Pythagorean Theorem Proofs

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**Proof # 1.** Inscribe objects inside the $c^2$ square, and add up their areas.

![Diagram of Proof # 1](image)

Total area = hypothenuse squared = $c^2$.
Inscribed area = four triangles + one square.
What is $s$?

$s = b - a$.

$c^2 = 4 \times \frac{ab}{2} + s^2$

$c^2 = 2ab + (b - a)^2 = 2ab + b^2 - 2ab + a^2$

$\implies c^2 = a^2 + b^2$.

**Proof # 2.** Use similar triangles repeatedly.

![Diagram of Proof # 2](image)

All triangles are similar, with same ratios of sides.

Do ratios.

\[
\frac{\text{short}}{\text{long}} = \frac{a}{b} = \frac{x}{y} = \frac{y}{z}
\]

\[
\frac{\text{short}}{\text{hyp}} = \frac{a}{c} = \frac{x}{a} = \frac{y}{b} \implies x = \frac{a^2}{c}
\]

\[
\frac{\text{long}}{\text{hyp}} = \frac{b}{c} = \frac{y}{a} = \frac{z}{b} \implies z = \frac{b^2}{c}
\]

Then combine,

$c = x + z = \frac{a^2}{c} + \frac{b^2}{c} \implies c^2 = a^2 + b^2$. 