1. Second Harmonic Generation in Potassium Dihydrogen Phosphate (KDP)

You wish to produce second harmonic generation (SHG) of a continuous wave Nd:YAG laser centered at 1064 nm. To do this you will use a KDP crystal that is cut to produce the second harmonic using Type I$^{(-)}$ (ooe) phase matching. A single laser provides the fundamental fields for the E_1 and E_2 fields at frequency $\omega = \omega_1 = \omega_2$ (corresponding to 1064 nm), the second harmonic field will be the E_3 field at $\omega_3 = 2\omega$. Thus, $E_1 = E_2$ and half of the total power is shared among these fields. The laser power is P=0.2 W and beam diameter (assuming a "top-hat" spatial profile) of the laser in the crystal is 10 μm. The length of the crystal is L=1.0 cm

Type I$^{(-)}$ phase matching implies that the fundamental fields ($E_1 = E_2$) are both orientated along the ordinary (o) axis and the second harmonic (E_3) is orientated along the extraordinary (e) axis of the negative uniaxial KDP crystal. The ordinary and extraordinary indices of refraction as a function of wavelength for KDP are given by the following Laurent series expressions (where λ is expressed in μm):

$$n_o^2(\lambda) = 2.2576 + \frac{1.7623\lambda^2}{\lambda^2 - 57.898} + \frac{0.0101}{\lambda^2 - 0.0142} \qquad n_e^2(\lambda) = 2.1295 + \frac{0.7580\lambda^2}{\lambda^2 - 127.0535} + \frac{0.0097}{\lambda^2 - 0.0014}$$
$$n_e(\theta,\lambda) = \left[\frac{\sin^2\theta}{n_e^2(\lambda)} + \frac{\cos^2\theta}{n_o^2(\lambda)}\right]^{-1/2} \tag{1}$$

For this process d_{eff} will have the form $d_{eff} = d_{ooe} = d_{36}\sin\theta\sin 2\phi$ where d_{36}=0.39 pm/V for KDP.

1. What is the wavelength of the second harmonic generated field (E_3)?
2. Assuming the phase matching process is Type I$^{(-)}$ (ooe), find the phase matching angle θ_{pm} where $\Delta k=0$.
3. Assuming the phase matching process is Type I$^{(-)}$ (ooe), what are the values of n_1, n_2, n_3 where $n_j = n(\lambda_j)$. Make sure to use the proper index n_j (either $n_e(\theta,\lambda)$ or $n_o(\lambda)$) when computing n_1, n_2, n_3

You try to orientate the crystal for perfect phase matching, however you make an error and set the crystal at angles $\theta = 0.995\theta_{pm}$ and $\varphi = 45°$.

4. Find d_{eff} under these conditions in units of pm/V
5. Compute the phase mismatch Δk under these conditions. Use the proper n_1, n_2, n_3.
6. Determine the initial electric field amplitudes $A_1(z=0)$ and $A_2(0)$ in V/m from the given total input power of P=0.2 W. Remember that irradiance (intensity) has units of W/m^2 and is given by

$$I_j = 2\varepsilon_0 n_j c A_j A_j^* \text{ in units of W/m}^2 \tag{2}$$

7. What is the initial amplitude of $A_3(0)$?
8. Numerically solve the three coupled differential equations derived in class for the amplitudes $A_1(z)$, $A_2(z)$ and $A_3(z)$. Assume the possibility of pump depletion, $\theta = 0.995\theta_{pm}$ and $\varphi = 45°$. Plot $I_3(z)$ and $I_1(z)$ for z=0 to L.
9. Is the fundamental power depleted at $z=L$?
10. Using your numerical solution, determine the output SHG power in Watts at $z=L$=1.0 cm. Is the power at $z=L$ the maximum SHG power produced at any position z in the crystal?
11. Determine the SHG conversion efficiency $\eta_{SHG}(z) \equiv I_3(z)/\left[I_1(0)+I_2(0)\right]$ at $z=L$.

Now you set the angle θ for perfect phasematching $\theta = \theta_{pm}$ thus setting the phase mismatch Δk to zero.

12. Solve the coupled differential equations again with $\theta = \theta_{pm}$ and $\Delta k=0$, using the correct values of n_1, n_2, n_3.
13. What SHG power and SHG conversion efficient at $z=L$? Is it larger than before?

We can define a nonlinear length L_{NL} which is a length scale that determines the strength of the nonlinearity. Note that $\eta_{SHG}(z = L_{NL}) \simeq 0.58$ for perfect phase matching. A form for the nonlinear length is given by

$$L_{NL} = \frac{1}{4\pi d_{eff}}\sqrt{\frac{2\varepsilon_0 n_1 n_2 n_3 c \lambda_1^2}{I_1(0)}} \tag{3}$$

14. Compute L_{NL} using Eq. 3. How does it compare to L=1 cm?
15. Solve the coupled differential equations setting $L=4L_{NL}$ for ΔkL=10, ΔkL=1 and ΔkL=0. Plot the conversion efficiencies $\eta_{SHG}(z)$ and $\eta(z) \equiv \left[I_1(z)+I_2(z)\right]/\left[I_1(0)+I_2(0)\right]$ as a function of z for the three cases. Which case produced the most SHG power and the largest $\eta_{SHG}(L)$?

2. Second Harmonic Generation (SHG) of an Ultrashort Pulse

You wish to build a experiment to accurately measure the pulse duration of ultrashort pulses produced by a Chromium: Forsterite (Cr:F) laser. You do not need to know the details of the experiment, only that it needs second harmonic generated light to work. Thus a nonlinear crystal is needed to produce this SHG: the fundamental pulse (the pulse from the Cr:F laser) will be used to produce a SHG pulse using a nonlinear crystal. Phasematching in this nonlinear crystal will be obtained using angle tuning.

The Cr:F laser center wavelength is at 1275 nm, and it produces an average power of 0.5 W. The second harmonic light will be at 637.5 nm. The beam diameter is 50 μm in the crystal (assuming a "top-hat" spatial profile). A single pulse exits the laser every 10 ns thus the laser has a repetition rate of 100 MHz. An estimate of the pulse duration is roughly 20 fs full width at half maximum (FWHM).

Your job is choose a nonlinear crystal to generate second harmonic light at 637.5 nm from fundamental Cr:F laser pulses at 1275 nm.

1. What is the name of the crystal you would use? Find a common and easily purchased crystal that has the smallest absorption α (in units of 1/m) at the fundamental wavelength of 1275 nm.
2. Where could you buy this crystal? If you cannot find a vendor choose a different crystal. Use the internet.
3. Is the crystal uniaxial or biaxial? If your answer is biaxial, choose a different crystal.
4. Is the crystal negative or positive uniaxial?
5. What type of phase matching would you use? Type I or Type II? ooe or oeo or something else?
6. Given your choice of crystal and phase matching type, what would be the phase matching angle θ_{PM}?
7. What would be d_{eff} for your crystal in pm/V?

As discussed in class, each crystal has a finite phase matching bandwidth for pulsed SHG depending on the thickness of the crystal. This means that a given crystal cannot simultaneously phase match all spectral components of the pulse. For pulsed SHG you wish to have the longest crystal possible in order to get the most SHG power *but not at the cost of severely filtering the SHG spectrum*!

8. Given that the pulse duration approximately 20 fs FWHM, estimate the transform-limited spectral FWHM bandwidth of the fundamental pulse spectrum $I(\lambda)$ in nanometers?
9. Using the above pulse as the fundamental, what is the SHG spectral bandwidth (FWHM) in nanometers. The SHG spectrum $I_{SHG}(\omega)$ is proportional to the autoconvolution of the fundamental spectrum:

$$I_{SHG}(\omega) \propto \int I(\eta-\omega) I(\eta) d\eta \tag{4}$$

10. Make an educated guess for the optimal crystal thickness L needed for proper phase matching. Make your choice based on the longest crystal that does not severely filter the SHG spectrum. (Hint: the thickness should be between 0.001 and 1 mm). Remember, the spectral filter function $H(\omega)$ due to the phase mismatch is given by

$$H(\lambda) = \left(\frac{\sin\left(\Delta k(\theta,\lambda) L\right)}{\Delta k(\theta,\lambda) L} \right)^2 \text{ where } L \text{ is the crystal thickness.} \tag{5}$$

11. Determine the spectral width of the filtered SHG spectrum $H(\lambda) I_{SHG}(\lambda)$ in nanometers.