1. A gas of hydrogen atoms has been prepared in the 2p state $|n = 2, l = 1, m = 0\rangle$.

a) Calculate the electric dipole matrix element (in C·m) associated with the transition to the 1s ground state:
\[
\vec{P} = e \langle 1, 0, 0 | \vec{r} | 2, 1, 0 \rangle
\]  
(1)
b) Determine the spontaneous transition rate $A$ (in s$^{-1}$) for this transition.

2. Suppose the atoms of question 1 are bathed in incoherent unpolarized incident light from all directions with energy density per unit frequency given by
\[
\rho(\omega) = \rho_0 e^{-\left(\frac{\omega_0 - \omega}{1000\omega_0}\right)^2}
\]  
(2)
where $\omega_0$ is the 2p $\rightarrow$ 1s transition frequency and $\rho_0$ is a constant.

a) What value of $\rho_0$ (in J·s/m$^3$) will produce a stimulated transition rate $R_{2p \rightarrow 1s}$ equal to $12.5 \times 10^8$ s$^{-1}$?
b) What is the frequency-averaged rms electric field intensity $E_{rms}$ of the incident light in a), defined by equating energy densities,
\[
\frac{1}{2} \epsilon_0 E_{rms}^2 = \int d\omega \rho(\omega).
\]  
(3)
Give $E_{rms}$ in Volts/m.

3. A quantum particle of momentum $\hbar k$ scatters from an attractive square well potential,
\[
V(r) = \begin{cases} 
-V_0, & \text{for } 0 < r < a \\
0, & \text{for } r > a
\end{cases}
\]  
(4)
a) Calculate the differential scattering cross section $(d\sigma/d\Omega)$ using the first Born approximation. Give the result in terms of the magnitude of the momentum transfer $\kappa = 2k \sin(\theta/2)$.
b) Carefully sketch $(d\sigma/d\Omega)$ vs. $\kappa a$. [Hint: can it go to zero at any points? What does it do at low $\kappa a$, high $\kappa a$?]
c) By expanding for $\kappa a \ll 1$, determine the leading energy dependence of $(d\sigma/d\Omega)$ at low energy.
d) Find the total scattering cross section in the limit of low energy.
For a free particle the wavefunction \( e^{ikz} \) can be written

\[
e^{ikz} = \sum_{l=0}^{\infty} d(2l + 1) \, j_l(kr) \, P_l(\cos \theta)
\]  

As \( r \to \infty \), \( j_l(kr) \to \sin(kr - l\pi/2)/kr \), which is a linear combination of an outgoing wave \( (e^{ikr}) \) and an incoming wave \( (e^{-ikr}) \).

After scattering, the outgoing wave suffers a phase shift relative to the incoming wave, such that we REPLACE

\[
\sin(kr - l\pi/2)
\]

BY

\[
\left[ e^{2i\delta_l} e^{i(kr - l\pi/2)} - e^{-i(kr - l\pi/2)} \right]/2i
\]

a) Show that far from the scatterer we can then write the wavefunction as

\[
\Psi = \sum_{l=0}^{\infty} d(2l + 1) \frac{\sin(kr - l\pi/2)}{kr} \, P_l(\cos \theta) + \left[ \frac{1}{k} \sum_{l=0}^{\infty} (2l + 1) e^{ikr} \sin \delta_l \, P_l(\cos \theta) \right] \frac{e^{ikr}}{r}
\]

where the term in square brackets is the scattering amplitude \( f(\theta) \).

b) For scattering from a hard sphere of radius \( a \), the solution for \( \Psi \) outside the sphere is

\[
\Psi = \sum_{l=0}^{\infty} d(2l + 1) \left[ j_l(kr) - \frac{j_l(ka)}{h_1^{(1)}(ka)} \, h_1^{(1)}(kr) \right] \, P_l(\cos \theta)
\]

By comparing (8) and (9) determine the phase shifts \( \delta_l \).