### Prefixes

 $z=10^{-21}, \ a=10^{-18}, \ f=10^{-15}, \ p=10^{-12}, \ n=10^{-9}, \ \mu=10^{-6}, \ m=10^{-3}, \ c=10^{-2}, \ k=10^3, \ M=10^6, \ G=10^9, \ T=10^{12}, \ P=10^{15}, \ E=10^{18}, \ Z=10^{21}, \ z=10^{12}, \ p=10^{15}, \ E=10^{18}, \ Z=10^{21}, \ z=10^{18}, \ z=10^{1$ 

### Physical Constants

 $\begin{array}{ll} g=9.80~\text{m/s}^2~\text{(gravitational acceleration)} & G=6.67\times 10^{-11}~\text{N}\cdot\text{m}^2/\text{kg}^2~\text{(gravitational constant)} \\ M_E=5.98\times 10^{24}~\text{kg}~\text{(mass of Earth)} & R_E=6380~\text{km}~\text{(mean radius of Earth)} \\ m_e=9.11\times 10^{-31}~\text{kg}~\text{(electron mass)} & m_p=1.67\times 10^{-27}~\text{kg}~\text{(proton mass)} \\ c=299,792,458~\text{m/s}~\text{(speed of light)} & 1~\text{amu}=1~\text{u}=1.6605402\times 10^{-27}~\text{kg}~\text{(atomic mass unit)} \\ \end{array}$ 

### Units & Conversions

# Algebra

Quadratic equations:  $ax^2 + bx + c = 0$ , solved by  $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a)$ .

# Geometry

Triangles:  $A = \frac{1}{2}bh$ , Circles;  $C = 2\pi r$ ,  $A = \pi r^2$ , arc  $= s = r\theta$ . Spheres:  $A = 4\pi r^2$ ,  $V = \frac{4\pi}{3}r^3$ 

# Trigonometry

 $\sin\theta = (\text{opp})/(\text{hyp}), \qquad \cos\theta = (\text{adj})/(\text{hyp}), \qquad \tan\theta = (\text{opp})/(\text{adj}), \qquad (\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2.$   $\sin^2\theta + \cos^2\theta = 1, \qquad a^2 + b^2 - 2ab\cos\gamma = c^2, \qquad \frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}, \qquad \alpha + \beta + \gamma = 180^\circ = \pi \text{ rad.}$ 

## Chapter 2 - Vectors - Magnitude & Direction

magnitude =  $a = \sqrt{a_x^2 + a_y^2}$  direction  $\to \tan \theta = a_y/a_x$  $\vec{\mathbf{a}} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}}$ 2D Vectors:  $a_x = a\cos\theta$  $a_u = a \sin \theta$  $\theta$ =angle to +x-axis. Components: Subtraction:  $\vec{\mathbf{a}} - \vec{\mathbf{b}}$  is  $\vec{\mathbf{a}} + (-\vec{\mathbf{b}})$  $-\vec{\mathbf{b}}$  is  $\vec{\mathbf{b}}$  reversed.  $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ , head to tail. Addition:  $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z$  $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = 1, \ \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0, \text{ etc.}$  $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = ab\cos\phi$ Scalar product:  $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = ab\sin\phi$  $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$ . etc.  $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$ Cross product:

# Chapter 3 - 1D Kinematics - Straight-line motion

 $v(t) = \frac{dx}{dt}$  = slope of x(t)  $a(t) = \frac{dv}{dt}$  = slope of v(t)Velocity:  $v_{\rm avg} = \Delta x / \Delta t$  $\Delta x = x - x_0$  $\Delta v = v - v_0$  $a_{\rm avg} = \Delta v / \Delta t$ Acceleration:  $x(t) = x_0 + \int_0^t v(t')dt',$  $v(t) = v_0 + \int_0^t a(t')dt'.$ Integrals = areas: $v = v_0 + at$ 
$$\begin{split} \Delta x &= v_{\rm avg} \Delta t. \\ v^2 &= v_0^2 + 2a \Delta x. \end{split}$$
Constant acceleration:  $v_{\text{avg}} = \frac{1}{2}(v_0 + v),$  $x = x_0 + v_0 t + \frac{1}{2} a t^2$  $x = x_0 + v_{\text{avg}}t,$ (position from acceleration) (timeless equation) (using average velocity)  $y = y_0 + v_{0y}t - \frac{1}{2}gt^2,$  $v_y^2 = v_{0y}^2 - 2g\Delta y.$ Free fall (+y-axis is up):  $v_y = v_{0y} - gt,$ 

#### Chapter 5 - Newton's laws and forces

Newton's 1<sup>st</sup> Law:  $\vec{a} = d\vec{v}/dt = 0 \text{ unless } \vec{F}_{\rm net} \neq 0$   $\vec{F}_{\rm net} = \sum \vec{F}_i = \text{sum of all forces on a mass.}$  Newton's 2<sup>nd</sup> Law:  $\vec{F}_{\rm net} = m\vec{a}$   $F_{\rm net,x} = ma_x, \ F_{\rm net,y} = ma_y, \ F_{\rm net,z} = ma_z$  Newton's 3<sup>rd</sup> Law:  $\vec{F}_{AB} = -\vec{F}_{BA}$  Forces exist in action-reaction pairs.

Gravitational force near Earth:  $F_g = mg$ , downward. Apparent weight is force measured by a scales. Gravity components on inclines:  $F_{g,\parallel} = mg\sin\theta, \ F_{g,\perp} = mg\cos\theta$   $\leftarrow$  for incline at angle  $\theta$  to horizontal. Spring force:  $F_s = -kx$  x is the displacement from equilibrium.

# Chapter 6 - Friction, circular motion

Static friction (object is stuck):  $f_s \le \mu_s N$ Can balance other forces in any direction.  $f_k = \mu_k N$ Kinetic friction (object sliding): Acts against the relative motion of surfaces.

 $a_c = v^2/r$ Centripetal acceleration: Points towards the center of the circle.

### Chapter 7 - Work and kinetic energy

 $W_{AB} = \int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$  (along the path  $A \to B$ )  $dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = F \ dr \ \cos \theta$ Work done by a force:

 $W = \vec{\mathbf{F}} \cdot \Delta \vec{\mathbf{r}}$  $\Delta \mathbf{r} = \vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A = \text{displacement.}$ Work of a constant force:

$$\begin{split} W_{\mathrm{g}} &= -mg\Delta y \\ W_{s} &= -\frac{1}{2}k(x_{B}^{2}-x_{A}^{2}) \end{split}$$
Work done by gravity:  $\Delta y = y_B - y_A$  (final minus initial height)

Work done by a spring: B=final stretch, A =initial stretch.

Friction's work can be positive or negative!!  $\text{KE} = \frac{1}{2} m v^2, \quad P = \frac{dW}{dt}, \quad P_{\text{ave}} = \frac{\Delta W}{\Delta t}.$   $\leftarrow \text{ the rate of doing work by some force.}$ Work done by friction: Use formula for constant force.

 $\Delta KE = W_{\text{net}} = \text{all works on } m.$ Work-KE theorem:

Instantaneous power: P = dW/dt $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$ When  $\vec{\mathbf{F}}$  acts on m:  $\leftarrow$  instantaneous power only due to  $\mathbf{F}$ 

 $P_{\text{ave}} = \Delta W / \Delta t$ Average power:  $\leftarrow$  average over time interval  $\Delta t$ .

## Chapter 8 - Potential energy and Conservation of energy

 $\Delta U = mg\Delta y,$ PE for gravity: U(y) = mgy + constant.

 $\Delta U = \frac{1}{2}k(x_B^2 - x_A^2),$  $U(x) = \frac{1}{2}kx^2 + \text{constant.}$ PE for springs:

 $\Delta E_{\text{total}} = 0$ , Arbitrary system:  $E_{\text{total}} = E_{\text{mec}} + E_{\text{thermal}} + E_{\text{other}}.$ 

(nothing is left out of energy accounting).

# Chapter 9 - Linear momentum and collisions

 $\Delta \vec{\mathbf{p}} = \vec{\mathbf{J}} = \int \vec{\mathbf{F}}(t) dt = \vec{\mathbf{F}}_{\text{ave}} \Delta t.$ Linear Momentum:  $\vec{\mathbf{p}} = m\vec{\mathbf{v}},$ Impulse Theorem:

 $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$ ,  $\vec{\mathbf{F}}_{\text{ave}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t}$ . Average force: Instantaneous force:

Conservation (@  $\vec{\mathbf{F}}_{net} = 0$ ):  $\Delta \vec{\mathbf{p}}_{total} = 0,$ i=initial, f=final.  $\vec{\mathbf{p}}_{1\mathrm{i}} + \vec{\mathbf{p}}_{2\mathrm{i}} = \vec{\mathbf{p}}_{1\mathrm{f}} + \vec{\mathbf{p}}_{2\mathrm{f}},$ 

 $\vec{\mathbf{r}}_{\text{com}} = \frac{m_1 \vec{\mathbf{r}}_1 + m_2 \vec{\mathbf{r}}_2 + \dots}{m_1 + m_2 + \dots},$   $v_{1f} = 2v_{\text{com}} - v_{1i}$ Center of mass:

$$\begin{split} \vec{\mathbf{v}}_{\text{com}} &= \frac{m_1 \vec{\mathbf{v}}_1 + m_2 \vec{\mathbf{v}}_2 + \dots}{m_1 + m_2 + \dots} \\ v_{\text{2f}} &= 2v_{\text{com}} - v_{\text{2i}}, \end{split} \quad \text{Equal masses swap velocities}.$$
1D elastic collisions:

 $\vec{P}_{\text{total}} = M\vec{\mathbf{v}}_{\text{com}} = \text{const.}$  $\vec{P}_{\text{total}} = m_1 \vec{\mathbf{v}}_1 + m_2 \vec{\mathbf{v}}_2 = \text{const.}$ Other collisions:

The point  $\vec{\mathbf{r}}_{com}$  moves as a point mass  $M = \sum_{i} m_i$  subjected to net force  $\vec{F}_{net}$ . Extended objects:

#### Chapter 12 - Static equilibrium

Statics requirements:  $\sum F_x = \sum F_y = \sum F_z = 0, \qquad \sum \tau = 0,$  $\tau = rF\sin\theta$ .

stress =  $F_{\perp}/A$ , strain =  $\Delta L/L_0$ , stress =  $Y \times$  strain. Stress & strain: Shear forces: stress =  $F_{\parallel}/A$ , strain =  $\Delta x/L_0$ , stress =  $S \times$  strain. b-stress =  $\Delta p$ , b-strain =  $\Delta V/V_0$ , b-stress =  $B \times$  b-strain. Bulk modulus B: stress in Pa=N/m<sup>2</sup>, strain = % or no units, Y, S, B in Pa=N/m<sup>2</sup>. Units:

## Chapter 13 - Gravitation

 $\begin{array}{ll} F = Gm_1m_2/r^2, & F = mg, & g = GM/r^2, & v_{\rm escape} = \sqrt{2GM/r}. \\ U = -Gm_1m_2/r, & \Delta U + \Delta K = 0, & \Delta K = -\Delta U, & \Delta K = \frac{1}{2}m(v_{\rm f}^2 - v_{\rm i}^2). \\ F = mv^2/r, & v = 2\pi r/T, & v_{\rm orb} = \sqrt{GM/r}, & {\rm Kepler:} \ T^2 = \frac{4\pi^2}{GM}\bar{r}^3. \end{array}$ Gravitational force: Gravitational PE:

Orbits:

# Chapter 14 - Fluids

1 atmosphere = 1 atm =  $101.3 \text{ kPa} = 1.013 \text{ bar} = 760 \text{ torr} = 760 \text{ mm Hg} = 14.7 \text{ lb/in}^2$ .

 $1 \text{ bar} = 10^5 \text{ Pa},$  $1 \text{ Pa} = 1 \text{ N/m}^2$ , 1 mm Hg = 1 torr = 133.3 Pa.Units:

 $\rho_{\rm H_2O} = 10^3 \text{ kg/m}^3 \text{ (4°C)},$  $10^3 \text{ kg/m}^3 = 1 \text{ g/cm}^3$ .  $\rho = m/V$ , Density: p = F/A $p_2 = p_1 + \rho g d$  $p_{\rm abs} = p_{\rm atm} + p_{\rm gauge}.$ Pressure: Bernoulli energy conserv. $\rightarrow$  $p + \rho gy + \frac{1}{2}\rho v^2 = \text{const.}$ Archimedes:  $F_B = \rho_{\text{fluid}} g V_s$ ,

Q = Av,  $Q_m = \rho A v,$  $Q = (p_2 - p_1)\pi r^4 / (8\eta L).$ Flow rates:

Viscosity:  $F = \eta v A/L$ ,  $N_R = 2\rho v r / \eta,$  $N_R < 2000$  laminar,  $N_R > 3000$  turbulent.