Physics 709 Exam 3 (open book) KSU 98/12/14

1. A gas of hydrogen atoms has been prepared in the 2p state $|n=2, l=1, m=0\rangle$.

a) Calculate the electric dipole matrix element (in C·m) associated with the transition to the 1s ground state:

$$\vec{\mathcal{P}} = e\langle 1,0,0|\ \vec{r}\ |2,1,0\rangle \tag{1}$$

b) Determine the spontaneous transition rate A (in s^{-1}) for this transition.

2. Suppose the atoms of question 1 are bathed in incoherent unpolarized incident light from all directions with energy density per unit frequency given by

$$\rho(\omega) = \rho_0 e^{-(\frac{\omega-\omega_0}{1000\omega_0})^2} \tag{2}$$

where ω_0 is the 2p $\rightarrow$ 1s transition frequency and ρ_0 is a constant.

a) What value of ρ_0 (in $\mathrm{J{\cdot}s/m^3}$) will produce a stimulated transition rate $R_{2p\rightarrow 1s}$ equal to $12.5 \times 10^8\ \mathrm{s}^{-1}$?

b) What is the frequency-averaged rms electric field intensity E_{rms} of the incident light in a), defined by equating energy densities,

$$\frac{1}{2}\epsilon_0 E_{rms}^2 = \int d\omega\ \rho(\omega). \tag{3}$$

Give E_{rms} in Volts/m.

3. A quantum particle of momentum $\hbar k$ scatters from an attractive square well potential,

$$\begin{aligned} V(r) &= -V_0, \quad \text{for} \quad 0 < r < a \\ V(r) &= \ \ 0, \quad\ \ \text{for} \quad r > a \end{aligned} \tag{4}$$

a) Calculate the differential scattering cross section $(d\sigma/d\Omega)$ using the first Born approximation. Give the result in terms of the magnitude of the momentum transfer $\kappa = 2k\sin(\theta/2)$.

b) Carefully sketch $(d\sigma/d\Omega)$ vs. κa. [Hint: can it go to zero at any points? What does it do at low κa, high κa?]

c) By expanding for $\kappa a \ll 1$, determine the leading energy dependence of $(d\sigma/d\Omega)$ at low energy.

d) Find the total scattering cross section in the limit of low energy.

4. (optional) For a free particle the wavefunction e^{ikz} can be written

$$e^{ikz} = \sum_{l=0}^{\infty} i^l (2l+1)\ j_l(kr)\ P_l(\cos\theta) \tag{5}$$

As $r \to \infty$, $j_l(kr) \to \sin(kr - l\pi/2)/kr$, which is a linear combination of an outgoing wave (e^{+ikr}) and an incoming wave (e^{-ikr}).

After scattering, the outgoing wave suffers a phase shift relative to the incoming wave, such that we REPLACE

$$\sin(kr - l\pi/2) \tag{6}$$

BY

$$\left[(e^{2i\delta_l} e^{i(kr-l\pi/2)} - e^{-i(kr-l\pi/2)}\right]/2i \tag{7}$$

a) Show that far from the scatterer we can then write the wavefunction as

$$\Psi = \sum_{l=0}^{\infty} i^l (2l+1) \frac{\sin(kr - l\pi/2)}{kr}\ P_l(\cos\theta) + \left[\frac{1}{k}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l\ P_l(\cos\theta)\right]\frac{e^{ikr}}{r} \tag{8}$$

where the term in square brackets is the scattering amplitude $f(\theta)$.

b) For scattering from a *hard sphere* of radius a, the solution for Ψ outside the sphere is

$$\Psi = \sum_{l=0}^{\infty} i^l (2l+1)\ \left[j_l(kr) - \frac{j_l(ka)}{h_l^{(1)}(ka)}\ h_l^{(1)}(kr)\right]\ P_l(\cos\theta) \tag{9}$$

By comparing (8) and (9) determine the phase shifts δ_l.