Pythagorean Theorem Proofs

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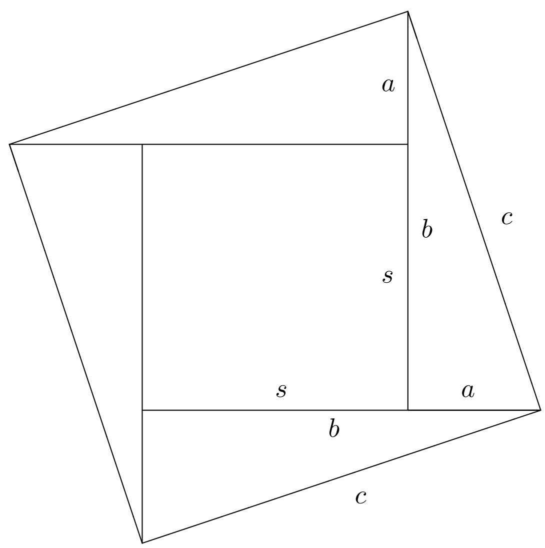
Proof # 1. Inscribe objects inside the c^2 square, and add up their areas.



Total area = hypothenuse squared = c^2.
Inscribed area = four triangles + one square.
What is s?

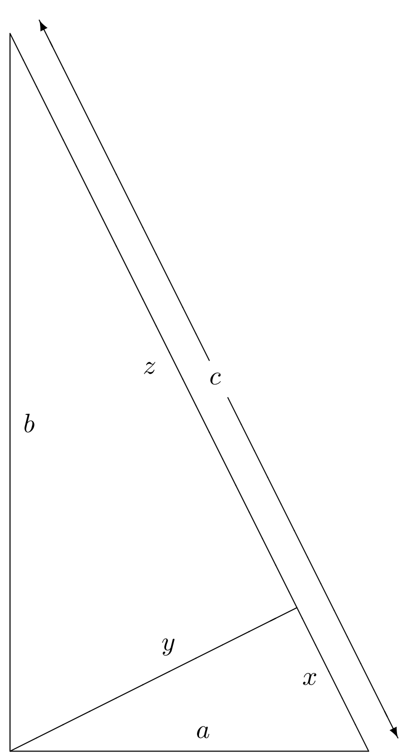
$$s = b - a.$$

$$c^2 = 4 \times \frac{ab}{2} + s^2$$

$$c^2 = 2ab + (b-a)^2 = 2ab + b^2 - 2ab + a^2$$

$$\Longrightarrow \quad c^2 = a^2 + b^2.$$

Proof # 2. Use similar triangles repeatedly.



All triangles are similar, with same ratios of sides.

Do ratios.

$$\frac{\text{short}}{\text{long}} = \frac{a}{b} = \frac{x}{y} = \frac{y}{z}$$

$$\frac{\text{short}}{\text{hyp}} = \frac{a}{c} = \frac{x}{a} = \frac{y}{b} \quad \Longrightarrow \quad x = \frac{a^2}{c}$$

$$\frac{\text{long}}{\text{hyp}} = \frac{b}{c} = \frac{y}{a} = \frac{z}{b} \quad \Longrightarrow \quad z = \frac{b^2}{c}$$

Then combine,

$$c = x + z = \frac{a^2}{c} + \frac{b^2}{c} \quad \Longrightarrow \quad c^2 = a^2 + b^2.$$