Name= **Studio Day/Time=**

Eng. Phys. I Exam 3 - Chs. 8,9,10,11 - Energy Conservation, Momentum, Rotation Oct. 22, 2021

Write **neat** & **clear** work. Show **formulas** used, essential steps, results with correct **units** and **significant figures**. Points shown in parenthesis. For TF and MC, choose the *best* answer. Use $g = 9.80$ m/s^2.

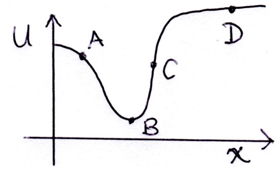
1. (3) A conservative force is one that

a. is associated with a potential energy.
b. depends on the position and velocity of a mass.
c. depends only on the velocity of a mass.
d. is a constant independent of the position.

2. (3) When a closed system decreases its mechanical energy, the energy in the system in other forms . . .

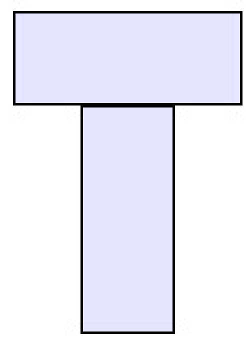
a. stays the same. b. increases. c. decreases. d. Not enough information is given to answer this.

3. (3) The potential energy associated with some force F acting on a mass is shown in the sketch. At which labeled point is the force on the mass to the right?

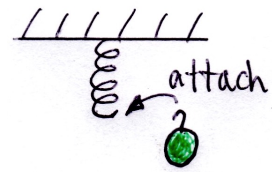


a. A b. B c. C d. D e. At none of the labeled points.

4. (8) Two uniform 1.00-kg rectangular 0.40 m × 1.00 m rods are mounted together in a T-shape. How far is the center of mass from the bottom of the T?



5. (18) An initially relaxed spring with spring constant $k = 125$ N/m hangs from the ceiling. Then a 2.40-kg mass is attached to the end and released. The mass will begin falling and bounce back up and down, repetitively.



a) (6) Assuming a conservative system, what is the maximum stretch of the spring, x_{max}?

b) (6) In reality, there will be dissipation of mechanical energy until the mass stops bouncing up and down. What is the stretch of the spring x_{f} when the mass stops moving?

c) (6) How much mechanical energy was converted to thermal energy by the time the mass stopped bouncing?

6. (2) **T** **F** The impulse experienced by a mass in some process is exactly the same as its change in momentum.

7. (2) **T** **F** An impulse involves smaller average force when the time interval is larger.

8. (2) **T** **F** In an inelastic collision, the total momentum is not conserved.

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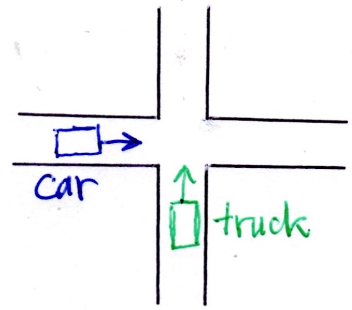
9. (12) In a billiard game the balls' masses are all 160 grams. After the cue ball is shot into the rack to start the game, the 1-ball traveling at 12.0 m/s east makes an elastic head-on collision with the 2-ball traveling at 4.5 m/s west. They are in contact for 2.5 ms.

a) (6) Determine the velocity of the 2-ball after the collision (magnitude and direction).

b) (6) Find the average force exerted by the 1-ball on the 2-ball (magnitude and direction).

10. (16) A 1800-kg car moving east at 18.0 m/s collides in an intersection with a 3200-kg truck moving north at 24 m/s. They crumple and stick together, and slide until kinetic friction with $\mu_k = 0.80$ brings them to rest.



a) (8) Immediately after they stick together, how fast are they moving?

b) (8) How far from the intersection do they come to rest?

11. (21) A uniform disk-shaped flywheel that rotates around its center has radius $R = 12.0$ cm, mass $M = 2.50$ kg. It is brought from rest to a final rotational speed of 3600 rpm in 10.0 seconds.

a) (6) What is the angular acceleration of the flywheel in rad/s?

b) (9) What magnitude torque (in N·m) caused the acceleration?

c) (6) What force applied tangentially to the edge of the flywheel could produce this angular acceleration?

12. (2) **T** **F** A force that points through an axis of rotation of an object makes no torque.

13. (2) **T** **F** An object with small rotational inertia is easier to spin than one with large rotational inertia.

14. (2) **T** **F** The acceleration vector of a point on a spinning disk always points towards the center of the disk.

15. (2) **T** **F** The angular momentum of a rotating object is its mass times its angular velocity.

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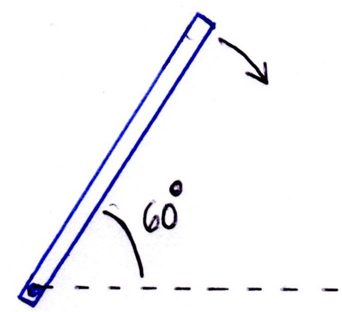
16. (8) A uniform 1.00 kg rod of length 2.00 m, free to rotate about the lower end, starts at rest in the position shown. Use conservation of energy to find the angular speed of the rod as it passes the horizontal position.



Score = ______________/106.

Prefixes

z=10^{-21},	a=10^{-18},	f=10^{-15},	p=10^{-12},	n=10^{-9},	$\mu = 10^{-6}$,	m=10^{-3},	c=10^{-2},	k=10^{3},	M=10^{6},	G=10^{9},	T=10^{12},	P=10^{15},	E=10^{18},	Z=10^{21}
zepto,	atto,	femto,	pico,	nano,	micro,	milli,	centi,	kilo,	mega,	giga,	tera,	peta,	exa,	zeta.

Physical Constants

$g = 9.80$ m/s^2 (gravitational acceleration) $\quad$ $G = 6.67 \times 10^{-11}$ N·m^2/kg^2 (gravitational constant)

$M_E = 5.98 \times 10^{24}$ kg (mass of Earth) $\quad$ $R_E = 6380$ km (mean radius of Earth)

$m_e = 9.11 \times 10^{-31}$ kg (electron mass) $\quad$ $m_p = 1.67 \times 10^{-27}$ kg (proton mass)

$c = 299\ 792\ 458$ m/s (speed of light) $\quad$ 1 amu = 1 u = $1.6605402 \times 10^{-27}$ kg (atomic mass unit)

Units & Conversions

1 inch = 1 in = 2.54 cm $\quad$ 1 foot = 1 ft = 12 in = 0.3048 m

1 mile = 5280 ft =1760 yards $\quad$ 1 mile = 1609.344 m = 1.609344 km

1 m/s = 3.6 km/hour $\quad$ 88 ft/s = 60 mile/hour

1 acre = (1 mile)^2/640 = 43 560 ft^2 $\quad$ 1 hectare = (100 m)^2 = 10^4 m^2

1 lb = 4.45 N $\quad$ 1 N = 0.225 lb $\quad$ 1 J = 1 joule = 1 N·m

Geometry

Triangles: $A = \frac{1}{2}bh$, $\quad$ Circles; $C = 2\pi r$, $\quad A = \pi r^2$, $\quad$ arc $= s = r\theta$. $\quad$ Spheres: $A = 4\pi r^2$, $\quad V = \frac{4\pi}{3}r^3$

Trigonometry

$\sin\theta = \frac{\text{(opp)}}{\text{(hyp)}}$, $\quad \cos\theta = \frac{\text{(adj)}}{\text{(hyp)}}$, $\quad \tan\theta = \frac{\text{(opp)}}{\text{(adj)}}$.

$\text{(opp)}^2 + \text{(adj)}^2 = \text{(hyp)}^2$, $\quad a^2 + b^2 - 2ab\cos\gamma = c^2$, $\quad \frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$.

Chapter 2 - Vectors - Magnitude & Direction

2D Vectors:	$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$,	magnitude = $a = \sqrt{a_x^2 + a_y^2}$,	direction $\to \tan\theta = a_y/a_x$.
Components:	$a_x = a\cos\theta$,	$a_y = a\sin\theta$,	θ=angle to $+x$-axis.
Addition:	$\vec{\mathbf{a}} + \vec{\mathbf{b}}$, head to tail.	Subtraction: $\vec{\mathbf{a}} - \vec{\mathbf{b}}$ is $\vec{\mathbf{a}} + (-\vec{\mathbf{b}})$,	$-\vec{\mathbf{b}}$ is $\vec{\mathbf{b}}$ reversed.
Scalar product:	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = ab\cos\phi$,	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_xb_x + a_yb_y + a_zb_z$,	$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = 1$, $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0$, etc.
Cross product:	$\lvert\vec{\mathbf{a}} \times \vec{\mathbf{b}}\rvert = ab\sin\phi$,	$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, etc.	$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$.

Chapter 3 - 1D Kinematics - Straight-line motion

Velocity:	$v_{\text{ave}} = \frac{\Delta x}{\Delta t}$,	$\Delta x = x - x_0$,	$v(t) = \frac{dx}{dt}$= slope of $x(t)$.
Acceleration:	$a_{\text{ave}} = \frac{\Delta v}{\Delta t}$,	$\Delta v = v - v_0$,	$a(t) = \frac{dv}{dt}$= slope of $v(t)$.
Constant acceleration:	$v = v_0 + at$,	$v_{\text{ave}} = \frac{1}{2}(v_0 + v)$.	
	$x = x_0 + v_0t + \frac{1}{2}at^2$.	$x = x_0 + v_{\text{ave}}t$,	$v^2 = v_0^2 + 2a\Delta x$.
Free fall ($+y$-axis is up):	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$,	$v_y = v_{0y} - gt$,	$v_y^2 = v_{0y}^2 - 2g\Delta y$.

Chapter 4 - 2D and 3D Motion - Vector displacement, velocity, acceleration

Position:	$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$,	$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.	$\vec{\mathbf{r}} = (x, y, z)$.
Velocity:	$\vec{\mathbf{v}}_{\text{ave}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$,	$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt}$,	$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0$.
Acceleration:	$\vec{\mathbf{a}}_{\text{ave}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$,	$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt}$,	$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0$.

Chapter 5 - Newton's laws and forces

Newton's 1$^{\text{st}}$ Law:	$\vec{a} = \frac{d\vec{v}}{dt} = 0$ unless $\vec{F}_{\text{net}} \neq 0$,	$\vec{F}_{\text{net}} = \sum \vec{F}_i$ = sum of all forces on a mass.
Newton's 2$^{\text{nd}}$ Law:	$\vec{F}_{\text{net}} = m\vec{a}$,	$F_{\text{net},x} = ma_x,\ F_{\text{net},y} = ma_y,\ F_{\text{net},z} = ma_z$.
Newton's 3$^{\text{rd}}$ Law:	$\vec{F}_{AB} = -\vec{F}_{BA}$,	Forces exist in action-reaction pairs.
Gravitational force near Earth:	$F_G = mg$, downward.	Apparent weight is force measured by a scales.
Gravity components on inclines:	$F_\parallel = mg\sin\theta,\ F_\perp = mg\cos\theta$,	$\leftarrow$ for incline at angle θ to horizontal.
Spring force:	$F_s = -kx$,	x is the displacement from equilibrium.

Chapter 6 - Friction, circular motion

Static friction (object is stuck):	$f_s \le \mu_s N$,	Can balance other forces in any direction.
Kinetic friction (object sliding):	$f_k = \mu_k N$,	Acts **against** the relative motion of surfaces.
Centripetal acceleration:	$a_c = \frac{v^2}{r}$,	Points towards the center of the circle.
Centripetal force:	$F_{\text{net,inward}} = m a_c$,	Centripetal force is the **sum** of inward forces.
Rates of circular motion:	speed $v = \frac{2\pi r}{T} = 2\pi r f$,	frequency $f = \frac{1}{T}$, T=period of one revolution.

Chapter 7 - Work and kinetic energy

Work done by a force:	$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = F\, dr\, \cos\theta$,	$W_{AB} = \int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ (along the path $A \to B$).
Work of a constant force:	$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$,	$\Delta\mathbf{r} = \vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A$ = displacement.
Work done by gravity:	$W_{\text{G}} = -mg\Delta y$,	$\Delta y = y_B - y_A$ (final minus initial height).
Work done by a spring:	$W_s = -\frac{1}{2}k(x_B^2 - x_A^2)$,	B=final stretch, A =initial stretch.
Work done by friction:	Use formula for constant force.	Friction's work can be positive or negative!!
Work-KE theorem:	$\Delta\text{KE} = W_{\text{net}}$ = all works on m.	$\text{KE} = \frac{1}{2}mv^2$, $\Delta\text{KE} = \frac{1}{2}m(v_B^2 - v_A^2)$.
Instantaneous power:	$P = \frac{dW}{dt}$,	← the rate of doing work by some force.
When $\vec{\mathbf{F}}$ acts on m:	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$,	← power only due to $\vec{\mathbf{F}}$.
Average power:	$P_{\text{ave}} = \frac{\Delta W}{\Delta t}$,	← average over time interval Δt.

Chapter 8 - Potential energy and Conservation of energy

Potential energy:	$\Delta U = U_B - U_A = -W_{A\to B}$,	$W_{A\to B}$ = work done by a conservative force.
PE for gravity:	$\Delta U = mg\Delta y$,	$U(y) = mgy+$ constant.
PE for springs:	$\Delta U = \frac{1}{2}k(x_B^2 - x_A^2)$,	$U(x) = \frac{1}{2}kx^2$ + constant.
Force from potential:	$F_x = -\frac{dU}{dx}$,	← the force component along x-axis.
Conservative system:	$\Delta E_{\text{mec}} = 0$,	$E_{\text{mec}} = K + U$.
Non-conservative system:	$\Delta E_{\text{mec}} = W_{\text{nc}}$,	W_{nc} = work of nonconservative forces.
Any arbitrary system:	$\Delta E_{\text{total}} = 0$,	$E_{\text{total}} = E_{\text{mec}} + E_{\text{thermal}} + E_{\text{other}}$.

Chapter 9 - Linear momentum and collisions

Linear Momentum:	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$,	Impulse Theorem:	$\Delta\vec{\mathbf{p}} = \vec{\mathbf{J}} = \int \vec{\mathbf{F}}(t)\, dt = \vec{\mathbf{F}}_{\text{ave}}\Delta t$.
Instantaneous force:	$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$,	Average force:	$\vec{\mathbf{F}}_{\text{ave}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$.
Conservation (@ $\vec{\mathbf{F}}_{\text{net}} = 0$):	$\Delta\vec{\mathbf{p}}_{\text{total}} = 0$,	$\vec{\mathbf{p}}_{1\text{i}} + \vec{\mathbf{p}}_{2\text{i}} = \vec{\mathbf{p}}_{1\text{f}} + \vec{\mathbf{p}}_{2\text{f}}$,	i=initial, f=final.

Center of mass:	$\vec{\mathbf{r}}_{\text{com}} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2 + \ldots}{m_1 + m_2 + \ldots}$,	$\vec{\mathbf{v}}_{\text{com}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + \ldots}{m_1 + m_2 + \ldots}$	
1D elastic collisions:	$v_{1\text{f}} = 2v_{\text{com}} - v_{1\text{i}}$	$v_{2\text{f}} = 2v_{\text{com}} - v_{2\text{i}}$,	Equal masses swap velocities.
Other collisions:	$\vec{P}_{\text{total}} = M\vec{\mathbf{v}}_{\text{com}}$ = const.	$\vec{P}_{\text{total}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$ = const.	

Extended objects: The point $\vec{\mathbf{r}}_{\text{com}}$ moves as a point mass $M = \sum_i m_i$ subjected to net force $\vec{F}_{\text{net}}$.

Chapters 10,11 - Rotational motion, angular momentum

Coordinates:	1 rev = 2π rad	1 rev = $360°$,	$\omega = 2\pi f$,	$f = \frac{1}{T}$.
Averages:	$\omega_{\text{ave}} = \frac{\Delta\theta}{\Delta t}$,	$\Delta\theta = \omega_{\text{ave}}\Delta t$,	$\alpha_{\text{ave}} = \frac{\Delta\omega}{\Delta t}$,	$\Delta\omega = \alpha_{\text{ave}}\Delta t$.
Radius factors:	$l = \theta r$,	$v = \omega r$,	$a_{\tan} = \alpha r$,	$a_c = \omega^2 r$.
Const. acceleration:	$\omega = \omega_0 + \alpha t$,	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$,	$\omega_{\text{ave}} = \frac{1}{2}(\omega_0 + \omega)$,	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$.

Torque:	$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$,	$\tau = rF\sin\theta$,	$\tau = r_\perp F = rF_\perp$,	$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, etc.	
Dynamics, inertia:	$I = \Sigma\, m\, r^2$,	$I = \int dm\, r^2$,	$\tau_{\text{net}} = I\alpha$,	$K_{\text{rot}} = \frac{1}{2}I\omega^2$.	
Rotational inertias:	$I = MR^2$,	$I = \frac{1}{2}MR^2$,	$I = \frac{2}{5}MR^2$,	$I = \frac{1}{12}ML^2$,	$I = I_0 + md^2$.
(about centers)	thin hoop,	solid cylinder,	solid sphere,	thin rod,	parallel axis theorem.

Work, power:	$dW = \tau\, d\theta$,	$W = \int \tau\, d\theta$,	$W = \tau_{\text{ave}}\Delta\theta$,	$P = \tau\omega$.	
Angular momentum:	$\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$,	$l = rp\sin\theta$,	$l = r_\perp p = rp_\perp$,	$\vec{\mathbf{L}} = \int \vec{\mathbf{r}} \times \vec{\mathbf{v}}\, dm$,	$L = I\omega$.
Dynamics:	$\frac{d}{dt}\vec{\mathbf{L}} = \vec{\tau}_{\text{net}}$,	$\Delta\vec{\mathbf{L}} = \vec{\tau}_{\text{ave}}\Delta t$.			
Conservation (@ $\vec{\tau}_{\text{net}} = 0$):	$\Delta\vec{\mathbf{L}}_{\text{total}} = 0$,	$\vec{\mathbf{L}}_{\text{total}}$ =const.			