Name= **Studio Days/Time=**

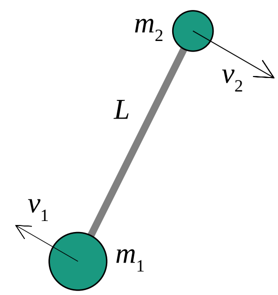
Eng. Phys. I **Exam 4 - Chs. 9,10,11 - Momentum, Rotation, Rolling** **Apr. 1, 2022**

Write **neat** & **clear** work. Show **formulas** used, essential steps, results with correct **units** and **significant figures**. Points shown in parenthesis. For TF and MC, choose the *best* answer. Use $g = 9.80 \text{ m/s}^2$. Ignore air resistance unless it is mentioned in a question. You are allowed to use only a calculator and the attached equation sheet.

1. (3) In a time interval Δt, some amount of hot gas is ejected from a rocket in deep space (zero gravity). Which has the larger magnitude change in momentum?

a. The rocket. b. The ejected gas. c. Its a tie. d. Not enough information to answer.

2. (16) A dumbbell has spherical masses $m_1 = 12.0$ kg and $m_2 = 8.00$ kg mounted at the ends of a 2.00-m long rod of negligible mass. It is rotating about its center of mass at 5.00 radians per second without any external forces acting on it.



a) (5) How far is the dumbbell's center of mass from the center of m_1?

b) (6) How large is the dumbbell's rotational inertia about its center of mass?

c) (5) How large is the rotational kinetic energy?

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3. (3) A collision between two masses that conserves total momentum and total kinetic energy is called

a. springy. b. explosive. c. elastic. d. inelastic. e. completely inelastic.

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4. (2) **T F** Impulse is the dot product of average force acting on a mass and its velocity.

5. (2) **T F** When a moving object is brought to rest over a shorter distance, the average force required is less.

6. (2) **T F** A completely inelastic collision of two masses does not conserve their total momentum.

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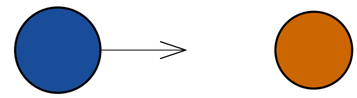
7. (12) A 5.0-kg medicine ball traveling on a frictionless surface at 16.0 m/s crashes head-on into a 0.62-kg basketball initially at rest. The collision is elastic and the balls are in contact for 38 ms.



a) (6) How fast is the basketball moving just after the collision?

b) (6) Find the magnitude of the average force exerted by the medicine ball on the basketball.

8. (2) **T** **F** The centripetal acceleration of points on a rotating disk decreases with their distance from the axis.

9. (2) **T** **F** The SI unit for torque is newtons $\times$ meters^2.

10. (2) **T** **F** All points on a circular disk rotating around its center have the same angular speed.

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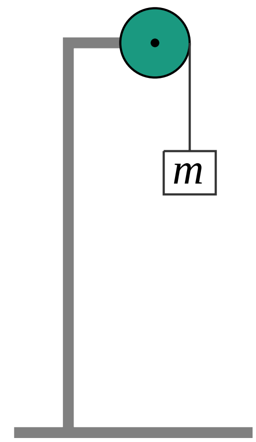
11. (16) A mass $m = 1.40$ kg hangs on a string of negligible mass that is wrapped around a disk of radius $R = 4.50$ cm mounted to rotate around its center as shown. When m is released from rest, it falls 2.25 m in 3.00 s while causing the disk to rotate. Ignore any friction.



a) (8) Analyze the acceleration of m to find the tension in the string.

b) (8) Analyze the angular acceleration of the disk to find its rotational inertia.

12. (8) A centrifuge accelerates at 475 rad/s^2 starting from rest for 4.00 s to get up to full speed. What is its final angular speed in rpm (revolutions per minute)?

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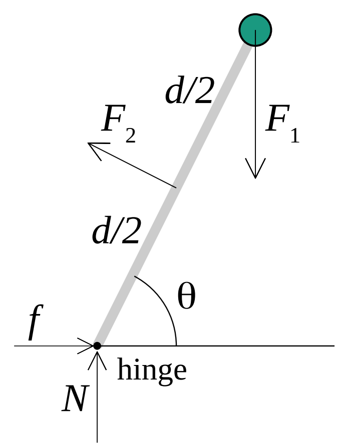
13. (8) A rod of length d free to rotate around a hinge at the lower end is subjected to the forces shown. F_2 is perpendicular to the rod. Write an expression for the net torque around the hinge point, using the given variables.



14. (2) **T F** A solid sphere will be beat a solid disk of equal radius when rolling down a hill from rest.

15. (2) **T F** A solid iron sphere of 1.0 cm radius will accelerate faster down a hill than one of 2.0 cm radius.

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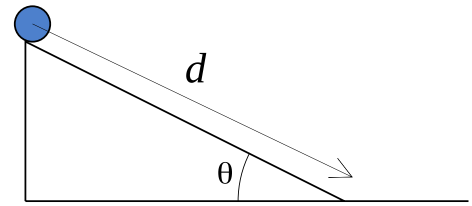
16. (10) Starting from rest, a solid sphere of 2.5 cm radius and 86 g mass rolls distance $d = 1.00$ m downhill on a $\theta = 30.0°$ incline. Use conservation of energy to find the final speed of its center.



17. (2) **T F** The net torque on an object with increasing angular momentum around some axis is nonzero.

18. (2) **T F** A particle undergoing straight-line motion does not have angular momentum about any axis.

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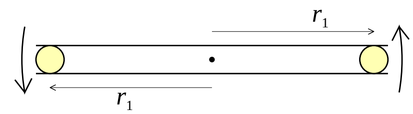
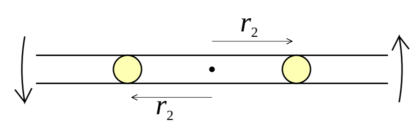
19. (12) Two small 0.250-kg spheres are mounted symmetrically at $r_1 = 50.0$ cm from the center of a hollow tube of negligible mass freely rotating (no friction) counterclockwise at 3.00 rad/s around an axis at its center.

a) (6) Determine the magnitude and direction of the system's angular momentum.



b) (6) While still rotating, a mechanism pulls the spheres inward until they are at $r_2 = 25.0$ cm from its center. What is the final angular speed in rad/s?



Prefixes

z=10^{-21},	a=10^{-18},	f=10^{-15},	p=10^{-12},	n=10^{-9},	$\mu=10^{-6}$,	m=10^{-3},	c=10^{-2},	k=10^{3},	M=10^{6},	G=10^{9},	T=10^{12},	P=10^{15},	E=10^{18},	Z=10^{21}
zepto,	atto,	femto,	pico,	nano,	micro,	milli,	centi,	kilo,	mega,	giga,	tera,	peta,	exa,	zeta.

Physical Constants

$g = 9.80$ m/s^2 (gravitational acceleration) $\quad$ $G = 6.67 \times 10^{-11}$ N·m^2/kg^2 (gravitational constant)
$M_E = 5.98 \times 10^{24}$ kg (mass of Earth) $\quad$ $R_E = 6380$ km (mean radius of Earth)
$m_e = 9.11 \times 10^{-31}$ kg (electron mass) $\quad$ $m_p = 1.67 \times 10^{-27}$ kg (proton mass)
$c = 299,792,458$ m/s (speed of light) $\quad$ 1 amu = 1 u = $1.6605402 \times 10^{-27}$ kg (atomic mass unit)

Units & Conversions

1 inch = 1 in = 2.54 cm $\quad$ 1 foot = 1 ft = 12 in = 0.3048 m
1 mile = 5280 ft =1760 yards $\quad$ 1 mile = 1609.344 m = 1.609344 km
1 m/s = 3.6 km/hour $\quad$ 88 ft/s = 60 mile/hour
1 acre = (1 mile)^2/640 = 43 560 ft^2 $\quad$ 1 hectare = (100 m)^2 = 10^4 m^2
1 lb = 4.45 N $\quad$ 1 N = 0.225 lb $\quad$ 1 J = 1 joule = 1 N·m

Algebra

Quadratic equations: $\quad ax^2 + bx + c = 0, \quad$ solved by $\quad x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a).$

Geometry

Triangles: $A = \frac{1}{2}bh$, $\quad$ Circles; $C = 2\pi r$, $\quad A = \pi r^2$, $\quad$ arc $= s = r\theta$. $\quad$ Spheres: $A = 4\pi r^2$, $\quad V = \frac{4\pi}{3}r^3$

Trigonometry

$\sin\theta = (\text{opp})/(\text{hyp})$, $\quad \cos\theta = (\text{adj})/(\text{hyp})$, $\quad \tan\theta = (\text{opp})/(\text{adj})$, $\quad (\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2$.
$\sin^2\theta + \cos^2\theta = 1$, $\quad a^2 + b^2 - 2ab\cos\gamma = c^2$, $\quad \frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$, $\quad \alpha + \beta + \gamma = 180^\circ = \pi$ rad.

Chapter 2 - Vectors - Magnitude & Direction

2D Vectors:	$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$	magnitude = $a = \sqrt{a_x^2 + a_y^2}$	direction $\to \tan\theta = a_y/a_x$
Components:	$a_x = a\cos\theta$	$a_y = a\sin\theta$	θ=angle to $+x$-axis.
Addition:	$\vec{\mathbf{a}} + \vec{\mathbf{b}}$, head to tail.	Subtraction: $\vec{\mathbf{a}} - \vec{\mathbf{b}}$ is $\vec{\mathbf{a}} + (-\vec{\mathbf{b}})$	$-\vec{\mathbf{b}}$ is $\vec{\mathbf{b}}$ reversed.
Scalar product:	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = ab\cos\phi$	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_xb_x + a_yb_y + a_zb_z$	$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = 1$, $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0$, etc.
Cross product:	$\lvert\vec{\mathbf{a}} \times \vec{\mathbf{b}}\rvert = ab\sin\phi$	$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, etc.	$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$

Chapter 3 - 1D Kinematics - Straight-line motion

Velocity:	$v_{\text{avg}} = \Delta x/\Delta t$	$\Delta x = x - x_0$	$v(t) = \frac{dx}{dt}$= slope of $x(t)$
Acceleration:	$a_{\text{avg}} = \Delta v/\Delta t$	$\Delta v = v - v_0$	$a(t) = \frac{dv}{dt}$= slope of $v(t)$
Integrals = areas:	$x(t) = x_0 + \int_0^t v(t')dt'$,	$v(t) = v_0 + \int_0^t a(t')dt'$.	
Constant acceleration:	$v = v_0 + at$,	$v_{\text{avg}} = \frac{1}{2}(v_0 + v)$,	$\Delta x = v_{\text{avg}}\Delta t$.
	$x = x_0 + v_0t + \frac{1}{2}at^2$,	$x = x_0 + v_{\text{avg}}t$,	$v^2 = v_0^2 + 2a\Delta x$.
	(position from acceleration)	(using average velocity)	(timeless equation)
Free fall ($+y$-axis is up):	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$,	$v_y = v_{0y} - gt$,	$v_y^2 = v_{0y}^2 - 2g\Delta y$.

Chapter 5 - Newton's laws and forces

Newton's 1st Law:	$\vec{a} = d\vec{v}/dt = 0$ unless $\vec{F}_{\text{net}} \neq 0$	$\vec{F}_{\text{net}} = \sum \vec{F}_i$ = sum of all forces on a mass.
Newton's 2nd Law:	$\vec{F}_{\text{net}} = m\vec{a}$	$F_{\text{net},x} = ma_x$, $F_{\text{net},y} = ma_y$, $F_{\text{net},z} = ma_z$
Newton's 3rd Law:	$\vec{F}_{AB} = -\vec{F}_{BA}$	Forces exist in action-reaction pairs.
Gravitational force near Earth:	$F_g = mg$, downward.	Apparent weight is force measured by a scales.
Gravity components on inclines:	$F_{g,\parallel} = mg\sin\theta$, $F_{g,\perp} = mg\cos\theta$	$\leftarrow$ for incline at angle θ to horizontal.
Spring force:	$F_s = -kx$	x is the displacement from equilibrium.

Chapter 6 - Friction, circular motion

Static friction (object is stuck):	$f_s \le \mu_s N$	Can balance other forces in any direction.
Kinetic friction (object sliding):	$f_k = \mu_k N$	Acts **against** the relative motion of surfaces.
Centripetal acceleration:	$a_c = v^2/r$	Points towards the center of the circle.

Chapter 7 - Work and kinetic energy

Work done by a force:	$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = F\ dr\ \cos\theta$	$W_{AB} = \int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ (along the path $A \to B$)
Work of a constant force:	$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$	$\Delta\mathbf{r} = \vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A =$ displacement.
Work done by gravity:	$W_{\mathrm{g}} = -mg\Delta y$	$\Delta y = y_B - y_A$ (final minus initial height)
Work done by a spring:	$W_s = -\frac{1}{2}k(x_B^2 - x_A^2)$	B=final stretch, A =initial stretch.
Work done by friction:	Use formula for constant force.	Friction's work can be positive or negative!!
Work-KE theorem:	$\Delta\mathrm{KE} = W_{\mathrm{net}} =$ all works on m.	$\mathrm{KE} = \frac{1}{2}mv^2$, $P = \frac{dW}{dt}$, $P_{\mathrm{ave}} = \frac{\Delta W}{\Delta t}$.
Instantaneous power:	$P = dW/dt$	$\leftarrow$ the rate of doing work by some force.
When $\vec{\mathbf{F}}$ acts on m:	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$	$\leftarrow$ instantaneous power only due to $\vec{\mathbf{F}}$
Average power:	$P_{\mathrm{ave}} = \Delta W/\Delta t$	$\leftarrow$ average over time interval Δt.

Chapter 8 - Potential energy and Conservation of energy

PE for gravity:	$\Delta U = mg\Delta y$,	$U(y) = mgy +$ constant.
PE for springs:	$\Delta U = \frac{1}{2}k(x_B^2 - x_A^2)$,	$U(x) = \frac{1}{2}kx^2$ + constant.
Arbitrary system:	$\Delta E_{\mathrm{total}} = 0$,	$E_{\mathrm{total}} = E_{\mathrm{mec}} + E_{\mathrm{thermal}} + E_{\mathrm{other}}$.
		(nothing is left out of energy accounting).

Chapter 9 - Linear momentum and collisions

Linear Momentum:	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$,	Impulse Theorem:	$\Delta\vec{\mathbf{p}} = \vec{\mathbf{J}} = \int \vec{\mathbf{F}}(t)\,dt = \vec{\mathbf{F}}_{\mathrm{ave}}\Delta t$.
Instantaneous force:	$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$,	Average force:	$\vec{\mathbf{F}}_{\mathrm{ave}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$.
Conservation (@ $\vec{\mathbf{F}}_{\mathrm{net}} = 0$):	$\Delta\vec{\mathbf{p}}_{\mathrm{total}} = 0$,	$\vec{\mathbf{p}}_{1\mathrm{i}} + \vec{\mathbf{p}}_{2\mathrm{i}} = \vec{\mathbf{p}}_{1\mathrm{f}} + \vec{\mathbf{p}}_{2\mathrm{f}}$,	i=initial, f=final.

Center of mass:	$\vec{\mathbf{r}}_{\mathrm{com}} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2 + \ldots}{m_1 + m_2 + \ldots}$,	$\vec{\mathbf{v}}_{\mathrm{com}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + \ldots}{m_1 + m_2 + \ldots}$	
1D elastic collisions:	$v_{1\mathrm{f}} = 2v_{\mathrm{com}} - v_{1\mathrm{i}}$	$v_{2\mathrm{f}} = 2v_{\mathrm{com}} - v_{2\mathrm{i}}$,	Equal masses swap velocities.
Other collisions:	$\vec{P}_{\mathrm{total}} = M\vec{\mathbf{v}}_{\mathrm{com}} =$ const.	$\vec{P}_{\mathrm{total}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 =$ const.	

Extended objects: The point $\vec{\mathbf{r}}_{\mathrm{com}}$ moves as a point mass $M = \sum_i m_i$ subjected to net force $\vec{F}_{\mathrm{net}}$.

Chapters 10,11 - Rotational motion, angular momentum

Coordinates:	1 rev = 2π rad	1 rev = $360°$,	$\omega = 2\pi f$,	$f = \frac{1}{T}$.
Averages:	$\omega_{\mathrm{ave}} = \frac{\Delta\theta}{\Delta t}$,	$\Delta\theta = \omega_{\mathrm{ave}}\Delta t$,	$\alpha_{\mathrm{ave}} = \frac{\Delta\omega}{\Delta t}$,	$\Delta\omega = \alpha_{\mathrm{ave}}\Delta t$.
Radius factors:	$l = \theta r$,	$v = \omega r$,	$a_{\tan} = \alpha r$,	$a_c = \omega^2 r$.
Const. acceleration:	$\omega = \omega_0 + \alpha t$,	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$,	$\omega_{\mathrm{ave}} = \frac{1}{2}(\omega_0 + \omega)$,	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$.

Torque:	$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$,	$\tau = rF\sin\theta$,	$\tau = r_\perp F = rF_\perp$,	$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, etc.	
Dynamics, inertia:	$I = \Sigma m r^2$,	$I = \int dm\, r^2$,	$\tau_{\mathrm{net}} = I\alpha$,	$K_{\mathrm{rot}} = \frac{1}{2}I\omega^2$.	
Rotational inertias:	$I = MR^2$,	$I = \frac{1}{2}MR^2$,	$I = \frac{2}{5}MR^2$,	$I = \frac{1}{12}ML^2$,	$I = I_0 + md^2$.
(about centers)	thin hoop,	solid cylinder,	solid sphere,	thin rod,	parallel axis theorem.

Work, power:	$dW = \tau\, d\theta$,	$W = \int \tau\, d\theta$,	$W = \tau_{\mathrm{ave}}\Delta\theta$,	$P = \tau\omega$.	
Angular momentum:	$\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$,	$l = rp\sin\theta$,	$l = r_\perp p = rp_\perp$,	$\vec{\mathbf{L}} = \int \vec{\mathbf{r}} \times \vec{\mathbf{v}}\, dm$,	$L = I\omega$.
Dynamics:	$\frac{d}{dt}\vec{\mathbf{L}} = \vec{\tau}_{\mathrm{net}}$,	$\Delta\vec{\mathbf{L}} = \vec{\tau}_{\mathrm{ave}}\Delta t$.			
Conservation (@ $\vec{\tau}_{\mathrm{net}} = 0$):	$\Delta\vec{\mathbf{L}}_{\mathrm{total}} = 0$,	$\vec{\mathbf{L}}_{\mathrm{total}}$ =const.			