For: Feb. 11, 2016 and later.

Remember, you can volunteer to explain any of these at the board. You don't need to work them all out, these are the problems most closely related to the recent lectures.

Problems in Jackson's 3rd ed.:

10.2 Scattering of elliptically polarized light by a perfectly conducting sphere ($\lambda \gg a$).
10.4 Scattering of unpolarized light by a *slightly lossy* dielectric sphere ($\lambda \gg a$).
10.5 Correction to 10.4 due to magnetic dipole contribution.
10.7 Multipole scattering by a dielectric sphere for arbitrary ka.

Other Problems:

W5. (Based on Jackson's 2nd ed., Problem 9.25). In the long-wavelength approximation, analyze the scattering of unpolarized light by a solid sphere of radius R, permittivity ϵ and conductivity σ. Assume a frequency is selected such that the skin depth δ is *much smaller* than R. If necessary, make approximations to leading order in δ/R.

a) Calculate the total scattering cross section $\sigma_{\rm sc}$. How does it behave with frequency ω of the incident light?

b) Calculate the total absorption cross section $\sigma_{\rm abs}$. It is defined as the total power absorbed by the sphere, divided by the incident power per area. Show that it varies as $\omega^{1/2}$ when the conductivity is independent of frequency.

c) Verify that the ratio of these cross sections varies as $\frac{\sigma_{\rm sc}}{\sigma_{\rm abs}} \propto \sqrt{\sigma}\frac{R^4}{c^4}$.

W6. Consider an experiment where light propagates in glass with impurities, such that the permittivity near an impurity is

$$\epsilon(\vec{x}) = \bar{\epsilon} + \epsilon_1 e^{-r^2/2l^2}.$$

r is measured from the impurity, $\bar{\epsilon}$ is the permittivity of the pure glass, and ϵ_1, l characterize the impurity.

a) Assume that the incident light has polarization $\hat{\varepsilon}_0$, and the scattered light is detected at polarization $\hat{\varepsilon}$. Calculate the differential scattering cross section due to one impurity, in the 1st Born approximation.

b) Determine the total cross section for scattering from an impurity, assuming unpolarized incident light, and summing over outgoing polarizations.

c) Suppose there are $N = 10^6$ impurities/cm^3, with $\bar{\epsilon} = 2.0\epsilon_0$, $\epsilon_1 = 0.1\epsilon_0$, and $l = 10^{-4}$ cm. Give numerical estimates for the single impurity scattering cross sections, and for the attenuation coefficient α (in inverse cm), for light with vacuum wavelengths 400 nm and 700 nm.

W7. Light is scattered from matter that possesses a space- and time-dependent dielectric function $\epsilon(\vec{x}, t)$. Suppose the variations in ϵ are caused by collective excitations in the matter, that have wavevector $\vec{Q}$ and frequency $\omega_{\vec{Q}}$, such that

$$\epsilon(\vec{x}, t) = \bar{\epsilon} + \delta\epsilon(\vec{x}, t), \qquad \delta\epsilon(\vec{x}, t) = \sum_{\vec{Q}} F_{\vec{Q}} \; e^{i(\vec{Q}\cdot\vec{x} - \omega_Q t)}.$$

$F_{\vec{Q}}$ is amplitude for collective mode $\vec{Q}$. Function $\omega_{\vec{Q}}$ is the dispersion relation of these modes.

a) Starting from the wave equation for displacement [Jackson Eqn. 10.22], and the appropriate retarded Green function, apply the first Born approximation. Show that the displacement field can be written in the form,

$$\vec{D}(\vec{x}, t) = \vec{D}^0(\vec{x}, t) + \frac{1}{r} \sum_{\vec{Q}} \vec{A}_{\vec{Q}} \; e^{i(kr - \omega t)}, \qquad \vec{D}^0(\vec{x}, t) = D^0 \hat{\varepsilon}_0 \; e^{i(\vec{k}_0\cdot\vec{x} - \omega_0 t)}.$$

Note that k, ω, of the outgoing waves, are different from k_0, ω_0, of the incident waves. Determine k and ω in terms of k_0, ω_0 and $\omega_{\vec{Q}}$.

b) Give an expression for the scattering amplitude $\vec{A}_{\vec{Q}}$. Show that this implies the relation giving the outgoing wavevector,

$$\vec{k} = \vec{k}_0 + \vec{Q}.$$

That is, the light gains momentum from a collective oscillation in the medium, and also changes frequency.

W8. Continue the discussion of W7. Consider how the dispersion relation of the collective modes affects the light scattering (inelastic scattering).

a) Suppose the collective modes have an *acoustic* dispersion,

$$\omega_{\vec{Q}} \approx uQ, \qquad \text{with} \quad u \ll c.$$

Here u is the speed of propagation of the collective modes (i.e., speed of sound). If the light scatters through angle θ, what values of Q are causing the scattering? Note also that $Q < 0$ is also allowed. What do the different signs of Q mean? What wavevectors $\vec{Q}$ are involved?

b) Suppose the collective modes have an *optical* dispersion,

$$\omega_{\vec{Q}} \approx \Omega = \text{constant}.$$

Again, for light scattered through angle θ, what values of Q are causing the scattering? What wavevectors $\vec{Q}$ are involved?