Eng.	Phys. I	Exam 3 -	Chs. 7	& 8 -	Work &	Conservation of	of Energy
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Mar. 4, 2022

Write **neat** & **clear** work. Show **formulas** used, essential steps, results with correct **units** and **significant figures**. Points shown in parenthesis. For TF and MC, choose the *best* answer. Use  $g = 9.80 \text{ m/s}^2$ . Ignore air resistance unless it is mentioned in a question. You are allowed to use only a calculator and the attached equation sheet.

- 1. (3) The work done by a constant force that is perpendicular to the displacement of a mass
  - a. is negative.
- b. is positive.
- c. is zero.
- d. can be anything.
- 2. (3) If a person weighing 600 N climbs up stairs and increases altitude by 20 m while moving at 2.0 m/s, the work done by gravity is
  - a. -12 kJ.
- b. -1.2 kJ.
- c. zero.
- d. 1.2 kJ.
- e. 12 kJ.
- 3. (3) The net force on a mass points along the  $\hat{i}$ -axis. If the kinetic energy is increasing, in which direction should the velocity point?
- a. Along  $-\hat{\mathbf{i}}$ .
- b. Along  $+\hat{\mathbf{i}}$ .
- c. Along  $-\hat{\mathbf{j}}$ .
- d. Along  $+\hat{\mathbf{j}}$ .
- e. Along  $-\hat{\mathbf{k}}$
- f. Along  $+\hat{\mathbf{k}}$
- 4. (8) For each of the listed forces, write a minus (-), zero (0) or plus (+) in the space according to whether the work done by that force is negative, zero, or positive, respectively.
  - a) \_\_\_\_ Frictional force on the tires slows a car as it skids to a stop.
  - b) \_\_\_\_ A rock falls to the ground under the influence of gravitational force.
  - c) \_\_\_\_ The normal force acts on your shoes as you walk to class.
  - d) \_\_\_\_ Static friction force acts on your car tires while accelerating up a hill.
- 5. (10) A constant force  $\vec{\mathbf{f}} = (32.0\,\hat{\mathbf{i}} + 24.0\,\hat{\mathbf{j}})$  N is acting on a mass that moves from  $\vec{\mathbf{r}}_1 = (-8.00\,\hat{\mathbf{i}} + 15.0\,\hat{\mathbf{j}})$  m along a straight line to  $\vec{\mathbf{r}}_2 = (0.0\,\hat{\mathbf{i}} + 0.0\,\hat{\mathbf{j}})$  m in 2.50 s.
- a) (6) Calculate the work done by this force, in joules.

b) (4) Calculate the average power exerted by this force during the motion.

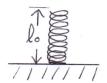
- 6. (2) **T F** If the kinetic energy of a mass increases, the net work on it must be positive.
- 7. (2) **T F** A joule of work performed in a microsecond equates to a power of 1 MW.
- 8. (18) A bus of mass m=1900 kg travels 0.50 km along a 8.0° incline while the static friction force of the road on the tires is a constant value,  $f_s=0.24\,mg$ . Its initial speed is 8.0 m/s.
  - a) (6) Calculate the work done by  $f_s$  acting on the bus.



b) (6) Calculate the work done by the gravitational force acting on the bus.

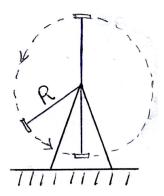
c) (6) How fast is the bus moving after traveling the 0.50 km distance?

- 9. (2) **T F** When the distance a spring is compressed is doubled, its stored potential energy is quadrupled.
- 10. (3) As you run up a hill, which of the following is a conservative force acting on you? Check all that apply.
  - a. The normal force on your shoes.
- b. The static friction force on your shoes.
- c. Gravitational force on your body.
- d. The force of air resistance on your body.
- 11. (14) A spring has relaxed length  $\ell_0 = 50.0$  cm and is mounted vertically on the floor as shown. When a 2.50 kg ball is placed on top of it and allowed to settle to equilibrium, the spring compresses by 2.00 cm.
  - a) (6) How large is the spring constant?



b) (8) Now the ball is pushed downward (on the spring) until it is 20.0 cm above the floor, and then released. What maximum height above the floor does the ball reach?

- 12. (2) **T F** When the potential energy of a conservative system increases, the kinetic energy decreases.
- 13. (2) T F Just as kinetic energy cannot be negative, potential energy cannot be negative.
- 14. (2) **T F** The potential energy change associated with a force acting over some distance is the negative of the work done by that force.
- 15. (18) With a big initial push from her friends, a child of mass m swings all the way around in a full circle of radius R on a swing set. The friends have pushed so hard that the normal force of the chair acting on her at the top of the circle is mg. After the push, her mechanical energy is conserved. In terms of the given symbols,
  - a) (6) How large is her kinetic energy at the top of the circle?



b) (6) How large is her kinetic energy at the bottom of the circle?

c) (6) How large is the normal force of the chair acting on her at the bottom of the circle?

- 16. (2) **T** F The negative slope of a potential energy function U(x) gives the force component  $F_x$ .
- 17. (2) **T** F The minima of a potential energy function U(x) are turning points or limits of the motion.

18. (12) On a level highway a 980-kg vehicle comes to a complete stop over a distance of 250 m, starting from an initial speed of 36 m/s. Assuming smooth braking and constant deceleration, calculate the average rate at which mechanical energy was converted into thermal energy by braking (a power).

5. \_\_\_\_\_ / 16 Score = \_\_\_\_ / 108.

#### Prefixes

 $z = 10^{-21}, \ a = 10^{-18}, \ f = 10^{-15}, \ p = 10^{-12}, \ n = 10^{-9}, \ \mu = 10^{-6}, \ m = 10^{-3}, \ c = 10^{-2}, \ k = 10^{3}, \ M = 10^{6}, \ G = 10^{9}, \ T = 10^{12}, \ P = 10^{15}, \ E = 10^{18}, \ Z = 10^{21}, \ E = 10^{10}, \ E$ 

#### Physical Constants

 $\begin{array}{ll} g=9.80 \text{ m/s}^2 \text{ (gravitational acceleration)} & G=6.67\times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 \text{ (gravitational constant)} \\ M_E=5.98\times 10^{24} \text{ kg (mass of Earth)} & R_E=6380 \text{ km (mean radius of Earth)} \\ m_e=9.11\times 10^{-31} \text{ kg (electron mass)} & m_p=1.67\times 10^{-27} \text{ kg (proton mass)} \\ c=299,792,458 \text{ m/s (speed of light)} & 1 \text{ amu}=1 \text{ u}=1.6605402\times 10^{-27} \text{ kg (atomic mass unit)} \end{array}$ 

#### Units & Conversions

## Algebra

Quadratic equations:  $ax^2 + bx + c = 0$ , solved by  $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a)$ .

#### Geometry

Triangles:  $A = \frac{1}{2}bh$ , Circles;  $C = 2\pi r$ ,  $A = \pi r^2$ , arc  $= s = r\theta$ . Spheres:  $A = 4\pi r^2$ ,  $V = \frac{4\pi}{3}r^3$ 

### Trigonometry

 $\sin \theta = (\text{opp})/(\text{hyp}),$   $\cos \theta = (\text{adj})/(\text{hyp}),$   $\tan \theta = (\text{opp})/(\text{adj}),$   $(\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2.$   $\sin^2 \theta + \cos^2 \theta = 1,$   $a^2 + b^2 - 2ab\cos \gamma = c^2,$   $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c},$   $\alpha + \beta + \gamma = 180^\circ = \pi \text{ rad.}$ 

## Chapter 1 - Units, measurements, errors or uncertainties

(old units)  $\times \left(\frac{\text{new units}}{\text{old units}}\right) = (\text{new units}).$ Unit conversions: value = # (old units), Significant figures:  $\div$  or  $\times$ , use the least no. of sig. figs., + or -, drop the insignificant digits. Sig. figs. "1" rule: if 1st digit=1, keep 1 extra digit,  $\leftarrow$  for division or multiplication only. Measurements: measurement =  $x \pm \delta x$ , x = observed value,  $\delta x =$  error or uncertainty. Percent error: measurement = value  $\pm$  error, percent error =  $(error / value) \times 100\%$ . + or -,  $\delta x = \sqrt{(\delta x_1)^2 + (\delta x_2)^2 + \dots}$ . Combining errors:  $\div$  or  $\times$ , add the % errors,

## Chapter 2 - Vectors - Magnitude & Direction

magnitude =  $a = \sqrt{a_x^2 + a_y^2}$  direction  $\rightarrow \tan \theta = a_y/a_x$  $\vec{\mathbf{a}} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}}$ 2D Vectors:  $a_x = a\cos\theta$  $a_y = a \sin \theta$  $\theta$ =angle to +x-axis. Components: Subtraction:  $\vec{\mathbf{a}} - \vec{\mathbf{b}}$  is  $\vec{\mathbf{a}} + (-\vec{\mathbf{b}})$   $-\vec{\mathbf{b}}$  is  $\vec{\mathbf{b}}$  reversed.  $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ , head to tail. Addition:  $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z$  $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = 1$ ,  $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0$ , etc.  $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = ab\cos\phi$ Scalar product:  $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$  $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$ , etc.  $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = ab\sin\phi$ Cross product:

#### Chapter 3 - 1D Kinematics - Straight-line motion

Velocity:  $v_{\text{avg}} = \Delta x/\Delta t$   $\Delta x = x - x_0$   $v(t) = \frac{dx}{dt} = \text{slope of } x(t)$  Acceleration:  $a_{\text{avg}} = \Delta v/\Delta t$   $\Delta v = v - v_0$   $a(t) = \frac{dv}{dt} = \text{slope of } v(t)$  Integrals = areas:  $x(t) = x_0 + \int_0^t v(t')dt',$   $v(t) = v_0 + \int_0^t a(t')dt'.$ 

Constant acceleration:  $v = v_0 + at$ ,  $v_{\text{avg}} = \frac{1}{2}(v_0 + v)$ ,  $\Delta x = v_{\text{avg}} \Delta t$ .  $x = x_0 + v_0 t + \frac{1}{2}at^2$ ,  $x = x_0 + v_{\text{avg}}t$ ,  $v^2 = v_0^2 + 2a\Delta x$ . (position from acceleration) (using average velocity) (timeless equation)

Free fall (+y-axis is up):  $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ ,  $v_y = v_{0y} - gt$ ,  $v_y^2 = v_{0y}^2 - 2g\Delta y$ .

# Chapter 4 - 2D and 3D Motion - Vector displacement, velocity, acceleration

Position:	$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$	$\vec{\mathbf{r}} = (x, y, z)$	$\Delta \vec{\mathbf{r}} = (\Delta x, \Delta y, \Delta z)$
Velocity:	$\vec{\mathbf{v}}_{\mathrm{avg}} = \Delta \vec{\mathbf{r}} / \Delta t$	$\vec{\mathbf{v}} = d\vec{\mathbf{r}}/dt$	$\Delta ec{\mathbf{r}} = ec{\mathbf{r}} - ec{\mathbf{r}}_0$
Acceleration:	$\vec{\mathbf{a}}_{\mathrm{avg}} = \Delta \vec{\mathbf{v}} / \Delta t$	$\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$	$\Delta ec{\mathbf{v}} = ec{\mathbf{v}} - ec{\mathbf{v}}_0$

Projectiles: 
$$a_x = 0$$
  $v_x = v_{0x}$   $x = x_0 + v_{0x}t$   $(+y\text{-axis is up})$   $a_y = -g$   $v_y = v_{0y} - gt$   $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$  Range:  $R = (v_0^2/g)\sin(2\theta_0)$   $\leftarrow$  (range on level ground only)

Range: 
$$R = (v_0^2/g)\sin(2\theta_0) \leftarrow \text{(range on level ground only)}$$

Relative Motion: 
$$\vec{\mathbf{v}}_{\mathrm{BS}} = \vec{\mathbf{v}}_{\mathrm{BW}} + \vec{\mathbf{v}}_{\mathrm{WS}}$$
 B=Boat, S=Shore, W=Water. BS is "boat relative to shore", etc.  $a_c = v^2/r = \omega^2 r$   $v = 2\pi r/T = \omega r$   $\omega = 2\pi/T$ ,  $T = \mathrm{period}$  of 1 rev.

## Chapter 5 - Newton's laws and forces

Newton's 1 <sup>st</sup> Law:	$\vec{a} = d\vec{v}/dt = 0$ unless $\vec{F}_{\rm net} \neq 0$	$\vec{F}_{\rm net} = \sum \vec{F}_i = \text{sum of all forces on a mass.}$
Newton's 2 <sup>nd</sup> Law:	$ec{F}_{ m net} = m ec{a}$	$F_{\text{net},x} = ma_x, \ F_{\text{net},y} = ma_y, \ F_{\text{net},z} = ma_z$
Newton's 3 <sup>rd</sup> Law:	$ec{F}_{AB} = -ec{F}_{BA}$	Forces exist in action-reaction pairs.

Gravitational force near Earth:	$F_q = mg$ , downward.	Apparent weight is force measured by a scales.
Gravity components on inclines:	$F_{g,\parallel} = mg\sin\theta, \ F_{g,\perp} = mg\cos\theta$	$\leftarrow$ for incline at angle $\theta$ to horizontal.
Spring force:	$F_{r} = -kx$	r is the displacement from equilibrium

# Chapter 6 - Friction, circular motion

Static friction (object is stuck):	$f_s \le \mu_s N$	Can balance other forces in any direction.
Kinetic friction (object sliding):	$f_k = \mu_k N$	Acts <b>against</b> the relative motion of surfaces.

Centripetal acceleration:	$a_c = v^2/r$	Points towards the center of the circle.
Centripetal force:	$F_{\rm net,inward} = ma_c$	"Centripetal force" is the <b>sum</b> of forces inward.
Rates of circular motion:	$v = 2\pi r/T = 2\pi r f$	frequency $f = 1/T$ , $T$ =period of one revolution.

## Chapter 7 - Work and kinetic energy

Work-KE theorem:

Work done by a force:	$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = F \ dr \ \cos \theta$	$W_{AB} = \int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ (along the path $A \to B$ )
Work of a constant force:	$W = ec{\mathbf{F}} \cdot \Delta ec{\mathbf{r}}$	$\Delta \mathbf{r} = \vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A = \text{displacement.}$
Work done by gravity:	$W_{\rm g} = -mg\Delta y$	$\Delta y = y_B - y_A$ (final minus initial height)
Work done by a spring:	$W_s = -\frac{1}{2}k(x_B^2 - x_A^2)$	B=final stretch, $A$ =initial stretch.
Work done by friction:	Use formula for constant force.	Friction's work can be positive or negative!!

 $KE = \frac{1}{2}mv^2$ ,  $\Delta KE = \frac{1}{2}m(v_B^2 - v_A^2)$ 

Instantaneous power:	P = dW/dt	$\leftarrow$ the rate of doing work by some force.
When $\vec{\mathbf{F}}$ acts on $m$ :	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$	$\leftarrow$ instantaneous power only due to $ec{\mathbf{F}}$
Average power:	$P_{\rm ave} = \Delta W / \Delta t$	$\leftarrow$ average over time interval $\Delta t$ .

 $\Delta KE = W_{\text{net}} = \text{all works on } m.$ 

## Chapter 8 - Potential energy and Conservation of energy

Potential energy: PE for gravity: PE for springs: Force from potential:	$\Delta U = U_B - U_A = -W_{A \to B},$ $\Delta U = mg\Delta y,$ $\Delta U = \frac{1}{2}k(x_B^2 - x_A^2),$ $F_x = -dU/dx,$	$W_{A\to B}=$ work done by a conservative force. U(y)=mgy+ constant. $U(x)=\frac{1}{2}kx^2+$ constant. $\leftarrow$ the force component along $x$ -axis.
Conservative system: Non-conservative system: Isolated system:	$\Delta E_{ m mec} = 0,$ $\Delta E_{ m mec} = W_{ m nc},$ $\Delta E_{ m total} = 0,$	$E_{\text{mec}} = K + U$ only. $W_{\text{nc}} = \text{work of nonconservative forces.}$ $E_{\text{total}} = E_{\text{mec}} + E_{\text{thermal}} + E_{\text{other}}.$ (nothing is left out of energy accounting).