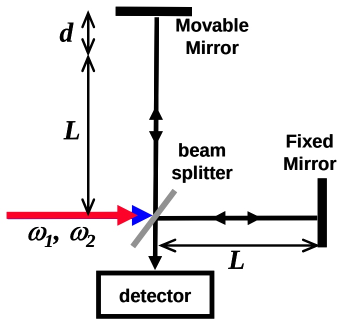
The purpose of the mini-projects is to offer problems in nonlinear and quantum optics in a format that mimics problem-solving scenarios found in a research environment. Buried in the mini-projects are questions that I do not expect you to know or are the solution easily found in the book. This mini-project consists of problems that should be a review of topics that will be important for our initial introduction to nonlinear optics.

1. Interference of two continuous wave lasers in a Michelson interferometer

We wish to build a Michelson interferometer to measure the frequency difference between two continuous wave lasers. The first laser has a center frequency of ω_1 (632.8 nm) and the second has a close but unknown center wavelength ($\omega_2=\omega_1+\delta\omega$). Both lasers have a power of 1 mW, and the polarizations are both vertical. The first step in building our interferometer is to choose a proper beam splitter that has 50% power reflectivity at an orientation of 45 degrees with respect to the input beam.



In the lab we have a beam splitter is made of fused silica of thickness 0.5 cm with an unknown transmission. We need to determine if this beam splitter will work for our interferometer.

1. Is the laser's polarization TE or TM?
2. Compute the power transmitted and reflected from the first surface of the beam splitter. Ignore any absorption.
3. Compute the power transmitted and reflected from the second surface of the beam splitter.
4. Give reasons why this beam splitter a bad choice for our Michelson interferometer.

To make our interferometer a second splitter is used that is very thin and has a power reflectivity of 50% at 632.8 nm for an angle of 45 degrees. If only one laser for frequency ω is used and the beam splitter is 50%, the interference is given by

$$I(\tau)=I_0\left[1+\cos(\omega\tau)\right] \tag{1}$$

where τ is the optical delay between the two arms in the interferometer, I_0 is the intensity of the input laser,.

5. Show that τ relates to d in the figure by $\tau=2d/c$, where c is the speed of light.
6. Derive the interference equation for the Michelson interferometer defining I_1 as the intensity in the fixed arm of the interferometer and I_2 as the intensity in the variable arm. Then derive Equation (1) by setting $I_1= I_2=1/2\ I_{0.}$

Now the two lasers of frequencies ω_2 andω_1 are input into the Michelson interferometer. The resulting interference relation will be

$$I(d)=I_0\left[1+\cos\left(\frac{\omega_1+\omega_2}{2}\frac{2d}{c}\right)\cos\left(\frac{\omega_1-\omega_2}{2}\frac{2d}{c}\right)\right] \tag{2}$$

7. Using (2) determine a method by varying d to measure the $\delta\omega$ between the two lasers input into the interferometer.

2. Quartz as a birefringent material

Crystalline quartz is a birefringent material used in many polarization optics.

1. Describe the crystal type and its birefringent properties.
2. Plot the ordinary and extraordinary indices of refraction at from 500 nm to 1000 nm.
3. To make a quartz quarter-wave zero-order retardation plate at 800 nm, how thick does the plate need to be?

3. Ultrashort pulse dispersion in fused silica

A train of ultrashort optical pulses is produced by a mode-locked Ti:sapphire laser. Each pulse has an electric field profile of hyperbolic secant, is transform limited, and each have a duration of 10 fs full-width half maximum (FWHM). The laser's repetition rate is 100 MHz and the average power from the laser is 100 mW.

1. What is the pulse energy? The peak power?
2. Plot the temporal intensity and phase of the pulse.
3. Plot the spectral intensity and phase of the pulse.

The pulse propagates through a fused-silica window of thickness 1 cm. The dispersion of the fused-silica causes the pulse duration to increase. Consider only quadratic phase distortion (β_2) due to the fused-silica window.

4. Compute and plot final temporal intensity $I_{out}(t)$ and phase $\varphi_{out}(t)$ after propagation through the window.
5. Compute and plot final spectral intensity $I_{out}(\omega)$ and phase $\varphi_{out}(\omega)$ after propagation through the window.
6. Is the "chirp" of the pulse positive or negative?
7. Does the pulse have the same spectral bandwidth before and after the window?
8. What is the final pulse duration (FWHM) after the fused silica window?

Now, ignore the quadratic phase distortion but let the fused silica window have only cubic phase distortion β_3.

9. Compute and plot final temporal intensity $I_{out}(t)$ and phase $\varphi_{out}(t)$ after propagation through the window. Consider only quadratic phase distortion due to the fused-silica window.
10. Compute and plot final spectral intensity $I_{out}(\omega)$ and phase $\varphi_{out}(\omega)$ after propagation through the window.
11. Set $\beta_3=-\beta_{3,\text{fused silica}}$ and find $I_{out}(t)$. How is the temporal intensity different than in Question 9?

4. One dimensional anharmonic oscillator

The Lorentz model of the atom, which treats a solid as a collection of harmonic oscillators, is a good classical model that describes the linear optical properties of a dielectric material. This model can be extended to nonlinear optical media by adding anharmonic terms to the atomic restoring force. In the lecture we will look closely at this model but let's first solve the differential equations for a one-dimensional anharmonic oscillator.

Consider a one-dimension anharmonic oscillator of mass *m* under the influence of the nonlinear restoring force:

$$F(x) = -kx - \alpha x^2 - \beta x^3$$

where $\omega_0^2 = k/m$ is the natural frequency sans any anharmonic terms. Let $m = 1$ kg and k=0.1 N/m .

1. Plot the potential energy for the above force using $\alpha = 0.01\ \text{N/m}^2$ and $\beta = 0\ \text{N/m}^3$. Compare it to the potential energy of a simple harmonic oscillator.
2. Plot the potential energy for the above force using $\alpha = 0\ \text{N/m}^2$ and $\beta = 0.01\ \text{N/m}^3$.
3. Now, let $\alpha = 0.01\ \text{N/m}^2$ and $\beta = 0.01\ \text{N/m}^3$. Numerically solve the 2nd order differential equation of motion, solving for $x(t)$ for t=0 to 20 seconds assuming that $x_0 \equiv x(0) = 0.1$ m and $\dot{x}(0) = 0$ m/s. By plotting $x(t)$ determine the frequency of oscillation ω. How does it compare to ω_0?
4. Find $x(t)$ for $x_0 = 10$ m and $\dot{x}(0) = 0$ m/s, letting $\alpha = 0.01\ \text{N/m}^2$ and $\beta = 0.01\ \text{N/m}^3$. What is the new frequency of oscillation and how does it compare to ω_0.
5. An analytic approximation for $\omega(x_0)$, derived using the method of successive approximations (see Landau's *Mechanics*), is given by

   $$\omega(x_0) = \omega_0 + \left(\frac{3\beta}{8\omega_0} - \frac{5\alpha^2}{12\omega_0^3}\right)x_0^2$$

   Compare your numerical $\omega(x_0)$ to the analytic approximation expression for $x_0 = 0.1$ m to 10 m .
6. The Fourier series for $x(t)$ is given by the expression

   $$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi}{T}t\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2n\pi}{T}t\right)$$

   where the period $T = 2\pi/\omega$ and the Fourier series coefficients are given by

   $$a_n \equiv \frac{2}{T}\int_0^T x(t)\cos\left(\frac{2n\pi}{T}t\right)dt \text{ and } b_n \equiv \frac{2}{T}\int_0^T x(t)\sin\left(\frac{2n\pi}{T}t\right)dt$$

   Numerically solve for $x(t)$ with $x_0 = 10$ m using $\alpha = 0\ \text{N/m}^2$ and $\beta = 0.01\ \text{N/m}^3$. Find the first five Fourier series coefficients a_n (where n=0, … , 4) of the solution $x(t)$. Explain why $b_n = 0$ for all n.
7. Numerically solve for $x(t)$ with $x_0 = 10$ m using $\alpha = 0.01\ \text{N/m}^2$ and $\beta = 0\ \text{N/m}^3$. Find the first five Fourier series coefficients a_n of the solution $x(t)$.
8. Compare the odd terms of a_n for the case where $\alpha = 0, \beta \neq 0$. Compare the even terms of a_n for the case where $\alpha \neq 0, \beta = 0$. How does the symmetry of the restoring force predetermine which order harmonics are produced by the nonlinear oscillator?

Informal Survey for PHYS953: Name:________________________________

Please answer the following questions completely.
What classes in optics and quantum mechanics have you taken? Where have you taken these classes?

Briefly describe your research interests.

Why do want to take this class?

How many hours per week can you spend on homework for this class?

Which of the topics listed in the syllabus seem most interesting to you?

Are there other topics that we should cover in this class?