Name____________________________________ KSU 2016/04/03

Instructions: Use SI units. Only brief derivations here. State your responses clearly, and define your variables in words. Write on other side if needed.

1. (10) Radiation is being produced by a localized time-dependent source with $\rho(\mathbf{x}, t), \mathbf{J}(\mathbf{x}, t)$. Write the wave equation for the vector potential $\mathbf{A}(\mathbf{x}, t)$ that is produced outside the source.

2. (10) Give an expression for the retarded Green's function that applies to the wave equation of the previous question.

3. (10) If the sources have harmonic time dependence $\sim e^{-i\omega t}$, what expression results for the vector potential $\mathbf{A}(\mathbf{x}, t)$ in a general case (without approximations)?

4. (10) If the source is a harmonically oscillating electric dipole $\mathbf{p}(t) = \mathbf{p}e^{-i\omega t}$, with $\mathbf{p} = \int d^3x \; \mathbf{x}\,\rho$, what results for $\mathbf{A}(\mathbf{x}, t)$ very far away?

5. (10) For a harmonically oscillating electric dipole, show how an expression for the magnetic field $\mathbf{H}$ very far away is obtained.

6. (10) If a radiation source is producing magnetic field $\mathbf{H}(\mathbf{x}, t)$ outside the source, how is the electric field $\mathbf{E}(\mathbf{x}, t)$ obtained?

7. (10) A radiation source produces fields $\mathbf{E}$ and $\mathbf{H}$ far from the source. How do you use them to obtain the power radiated per unit solid angle, along a direction $\hat{\mathbf{n}}$?

8. (10) What is the definition of a "vector spherical harmonic?" List one orthogonality property.

9. (10) Write out the general solution of the source-free scalar Helmholtz equation, $(\nabla^2 + k^2)\psi(\mathbf{x}, \omega) = 0$, in spherical coordinates.

10. (10) Light of wave vector $\mathbf{k}_0 = k\hat{\mathbf{n}}_0$ and polarization $\hat{\epsilon}_0$ is scattered into a new wave vector $\mathbf{k} = k\hat{\mathbf{n}}$ and polarization $\hat{\epsilon}$. Write an expression for the differential cross section $d\sigma/d\Omega$ for this process.

11. (10) Describe how the total scattering cross section of a nonconducting dieletric sphere of permittivity ε varies with its radius a and the wavelength λ of the light.

12. (10) Describe how the total cross section (scattering plus absorption) determines the extinction (or attentuation) coefficient α for a medium.

Part A Score = ____________/120

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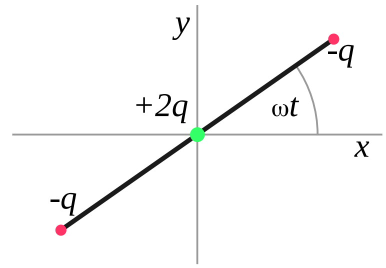
Instructions: **There is only one Problem.** Use SI units. Please show the details of your derivations here. Explain your reasoning for full credit. Open-book only, no notes.

1. (130) Charges $-q$, $-q$ and $+2q$ are fixed at the two ends and center, respectively, of a rod of length $2a$ that rotates in the xy-plane at angular speed ω around the z-axis, forming a rotating electric quadrupole. The position of the charge at one end can be written $x = a\cos\omega t$, $y = a\sin\omega t$; the other is directly opposite this point.



a) (20) Write out the time-dependent charge density $\rho(\mathbf{x}, t)$ in terms of delta functions in Cartesian coordinates.

b) (40) Determine the nonzero Cartesian components of its time-dependent electric quadrupole tensor $\tilde{\mathbf{Q}}(t)$.

c) (20) Express the result for $\tilde{\mathbf{Q}}(t)$ in complex form with a harmonic time dependent part. What is the frequency of the oscillating part? What will be the frequency of the radiation it produces? What is the wavelength?

d) (20) Find the radiated magnetic field $\mathbf{H}$ in the radiation zone. Give the xyz components of $\mathbf{H}$ as functions of the angular direction θ, ϕ of unit wave vector $\hat{\mathbf{n}}$.

e) (10) At a point on the y-axis at radius $r \gg \lambda$, what are the directions of the magnetic and electric field vectors?

f) (20) Find the formula for the time-averaged power radiated per unit solid angle, $dP/d\Omega$, in the far field, as a function of spherical angles θ, ϕ.