

Name _____

Instructions: Use SI units. No derivations here, just state your responses clearly, and define your variables in words.

1. (3) Write Gauss' Law in differential form.

2. (3) Write Poisson's equation for the potential $\Phi(\vec{r})$ produced by a charge density $\rho(\vec{r})$.

3. (3) Write a differential equation that a Green function $G(\vec{r}, \vec{r}')$ must solve.

4. (3) A problem has no boundaries. How can you write the solution $\Phi(\vec{r})$ to your above Poisson equation using a Green function?

5. (4) A problem has boundaries with Dirichlet boundary conditions. Now how can you write the solution to the Poisson equation using a Green function?

6. (3) What is the formula for the free-field Green function in three dimensions?

7. (3) Once you know the solution for potential $\Phi(\vec{r})$, how can you obtain the surface charge density σ on a conductor boundary?

8. (3) How can you use $\Phi(\vec{r})$ to obtain the energy density in an electric field in a linear dielectric medium?

9. (3) What is the capacitance of a conducting sphere of radius R ?
10. (9) Use delta-functions to express the charge density $\rho(\vec{r})$ for the following charge distributions, in the indicated coordinate systems:
- a) A charge Q distributed uniformly over a spherical shell of radius a , in spherical coordinates (r, ϕ, θ) .

 - b) A point charge q on the z -axis at $z = z_0$, in cylindrical coordinate (ρ, ϕ, z) .

 - c) The same charge, in spherical coordinates (r, ϕ, θ) .
11. (9) Give the orthogonality relations for
- a) Legendre polynomials

 - a) spherical harmonics

 - a) Bessel functions $J_m(k\rho)$ on $0 \leq \rho \leq a$
12. (3) Give an expression defining the electric dipole moment of a charge distribution:
13. (3) Give an expression defining the electric polarization in a medium:

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Instructions: Use SI units. Please show the details of your derivations here. Explain your reasoning for full credit. Open-book only, no notes.

14. Within a sphere of radius R there is a non-zero charge density,

$$\rho(\vec{r}) = \rho(r, \theta, \phi) = \rho_0 \frac{R}{r} \sin^2 \theta. \quad (1)$$

It is surrounded by an infinite vacuum.

- a) (6) Determine the total charge.
- b) (6) Express $\rho(\vec{r})$ in terms of Legendre polynomials. Why this is a good thing to do?
- c) (8) Determine the potential $\Phi(r, \theta, \phi)$ for points outside the sphere. Think about different ways to do this, perhaps, before proceeding.
- d) (12 Bonus pts) Try this after you finish the other problems. Determine the potential $\Phi(r, \theta, \phi)$ for points *inside* the sphere. Be more careful here, what equation are you solving?

15. A very long right circular cylinder of uniform permittivity ϵ , radius a , is placed into a vacuum containing a previously uniform electric field \vec{E}_0 oriented perpendicular to the axis of the cylinder.
- a) (4) Ignoring end effects, write general expressions for the potential inside and outside the cylinder.
 - b) (8) State and apply the appropriate boundary conditions at the surface of the cylinder.
 - c) (8) Determine the potential inside and outside the cylinder.
 - d) (8) Determine the electric field inside and outside the cylinder. How does the field strength inside compare to that outside, is it what you expect based on physical arguments?

Some Exciting Formulas

Spherical Harmonics

$$Y_0^0 = \sqrt{\frac{1}{4\pi}}, \quad (2)$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta \quad (3)$$

$$Y_1^{\pm 1} = \mp \sqrt{\frac{3}{8\pi}} e^{\pm i\varphi} \sin \theta \quad (4)$$

$$Y_2^0 = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1) \quad (5)$$

$$Y_2^{\pm 1} = \mp \sqrt{\frac{15}{8\pi}} e^{\pm i\varphi} \cos \theta \sin \theta \quad (6)$$

$$Y_2^{\pm 2} = \sqrt{\frac{15}{32\pi}} e^{\pm 2i\varphi} \sin^2 \theta \quad (7)$$