Electrodynamics I Final Exam - Part A - Closed Book KSU 2014/12/18

Name____

Instructions: Use SI units. Please make your answers brief and clear.

1. (2) Give an expression for the force $d\vec{F}$ on a current element $i\, d\vec{l}$ in a magnetic induction $\vec{B}$.

2. (2) A distribution of current $\vec{J}(\mathbf{r})$ exists in a region of magnetic induction $\vec{B}(\mathbf{r})$. Write an expression for the total torque on the current distribution.

3. (2) For a current distribution $\vec{J}(\mathbf{r})$, how does one express the vector potential $\vec{A}(\mathbf{r})$ that it produces?

4. (2) A flat coil (loops confined in some plane) has N turns of some arbitrary two-dimensional shape (along some path $r(\theta)$, for example). If it carries a current i, how do you write an expression for its magnetic dipole moment $\vec{m}$?

5. (2) A particle of charge q and mass M makes a periodic orbital motion. Write an expression relating its orbital magnetic moment $\vec{m}$ and its orbital angular momentum $\vec{L}$.

6. (2) What is the physical definition of magnetization $\vec{M}(\mathbf{r})$?

7. (2) Give the constitutive relation between magnetic induction, magnetization, and magnetic field.

8. (4) Use delta-functions in spherical coordinates to express the current density $\vec{J}(\mathrm{r})$ for the following situation: A spherical shell of radius a, with surface charge density $\sigma = \sigma_0 \sin\theta$, that is rotating around the z-axis at angular frequency ω.

9. (2) A square $a \times a$ coil has N turns. A uniform magnetic induction $B = B_0 \sin(\omega t)$ passes perpendicularly through the coil. Calculate the time-dependent emf produced in the coil.

10. (2) Give an expression for the time-dependent electric field in terms of scalar and vector potentials.

11. (4) In the presence of any time-dependent sources, what equation is obeyed by the vector potential, when using the Lorentz gauge?

12. (4) Consider a wave equation, $\nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi = -4\pi f(\mathbf{r}, t)$, where $f(\mathbf{r}, t)$ is the source that drives some waves $\Psi(\mathbf{r}, t)$. Write out the space- and time-dependent Green's function for this equation that applies to a problem where the source turns on at time $t = 0$.

13. (4) Write out an equation for Poynting's theorem in differential form. Explain in words what each term means physically.

14. (4) From consideration of Maxwell's equations, what are the symmetry properties (odd or even) of the electric polarization $\vec{P}$ under space inversion? What about time inversion?

15. (4) A plane EM wave is traveling in the z-direction in a medium with $\mu = \mu_0$ and $\epsilon = 4\epsilon_0$. With linearly polarized $\vec{E} = E_0 \hat{x}\ \exp[i(kz - \omega t)]$ write an expression for $\vec{B}(z,t)$ in this wave.

16. (4) A plane wave travels in the x-direction: $\vec{E}(\mathbf{r}, t) = E_0(\hat{y} - i\hat{z}) \exp[i(kx - \omega t)]$. Looking into the wave at a fixed point in space, in which direction does the electric field vector rotate (clockwise or couterclockwise)? Which circular polarization is this (right or left)?

17. (2) Write an expression for the dielectric function $\epsilon(\omega)$ in a plasma.

18. (2) What does $\epsilon(\omega)$ imply for EM waves of low frequency traveling in a plasma?

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Instructions: Use SI units. Please write your derivations and final answers on these pages. Explain your reasoning for full credit. Discuss the physics if the math is impossible.

1. (18) Consider a straight wire of radius a and length l in direction z, with a current density $\vec{J} = \sigma\vec{E}$ that is uniform over its cross section. $\vec{E}$ outside the wire is assumed to be negligible here.

   a) (6) For a DC current through the wire, find the magnetic induction $\vec{B}$ both inside and outside the wire.

   b) (6) Based on your result for the DC magnetic field, determine the Poynting vector $\vec{S}$ inside the wire, as a function of the radial coordinate ρ from the axis of the wire.

   c) (6) Show that the result for $\vec{S}$ satisfies Poynting's theorem applied to the whole volume of the wire segment. Comment on the physical significance of the terms in the equation.

2. (18) Consider again a straight wire as in the previous question, with $\vec{J} = \sigma \vec{E}$.

a) (6) Now suppose the current is driven through the wire at a high frequency ω, i.e, harmonic fields varying as $\exp(-i\omega t)$. Apply Maxwell's equations to such a situation to get the differential equation that E_z should solve inside the wire.

b) (6) Consider the case of a very good conductor. Explain physically why you should expect the electric field (and $\vec{J}$) to be nonuniform now within the wire. How should $|\vec{E}|$ vary with ρ inside the wire? I am looking for a very approximate answer; it may not require a calculation.

c) (6) For a copper wire of radius $a = 1.00$ mm, $\epsilon = \epsilon_0$ and $\mu = \mu_0$ and $\sigma = 5.95 \times 10^7$ $(\Omega \cdot \mathrm{m})^{-1}$, estimate the angular frequency ω above which one needs to account for this spatial variation of the fields inside the wire.

3. (16) Hydrogen gas (density $= 10^{12}$ H_2 molecules per cm^3) is heated to a very high temperature ($k_B T \gg 13.6$ eV) so that all molecules are broken apart and the atoms are ionized.

   a) (8) Estimate the range of angular frequencies ω of propagating EM waves in the plasma. Give a numerical result.

   b) (8) Suppose a plane EM wave of amplitude E_0 originally traveling in vacuum is incident on this plasma at normal incidence. The wave has a frequency $\omega = \omega_p/\sqrt{2}$, where ω_p is the plasma frequency. Use the Fresnel formulas to find the amplitude of the electric field after it travels a distance of 5λ into the plasma, where $\lambda = 2\pi c/\omega$ is the wavelength in vacuum.