General Physics I **Exam 3 - Chs. 7,8,9 - Momentum, Rotation, Equilibrium** **Oct. 28, 2009**

Name Rec. Instr. Rec. Time

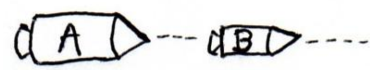
For full credit, make your work clear to the grader. Show the formulas you use, the essential steps, and results with correct units and correct number of significant figures. Partial credit is available if your work is clear. Points shown in parenthesis. For true-false and multiple choice questions, choose the *best* answer.
Use $g = 9.80$ m/s^2, $G = 6.67 \times 10^{-11}$ N m^2/kg^2. 1 inch = 2.54 cm, 1 ft = 12 in., 1 mile = 5280 ft, 1 hp = 746 W.
Prefixes: f=10^{-15}, p=10^{-12}, n=10^{-9}, $\mu = 10^{-6}$, m=10^{-3}, c=10^{-2}, k=10^3, M=10^6, G=10^9, T=10^{12}, P=10^{15}.

---

1. (2) A mass initially at rest in outer space explodes into two pieces of unequal mass moving in opposite directions. Which quantities (combined for the two rockets) are conserved in this event?

   a. kinetic energy and linear momentum. b. kinetic energy only. c. linear momentum only.

2. (6) Rocket A of mass $3m$ crashes into rocket B of mass m in outer space. For simplicity, assume they were both travelling initially in the same direction, coasting with their engines off.



   a) (2) Which has the larger magnitude change in momentum?

   a. rocket A. b. rocket B. c. it's a tie.

   b) (2) Which experiences the greater magnitude force due to the collision?

   a. rocket A. b. rocket B. c. it's a tie.

   c) (2) Which experiences the greater magnitude average acceleration during the collision?

   a. rocket A. b. rocket B. c. it's a tie.

3. (2) If you jump from a high point down to the floor, you bend your knees to land because this reduces your

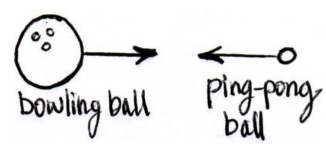
   a. momentum. b. impulse. c. kinetic energy. d. normal force. e. landing time interval.

4. (2) In any collision, $\vec{v}_A\,'$ and $\vec{v}_B\,'$ are the velocities of the masses after the collision. In a *completely inelastic collision*, which one of the following quantities is zero after the collision?

   a. total momentum. b. total kinetic energy. c. $\vec{v}_A\,' + \vec{v}_B\,'$. d. $\vec{v}_A\,' - \vec{v}_B\,'$. e. none of these.

---

5. (14) A 5.0-kg bowling ball travelling at 14 m/s due east collides head-on with a *much much lighter* ping-pong ball travelling at 16 m/s due west. Their collision is elastic. Use the $+x$-axis as the positive direction.



   a) (4) Before colliding, at what speed are the bowling ball and ping-pong ball approaching each other? (velocity of bowling ball relative to ping-pong ball)

   b) (4) After they collide, at what speed are the bowling ball and ping-pong ball separating from each other? (velocity of ping-pong ball relative to bowling ball)

   c) (6) After the collision, what is the velocity (magnitude and direction) of the ping-pong ball?

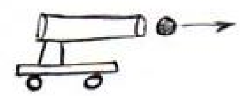
6. (14) A 190-gram softball leaves the bat at $v_0 = 26$ m/s, at 45° above horizontal, after contacting the bat for 5.4 ms. It flies to centerfield with negligible air resistance.

a) (6) What was the magnitude of the average force on the ball due to the bat?

b) (8) The centerfielder catches this fly ball without using a glove. Over what distance should the ball be brought to a stop to experience an average force of impact on her hands no larger than 250 N?

---

7. (8) A large cannon is mounted on a cart with frictionless wheels (total mass 420 kg). When it fires a 12.0-kg cannonball horizontally, the cart recoils with a speed of 8.00 m/s. What is the speed of the cannonball that was fired (relative to the ground)?



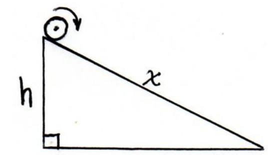
---

8. (14) The flywheel connected to a motor has an outer radius of 25.0 cm. Starting from rest, a constant angular acceleration during 8.25 s brings it to an angular speed of 2.50×10^3 rpm.

a) (8) How large is the flywheel's angular acceleration, in rad/s^2?

b) (6) Once it is running at 2.50×10^3 rpm, what is the magnitude of the centripetal acceleration of any point on the edge of the wheel?

9. (16) Starting from rest, a 10.0-kg wheel rolls without slipping down an incline. Its center is moving at 6.00 m/s after it loses $h = 3.00$ m altitude while rolling $x = 7.00$ m along the incline.



a) (2) The gravitational potential energy it lost ($mgh = 294$ J) is converted into

a. its translational KE only. b. its rotational KE only.
c. the sum of its translational KE plus rotational KE.

b) (6) When its center is moving at 6.00 m/s, how much *rotational KE* does the wheel have?

c) (8) If the radius of the wheel is 30.0 cm, how large is its rotational inertia?

---

10. (18) A figure skater (on frictionless ice) starts a spin using one foot to push and give herself an initial rotation rate of 1.0 rev every 2.5 s. As she ends the spin, she has increased her rotation rate to 1.0 rev every 0.50 s.

a) (2) How can she cause her rate of rotation to increase during the spin?

a. she pushes with her foot on every revolution, thereby applying a torque.
b. she starts with her arms and legs out from her body and brings them inward.
c. she starts with her arms and legs in close to the axis and then moves them outward.
d. she moves her arms alternately inward and outward, giving herself propulsion just like a flying pig.

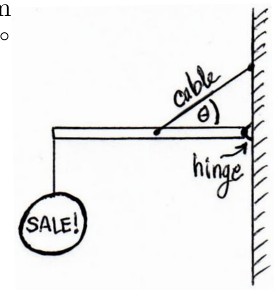
b) (2) Which one of the following quantities (for the skater) is conserved during her spin?

a. ω=angular speed. b. I=rotational inertia c. KE_{rotation} d. $L = I\omega$=angular momentum

c) (8) At the start of the spin, her rotational inertia was 4.4 kg·m^2. How large is her rotational inertia at the end of the spin?

d) (6) How large is her rotational kinetic energy at the end of the spin?

11. (20) A sign that weighs 680 N is mounted in stable equilibrium at the end of a 1.00-m long rod of negligible weight as shown. The supporting cable is connected at angle $\theta = 30.0°$ to the center of the rod.

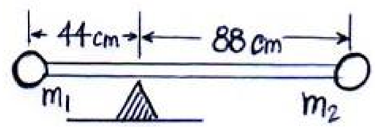


a) (8) Find the tension T in the supporting cable.

b) (12) Find the horizontal and vertical components of the force of the hinge on the rod, H_x and H_y.

---

12. (16) The rod in this diagram is of negligible mass, but has masses connected to its ends, with $m_1 = 5.00$ kg. The arrangment is in stable equlibrium.



a) (8) How large is m_2?

b) (8) How large is the rotational inertia of the rod with masses, taking the pivot point as the axis or origin.

Chapter 9 Equations

Static Equilibrium:

$$\Sigma F_x = \Sigma F_y = \Sigma F_z = 0, \qquad \Sigma\tau = 0, \qquad \tau = rFsin\theta.$$

Chapter 8 Equations

Rotational coordinates:

$$1 \text{ rev} = 2\pi \text{ radians} = 360°, \qquad \omega = 2\pi f, \quad f = \tfrac{1}{T}, \qquad \bar{\omega} = \tfrac{\Delta\theta}{\Delta t}, \qquad \bar{\alpha} = \tfrac{\Delta\omega}{\Delta t}, \qquad \Delta\theta = \bar{\omega}\Delta t.$$

Linear coordinates vs. rotation coordinates and radius:

$$l = \theta r, \qquad v = \omega r, \qquad a_{\text{tan}} = \alpha r, \qquad a_R = \omega^2 r, \qquad \text{(must use radians in these).}$$

Constant angular acceleration:

$$\omega = \omega_0 + \alpha t, \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2, \qquad \bar{\omega} + \tfrac{1}{2}(\omega_0 + \omega), \qquad \omega^2 = \omega_0^2 + 2\alpha\Delta\theta.$$

Torque & Dynamics:

$$\tau = rF\sin\theta, \qquad I = \Sigma mr^2, \qquad \tau_{\text{net}} = I\alpha, \qquad L = I\omega, \qquad \Delta L = \tau_{\text{net}}\Delta t, \qquad \text{KE}_{\text{rotation}} = \tfrac{1}{2}I\omega^2.$$

Rotational Inertias about centers:

$I = MR^2$,	$I = \frac{1}{2}MR^2$,	$I = \frac{2}{5}MR^2$,	$I = \frac{1}{12}ML^2$.
hoop	solid cylinder	sphere	thin rod

Chapter 7 Equations

Momentum & Impulse:

$$\text{momentum } \vec{p} = m\vec{v}, \qquad \text{impulse } \Delta\vec{p} = \vec{F}_{\text{ave}}\ \Delta t.$$

Conservation of Momentum:

$$\text{(2-body collision):} \quad m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v'}_A + m_B\vec{v'}_B.$$

1D elastic collision–conservation of energy:

$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2, \quad \text{or} \quad v_A - v_B = -(v_A' - v_B').$$

Center of Mass:

$$x_{\text{cm}} = \tfrac{m_1x_1 + m_2x_2 + \ldots}{m_1 + m_2 + \ldots}, \qquad v_{\text{cm}} = \tfrac{m_1v_1 + m_2v_2 + \ldots}{m_1 + m_2 + \ldots}.$$

Chapter 6 Equations

Work & Kinetic & Potential Energies:

$$W = Fd\cos\theta, \qquad \text{KE} = \tfrac{1}{2}mv^2, \qquad \text{PE}_{\text{gravity}} = mgy, \quad F_{\text{gravity}} = -mg. \qquad \text{PE}_{\text{spring}} = \tfrac{1}{2}kx^2, \quad F_{\text{spring}} = -kx.$$

Conservation or Transformation of Energy:

"work-KE theorem" $\Delta\text{KE} = W_{\text{net}}$, **or use conservation law**: $\Delta\text{KE} + \Delta\text{PE} = W_{\text{NC}}$. $E_2 = E_1 + W_{\text{NC}}$.

Power:

$$P_{\text{ave}} = \tfrac{W}{t}, \qquad \text{or use } P_{\text{ave}} = \tfrac{\text{energy}}{\text{time}}.$$

(over)

Chapter 5 Equations

Centripetal Acceleration:

$a_R = \frac{v^2}{r}$, towards the center of the circle.

Circular motion:

speed $v = \frac{2\pi r}{T} = 2\pi r f$, frequency $f = \frac{1}{T}$, where T is the period of one revolution.

Gravitation:

$F = G\frac{m_1 m_2}{r^2}$; $\quad g = \frac{GM}{r^2}$, $\quad$ where $G = 6.67 \times 10^{-11}$ Nm^2/kg^2;

Orbits:

$\frac{v^2}{r} = g = \frac{GM}{r^2}$; $\quad v = \sqrt{\frac{GM}{r}}$.

Chapter 4 Equations

Newton's Second Law:

$\vec{F}_{\text{net}} = m\vec{a}$, which means $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$.

Static friction (magnitude):

$f_s \le \mu_s N$ or $F_{\text{fr}} \le \mu_s F_N$.

Kinetic or sliding friction (magnitude):

$f_k = \mu_k N$ or $F_{\text{fr}} = \mu_k F_N$.

Gravitational force near Earth:

$F_G = mg$, downward.

Acceleration Equations

$\bar{v} = \frac{\Delta x}{\Delta t}$, $\quad \Delta x = x - x_0$, $\quad$ slope of $x(t) = v(t)$.

$\bar{a} = \frac{\Delta v}{\Delta t}$, $\quad \Delta v = v - v_0$, $\quad$ slope of $v(t) = a(t)$.

For constant acceleration in one-dimension:

$\bar{v} = \frac{1}{2}(v_0 + v)$, $\quad v = v_0 + at$, $\quad x = x_0 + v_0 t + \frac{1}{2}at^2$, $\quad v^2 = v_0^2 + 2a(x - x_0)$.

Vectors

Written $\vec{V}$ or $\mathbf{V}$, described by magnitude=V, direction=θ or by components (V_x, V_y).

$V_x = V\cos\theta$, $\quad V_y = V\sin\theta$,

$V = \sqrt{V_x^2 + V_y^2}$, $\quad \tan\theta = \frac{V_y}{V_x}$. $\qquad$ θ is the angle from $\vec{V}$ to $+x$-axis.

Addition: $\mathbf{A} + \mathbf{B}$, head to tail. $\quad$ Subtraction: $\mathbf{A} - \mathbf{B}$ is $\mathbf{A} + (-\mathbf{B})$, $\quad -\mathbf{B}$ is $\mathbf{B}$ reversed.

Trig summary

$\sin\theta = \frac{\text{(opp)}}{\text{(hyp)}}$, $\qquad \cos\theta = \frac{\text{(adj)}}{\text{(hyp)}}$, $\qquad \tan\theta = \frac{\text{(opp)}}{\text{(adj)}}$, $\qquad \text{(opp)}^2 + \text{(adj)}^2 = \text{(hyp)}^2$.

$\sin\theta = \sin(180^\circ - \theta)$, $\quad \cos\theta = \cos(-\theta)$, $\quad \tan\theta = \tan(180^\circ + \theta)$, $\quad \sin^2\theta + \cos^2\theta = 1$.