1. Soliton Propagation in a Single-Mode Optical Fiber

An optical soliton forms due to the interplay of anomalous group velocity dispersion (GVD) and self-phase modulation (SPM) in an optical fiber. For an ultrashort pulse injected into the fiber, GVD causes the pulse temporal envelope to broaden while SPM causes the spectral width to increase. A soliton forms when the two effects are balanced, which happens when the total amount of dispersion and nonlinearity is just right. We can define the nonlinear length (L_{NL}) and dispersion length (L_D) in the fiber in terms of the peak power P_0, the pulse duration FWHM Δt, group velocity dispersion β_2, and the effective nonlinearity γ by

$$L_{NL} = \frac{1}{\gamma P_0} \text{ and } L_D = \frac{T_0^2}{|\beta_2|} \quad \text{where } \mathrm{T}_0 = \frac{\Delta t}{2\ln(1+\sqrt{2})} \text{ and } \gamma = \frac{n_2 \omega}{c\pi r^2}.$$

A first order soliton occurs when $L_{NL}/L_D = 1$.

A hyperbolic secant pulse with center wavelength λ_0=1550 nm and pulse duration Δt=100 fs FWHM propagates through a length L_D of a single-mode optical fiber. The optical fiber has a core radius of r=4.1 μm and an index difference Δn=0.008 between the core and cladding index or refraction. The value for the nonlinear index of refraction is n_2=3 10^{-20} m^2/W. The fiber core consists of germanium-doped fused silica whose index of refraction is given by the three term Sellmeier equation (valid for wavelength in μm):

$$n^2(\lambda) = 1 + \sum_{i=1}^{3} \frac{B_i \lambda^2}{\lambda^2 - C_i^2} \text{ where } \begin{array}{l} \mathrm{B}_1 = 0.711040,\ \mathrm{B}_2 = 0.451885,\ \mathrm{B}_3 = 0.704048 \\ \mathrm{C}_1 = 0.064270,\ \mathrm{C}_2 = 0.129408,\ \mathrm{C}_3 = 9.45478 \end{array} \tag{1}$$

(The fiber cladding consists of fused silica, which has a smaller index of refraction than germanium-doped fused silica. We will not need to use its Sellmeier equation for the problem.) The wave guiding due to the fiber geometry changes the total dispersion that the pulse experiences. The propagation constant $\beta(\omega)$ for the fiber, which represents the z component of the wavevector $\mathbf{k}(\omega)$, is given by

$$\beta(\omega) = n(\omega)\sqrt{1 + 2\Delta n b(\omega)} .$$

The propagation constant is expressed where Δn is the index difference between core and cladding, r is the core radius, and $b(\omega)$ is the normalized mode propagation constant due to the fiber geometry given in terms of the normalized frequency $V(\omega)$. An approximate form for $b(\omega)$ is given by

$$b(\omega) = 1 - \left(\frac{1+\sqrt{2}}{1+\sqrt[4]{4+V(\omega)}} \right)^2 \text{ where } V(\omega) \equiv \frac{r\omega}{c} n(\omega)\sqrt{2\Delta n}.$$

1. Show that the value of the second order propagation constant β_2 (i.e. group velocity dispersion) at λ_0=1550 nm is -0.0000180 fs^2/nm. β_2 can be determined from

$$\beta_2(\omega_0) = \left. \frac{d^2\beta(\omega)}{d\omega^2} \right|_{\omega=\omega_0}$$

2. What is L_D? Determine the peak power P_0 for where $L_{NL}/L_D = 1$.
3. Consider the pulse propagating through L_D of fiber experiencing only group velocity dispersion (no nonlinear effects). Plot the temporal chirp $\omega_{GVD}(t) = \omega_0 - \partial_t \varphi_{GVD}(t)$ of the pulse due only to GVD after L_D.
4. Consider the pulse propagating through L_D of fiber experiencing only self-phase modulation (no dispersion). Plot the temporal chirp $\omega_{SPM}(t) = \omega_0 - \partial_t \varphi_{SPM}(t)$ of the pulse due only SPM after L_D.
5. By comparing $\omega_{GVD}(t)$ and $\omega_{SPM}(t)$, explain how the interaction of SPM and GVD leads to soliton formation.

2. Partially Degenerate Four Wave Mixing in a Single-Mode Optical Fiber

We wish to determine the pump, signal, and idler frequencies for partially degenerate four-wave mixing (FWM) in an optical fiber. Partially degenerate FWM is described by

$$2\omega_p - \omega_i - \omega_s = 0$$

where we use the terms pump (p), signal (s), and idler (i) as for difference frequency generation. Here we define $\omega_i > \omega_s$.

A strong continuous wave laser serves as the pump at ω_p of power P_0=0.5 MW. The pump is injected into an optical fiber with a germanium-doped fused silica core. The fiber has a core radius 4.1 µm and index difference Δn=0.008 between the core and cladding indices (as in Problem 1).

1. Determine the signal and idler wavelengths produced through partial degenerate four wave mixing for pump wavelengths from λ_p=900 to 2000 nm. To determine this for a given pump frequency ω_p you will need to find the signal ω_s and idler ω_i frequencies that satisfies both energy conservation and phase matching:

$$2\omega_p - \omega_i - \omega_s = 0$$
$$\Delta k = \Delta k_m + \Delta k_w + \Delta k_{NL} = 0$$

where

$$\Delta k_m = c^{-1}\left(n(\omega_s)\omega_s + n(\omega_i)\omega_i - 2n(\omega_p)\omega_p\right)$$
$$\Delta k_w = \Delta n c^{-1}\left(b(\omega_s)\omega_s + b(\omega_i)\omega_i - 2b(\omega_p)\omega_p\right)$$
$$\Delta k_{NL} = 2\gamma P_0$$

The phase mismatch Δk has contributions due to material dispersion (Δk_m), waveguide dispersion (Δk_w), and the fiber nonlinearity (Δk_{NL}). To determine the phase mismatch, you will need to use the Sellmeier equation and $b(\omega)$ from the previous problem.

2. Plot λ_s and λ_i versus λ_p.

The zero group velocity dispersion wavelength λ_{zGVD} is ~ 1345 nm for this fiber, which is determined using $\beta(\omega)$. Notice that the behavior of λ_s versus λ_p and λ_i versus λ_p is different on the long and short wavelength sides of λ_{zGVD}