Electrodynamics I | Exam 2 - Part A - Closed Book | KSU 2015/11/06

Name____

Instructions: Use SI units. Where appropriate, define all variables or symbols you use, in words. Try to tell about the physics involved, more than the mathematics, if possible.

1. (8) A charge density $\rho(\mathbf{x})$ is invariant when the system is rotated through any angle around the z-axis. How can you write the general solution of Poisson's equation for the potential $\Phi(\mathbf{x})$ in spherical coordinates in this situation?

2. (8) Write out the orthogonality condition for the Legendre polynomials $P_l(\cos\theta)$, when defined so that $P_l(0) = 1$ for all l.

3. (8) An electric dipole $\mathbf{p}$ at position $\mathbf{x}_0$ is exposed to an external electric field $\mathbf{E}(\mathbf{x})$. Write an expression for the interaction energy of the dipole with this field.

4. (12) The electric potential of an electric dipole $\mathbf{p}$ at the origin is $\Phi(\mathbf{x}) = k\frac{\mathbf{p}\cdot\mathbf{x}}{r^3}$. From this show how to write an expression for the electric field $\mathbf{E}(\mathbf{x})$ at point $\mathbf{x}$ due to an electric dipole at some position $\mathbf{x}_0$.

5. (8) Give a definition of electric polarization $\mathbf{P}$ of a medium.

6. (8) How is electric displacement $\mathbf{D}$ of a medium defined?

7. (8) What is a formula that gives the bound charge density ρ_b inside a dielectric medium?

8. (8) A dielectric medium has an electric susceptibility tensor with Cartesian components χ_{ij}, where i, j correspond to x, y, z. How is χ_{ij} used or what does it mean?

9. (8) A current density all over space is given by $\mathbf{J}(\mathbf{x})$. Write an integral expression that gives the magnetic induction $\mathbf{B}(\mathbf{x})$ due to that current density.

10. (8) Write an integral expression for the vector potential $\mathbf{A}(\mathbf{x})$ due to a current density $\mathbf{J}(\mathbf{x})$.

11. (16) A localized current density $\mathbf{J}(\mathbf{x})$ produces a magnetic dipole moment $\mathbf{m}$.

(a) (8) Write an integral expression for $\mathbf{m}$ in terms of $\mathbf{J}$.

(b) (8) Write an expression for the vector potential produced by $\mathbf{m}$.

12. (8) Give a definition of the magnetization $\mathbf{M}(\mathbf{x})$ of a magnetic material.

13. (8) How is an effective bound current density derived from some magnetization $\mathbf{M}(\mathbf{x})$?

14. (8) How is the magnetic field $\mathbf{H}$ defined?

Part A Score = __________/124

Name____

Instructions: Use SI units. Please show the details of your derivations here. Explain your reasoning for full credit. Open-book only, no notes.

1. (36) Inside a sphere of radius a the electric permittivity is a uniform value ϵ_1. Outside there is a another medium with uniform electric permittivity ϵ_2. A point charge q is placed at the center of the sphere (the origin).

(a) (12) How large is the electric field at $r = 2a$?

(b) (12) How large is the free surface charge density at $r = a$?

(c) (12) How large is the bound surface charge density at $r = a$?

2. (48) Consider the problem of finding an electrostatic potential $\Phi(\rho, \phi, z)$ in the region $z > 0$, above an infinite plane, on which the potential given in cylindrical coordinates is $V(\rho, \phi)$.

(a) (16) Using cylindrical coordinates, write an integral expression for the potential $\Phi(\rho, \phi, z)$ above the plane in terms of Bessel functions $J_m(x)$. It may contain some unknown expansion coefficients that depend on m.

(b) (16) Using appropriate orthogonality conditions, determine an integral expression for those expansion coefficients.

(c) (16) Now consider that the potential given on the plane is circularly symmetric,

$$V(\rho, \phi) = \begin{cases} V_0, & \rho < a \\ 0, & \rho > a \end{cases}$$

If possible, evaluate the expansion coefficients.

Hint: The recursion relations for the Bessel functions will be helpful.

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x).$$

$$J_{n-1}(x) - J_{n+1}(x) = 2\frac{d}{dx} J_n(x).$$

3. (48) A magnetic field is created by a localized distribution of permanent magnetization $\mathbf{M}$, without free currents, $\mathbf{J}$=0. Thus, assume $\mathbf{M}$ can be derived from a magnetic scalar potential by $\mathbf{H} = -\vec{\nabla}\Phi_M$. The total magnetic energy to assemble such a system is to be found.

(a) (16) Use the appropriate Maxwell's equations and get the differential equation that Φ_M obeys.

(b) (16) Show that the field energy integral over all space is zero:

$$W_0 = \frac{1}{2}\int d^3x\ \mathbf{B}\cdot\mathbf{H} = 0.$$

Integration by parts or vector integral calculus identities might be helpful.

(c) (16) Combine the integral in (b) with the interaction of the dipoles in a field, W_{int}, as would be based on a single-dipole interaction energy,

$$u = -\mathbf{m}\cdot\mathbf{B}.$$

Hint: Turn this into an integral W_{int} over the magnetization $\mathbf{M}$. Consider the process of starting with $\mathbf{M}$=0 everywhere and then bringing it to its final values, understanding that $\mathbf{H}$ and $\mathbf{B}$ are being generated by $\mathbf{M}$.

Show that the total system energy is

$$W = W_0 + W_{\text{int}} = \frac{1}{2}\mu_0\int d^3x\ \left(\mathbf{H}^2 - \mathbf{M}^2\right)$$

Part B Score = __________/132