).
[16] M.E. Gouvêa, G.M. Wysin, A.R. Bishop and F.G. Mertens, Phys. Rev. B **39**, 11840 (1989).
[17] G.M. Wysin, M.E Gouvêa, A.R. Bishop and F.G. Mertens, in *Computer Simulation Studies in Condensed Matter Physics*, edited by D.P. Landau, K.K. Mon and H.-B. Schüttler (Springer-Verlag, Berlin, 1988).
[18] G.M. Wysin, Phys. Rev. B **49**, 8780 (1994).
[19] B.A. Ivanov and D.D. Sheka, Low Temp. Phys. **21**, 881 (1995).
[20] B.A. Ivanov and G.M. Wysin, Phys. Rev. B **65**, 134434 (2002).
[21] T. Okuno, K. Shigeto, T. Ono, K. Mibu and T. Shinjo, J. Magn. Magn. Mater. **240**, 1 (2002).
[22] J.-G. Caputo, Y. Gaididei, F.G. Mertens and D.D. Sheka, Phys. Rev. Lett. **98**, 056604 (2007).
[23] Q.F. Xiao, J. Rudge, B.C. Choi, Y.K. Hong and G. Donohoe, Appl. Phys. Lett. **89**, 262507 (2006).
[24] T. Kimura, Y. Otani and J. Hamrle, Appl. Phys. Lett. **87**, 172506 (2005).
[25] J. Miltat, G. Albuquerque, A Thiaville and C. Vouille, J. Appl. Phys. **89**, 6982 (2001).
[26] A.R. Pereira, Phys. Lett. A **314**, 102 (2003).
[27] A.R. Pereira, L.A.S. Mól, S.A. Leonel, P.Z. Coura, and B.V. Costa, Phys. Rev. B **68**, 132409 (2003).
[28] F.M. Paula, A.R. Pereira, L.A.S. Mól, Phys. Lett. A **329**, 155 (2004).
[29] L.A.S. Mól, A.R. Pereira, and W.A. Moura-Melo, Phys. Rev. B **67**, 132403 (2003).
[30] M. Rahm, J. Biberger, V. Umansky, and D. Weiss, J. Appl. Phys. **93**, 7429 (2003).
[31] M. Rahm, R. Höllinger, V. Umansky, and D. Weiss, J. Appl. Phys. **95**, 6708 (2004).
[32] G.M. Wysin, Phys. Rev. B **68**, 184411 (2003).
[33] A.R. Pereira, J.Magn. Magn. Mater. **279**, 396 (2004).
[34] G.M. Wysin, Phys. Rev. B **71**, 094423 (2005).
[35] B. Berche, A.I. Farinas-Sanches, Y. Holovatch, and R. Paredes, Eur. Phys. J. B **36**, 91 (2003).
[36] G.M. Wysin, A.R. Pereira, I.A. Marques, S.A. Leonel, and P.Z. Coura, Phys. Rev. B **72**, 094418 (2005).
[37] A.R. Pereira, J. Appl. Phys. **97**, 094303 (2005).
[38] A.R. Pereira, Phys. Rev. B **71**, 224404 (2005).
[39] C.E. Zaspel, C.M. McKennan, and S.R. Snaric, Phys. Rev. B **53**, 11317 (1996).
[40] A.R. Pereira and G.M. Wysin, Phys. Rev. B **73**, 214402 (2006).
[41] C. Kawabata, M. Takeuchi and A.R. Bishop, J. Magn. Magn. Mater. **54–57**, 871 (1986).
[42] C. Kawabata, M. Takeuchi and A.R. Bishop, J. Stat. Phys. **43**, 869 (1986).
[43] G.M. Wysin and A.R. Bishop, Phys. Rev. B **42**, 810 (1990).
[44] M.E. Gouvêa and G.M. Wysin, Phys. Rev. B **56**, 14192 (1997).
[45] D.P. Landau, J. Magn. Magn. Mater. **200**, 231 (1999).