#### Prefixes

 $z = 10^{-21}, \ a = 10^{-18}, \ f = 10^{-15}, \ p = 10^{-12}, \ n = 10^{-9}, \ \mu = 10^{-6}, \ m = 10^{-3}, \ c = 10^{-2}, \ k = 10^{3}, \ M = 10^{6}, \ G = 10^{9}, \ T = 10^{12}, \ P = 10^{15}, \ E = 10^{18}, \ Z = 10^{21}, \ E = 10^{10}, \ E$ 

### Physical Constants

 $\begin{array}{ll} g=9.80~\text{m/s}^2~\text{(gravitational acceleration)} & G=6.67\times 10^{-11}~\text{N}\cdot\text{m}^2/\text{kg}^2~\text{(gravitational constant)} \\ M_E=5.98\times 10^{24}~\text{kg}~\text{(mass of Earth)} & R_E=6380~\text{km}~\text{(mean radius of Earth)} \\ m_e=9.11\times 10^{-31}~\text{kg}~\text{(electron mass)} & m_p=1.67\times 10^{-27}~\text{kg}~\text{(proton mass)} \\ c=299,792,458~\text{m/s}~\text{(speed of light)} & 1~\text{amu}=1~\text{u}=1.6605402\times 10^{-27}~\text{kg}~\text{(atomic mass unit)} \\ \end{array}$ 

#### Units & Conversions

### Algebra

Quadratic equations:  $ax^2 + bx + c = 0$ , solved by  $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a)$ .

## Geometry

Triangles:  $A = \frac{1}{2}bh$ , Circles;  $C = 2\pi r$ ,  $A = \pi r^2$ , arc  $= s = r\theta$ . Spheres:  $A = 4\pi r^2$ ,  $V = \frac{4\pi}{3}r^3$ 

### Trigonometry

 $\sin\theta = (\text{opp})/(\text{hyp}), \qquad \cos\theta = (\text{adj})/(\text{hyp}), \qquad \tan\theta = (\text{opp})/(\text{adj}), \qquad (\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2.$  $\sin^2\theta + \cos^2\theta = 1, \qquad a^2 + b^2 - 2ab\cos\gamma = c^2, \qquad \frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}, \qquad \alpha + \beta + \gamma = 180^\circ = \pi \text{ rad}.$ 

# Chapter 2 - Vectors - Magnitude & Direction

magnitude =  $a = \sqrt{a_x^2 + a_y^2}$  direction  $\to \tan \theta = a_y/a_x$  $\vec{\mathbf{a}} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}}$ 2D Vectors:  $a_x = a\cos\theta$  $a_u = a \sin \theta$  $\theta$ =angle to +x-axis. Components: Subtraction:  $\vec{\mathbf{a}} - \vec{\mathbf{b}}$  is  $\vec{\mathbf{a}} + (-\vec{\mathbf{b}})$  $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ , head to tail.  $-\vec{\mathbf{b}}$  is  $\vec{\mathbf{b}}$  reversed. Addition:  $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z$  $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = 1, \ \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0, \text{ etc.}$  $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = ab\cos\phi$ Scalar product:  $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = ab\sin\phi$  $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$ . etc.  $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$ Cross product:

### Chapter 3 - 1D Kinematics - Straight-line motion

 $v(t) = \frac{dx}{dt}$  = slope of x(t)  $a(t) = \frac{dv}{dt}$  = slope of v(t)Velocity:  $v_{\rm avg} = \Delta x / \Delta t$  $\Delta x = x - x_0$  $\Delta v = v - v_0$  $a_{\rm avg} = \Delta v / \Delta t$ Acceleration:  $v(t) = v_0 + \int_0^t a(t')dt'.$  $x(t) = x_0 + \int_0^t v(t')dt',$ Integrals = areas: $v = v_0 + at$  $v_{\text{avg}} = \frac{1}{2}(v_0 + v),$  $\Delta x = v_{\text{avg}} \Delta t.$  $v^2 = v_0^2 + 2a \Delta x.$ Constant acceleration:  $x = x_0 + v_0 t + \frac{1}{2} a t^2,$  $x = x_0 + v_{\text{avg}}t,$ (position from acceleration) (using average velocity) (timeless equation)  $y = y_0 + v_{0y}t - \frac{1}{2}gt^2,$  $v_y^2 = v_{0y}^2 - 2g\Delta y.$ Free fall (+y-axis is up):  $v_y = v_{0y} - gt,$ 

### Chapter 5 - Newton's laws and forces

Newton's  $1^{\rm st}$  Law:  $\vec{a} = d\vec{v}/dt = 0$  unless  $\vec{F}_{\rm net} \neq 0$   $\vec{F}_{\rm net} = \sum \vec{F}_i = {\rm sum~of~all~forces~on~a~mass.}$  Newton's  $2^{\rm nd}$  Law:  $\vec{F}_{\rm net} = m\vec{a}$   $F_{\rm net,x} = ma_x,~F_{{\rm net},y} = ma_y,~F_{{\rm net},z} = ma_z$  Newton's  $3^{\rm rd}$  Law:  $\vec{F}_{AB} = -\vec{F}_{BA}$  Forces exist in action-reaction pairs.

Gravitational force near Earth:  $F_g = mg$ , downward. Apparent weight is force measured by a scales. Gravity components on inclines:  $F_{g,\parallel} = mg\sin\theta, \ F_{g,\perp} = mg\cos\theta$   $\leftarrow$  for incline at angle  $\theta$  to horizontal. Spring force:  $F_{g,\parallel} = mg\sin\theta, \ F_{g,\perp} = mg\cos\theta$   $\Rightarrow$  x is the displacement from equilibrium.

### Chapter 6 - Friction, circular motion

Static friction (object is stuck):  $f_s \leq \mu_s N$ Can balance other forces in any direction.  $f_k = \mu_k N$ Kinetic friction (object sliding): Acts against the relative motion of surfaces.

 $a_c = v^2/r$ Centripetal acceleration: Points towards the center of the circle.

### Chapter 7 - Work and kinetic energy

 $W_{AB} = \int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$  (along the path  $A \to B$ )  $dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = F \ dr \ \cos \theta$ Work done by a force:

 $W = \vec{\mathbf{F}} \cdot \Delta \vec{\mathbf{r}}$  $\Delta \mathbf{r} = \vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A = \text{displacement.}$ Work of a constant force:

 $\begin{aligned} W_{\mathrm{g}} &= -mg\Delta y \\ W_{s} &= -\frac{1}{2}k(x_{B}^{2} - x_{A}^{2}) \end{aligned}$  $\Delta y = y_B - y_A$  (final minus initial height) Work done by gravity:

Work done by a spring: B=final stretch, A =initial stretch.

Work done by friction: Use formula for constant force. Friction's work can be positive or negative!!

KE =  $\frac{1}{2}mv^2$ ,  $P = \frac{dW}{dt}$ ,  $P_{\text{ave}} = \frac{\Delta W}{\Delta t}$ .  $\leftarrow$  the rate of doing work by some force.  $\Delta KE = W_{\text{net}} = \text{all works on } m.$ Work-KE theorem:

P = dW/dtInstantaneous power: When  $\vec{\mathbf{F}}$  acts on m:  $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$  $\leftarrow$  instantaneous power only due to  $\mathbf{F}$ 

 $P_{\text{ave}} = \Delta W / \Delta t$ Average power:  $\leftarrow$  average over time interval  $\Delta t$ .

### Chapter 8 - Potential energy and Conservation of energy

 $\Delta U = mg\Delta y,$ PE for gravity: U(y) = mgy +constant.

 $\Delta U = \frac{1}{2}k(x_B^2 - x_A^2),$  $U(x) = \frac{1}{2}kx^2 + \text{constant.}$ PE for springs:

 $\Delta E_{\text{total}} = 0,$ Arbitrary system:  $E_{\text{total}} = E_{\text{mec}} + E_{\text{thermal}} + E_{\text{other}}.$ 

(nothing is left out of energy accounting).

### Chapter 9 - Linear momentum and collisions

 $\Delta \vec{\mathbf{p}} = \vec{\mathbf{J}} = \int \vec{\mathbf{F}}(t) dt = \vec{\mathbf{F}}_{\text{ave}} \Delta t.$ Linear Momentum:  $\vec{\mathbf{p}} = m\vec{\mathbf{v}},$ Impulse Theorem:

 $\vec{\mathbf{F}}_{\mathrm{ave}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t}^{J}$ .  $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$ Instantaneous force: Average force:

Conservation (@  $\vec{\mathbf{F}}_{net} = 0$ ):  $\Delta \vec{\mathbf{p}}_{total} = 0,$  $\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f},$ i=initial, f=final.

 $\vec{\mathbf{r}}_{\text{com}} = \frac{m_1 \vec{\mathbf{r}}_1 + m_2 \vec{\mathbf{r}}_2 + \dots}{m_1 + m_2 + \dots},$   $v_{1f} = 2v_{\text{com}} - v_{1i}$ Center of mass:

$$\begin{split} \vec{\mathbf{v}}_{\text{com}} &= \frac{m_1 \vec{\mathbf{v}}_1 + m_2 \vec{\mathbf{v}}_2 + \dots}{m_1 + m_2 + \dots} \\ v_{\text{2f}} &= 2v_{\text{com}} - v_{\text{2i}}, \end{split} \quad \text{Equal masses swap velocities.}$$
1D elastic collisions:

 $\vec{P}_{\text{total}} = M\vec{\mathbf{v}}_{\text{com}} = \text{const.}$  $\vec{P}_{\text{total}} = m_1 \vec{\mathbf{v}}_1 + m_2 \vec{\mathbf{v}}_2 = \text{const.}$ Other collisions:

The point  $\vec{\mathbf{r}}_{com}$  moves as a point mass  $M = \sum_{i} m_i$  subjected to net force  $F_{net}$ . Extended objects:

#### Chapters 10,11 - Rotational motion, angular momentum

 $\begin{array}{lll} 1 \ \text{rev} = 2\pi \ \text{rad} & 1 \ \text{rev} = 360^{\circ}, & \omega = 2\pi f, \\ \omega_{\text{ave}} = \frac{\Delta\theta}{\Delta t}, & \Delta\theta = \omega_{\text{ave}} \Delta t, & \alpha_{\text{ave}} = \frac{\Delta\omega}{\Delta t}, \\ l = \theta r, & v = \omega r, & a_{\text{tan}} = \alpha r, \\ \omega = \omega_0 + \alpha t, & \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2, & \omega_{\text{ave}} = \frac{1}{2}(\omega_0 + \omega), \end{array}$ Coordinates: Averages: Radius factors:

Const. acceleration:

 $\begin{array}{ll} \tau = rF\sin\theta, & \tau = r_{\perp}F = rF_{\perp}, & \hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}, \text{ etc.} \\ I = \int dm\,r^2, & \tau_{\rm net} = I\alpha, & K_{\rm rot} = \frac{1}{2}I\omega^2. \\ I = \frac{1}{2}MR^2, & I = \frac{2}{5}MR^2, & I = \frac{1}{12}ML^2, \\ \text{solid cylinder,} & \text{solid sphere,} & \text{thin rod,} \end{array}$  $\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}},$  $I = \sum m r^2,$ Torque: Dynamics, inertia:

 $I = MR^2$ ,  $I = I_0 + md^2.$ Rotational inertias:

(about centers) thin hoop, parallel axis theorem.

Work, power:  $dW = \tau d\theta$ ,  $W = \int \tau d\theta$ ,  $W = \tau_{\text{ave}} \Delta \theta$ ,  $P = \tau \omega$ .

 $l = r_{\perp}p = rp_{\perp}, \qquad \vec{\mathbf{L}} = \int \vec{\mathbf{r}} \times \vec{\mathbf{v}} \, dm,$  $\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}},$  $l = rp\sin\theta$ , Angular momentum:

 $\frac{d}{dt}\vec{\mathbf{L}} = \vec{\tau}_{\mathrm{net}},$  $\Delta \vec{\mathbf{L}} = \vec{\tau}_{\text{ave}} \Delta t.$ Dynamics: Conservation (@  $\vec{\tau}_{net} = 0$ ):  $\vec{\Delta} \vec{\mathbf{L}}_{\text{total}} = 0,$  $\vec{\mathbf{L}}_{\text{total}} = \text{const.}$