Homework 7, Kansas State University PHYS522, Mechanics   Due: Friday March 16, 2007 before 5 pm.

The problems we will be doing are becoming more difficult, so to do well you will need to
1. Read the book before starting the homework and before the lecture.
2. Start the homework early.
3. Ask questions early. If you have problems with the numerical solutions, ask for help. Or look at the past solutions and the tutorials for help. It is a more efficient use of your time to ask questions early than to spend forever debugging some code.
4. Make sure you know how to do all the problems. Never surrender; giving up is not an option!

1. Double pendulum problem revisited. From solving Problem 7-7 we found the differential equations that describe the motion for the angles $\varphi_1(t)$ and $\varphi_2(t)$ (see the solution on kstate online for the differential equations).
   a. Express the two coupled, 2nd order differential as four 1st order coupled differential equations.
   b. Numerically solve the four 1st order coupled differential equations for the cases
      1. $\varphi_1(0) = \pi/9,\ \dot{\varphi}_1(0) = 0,\ \varphi_2(0) = 0,\ \dot{\varphi}_2(0) = 0,$
      2. $\varphi_1(0) = \pi/2,\ \dot{\varphi}_1(0) = 0,\ \varphi_2(0) = \pi/2,\ \dot{\varphi}_2(0) = 0,$
      3. $\varphi_1(0) = \pi/2,\ \dot{\varphi}_1(0) = 0,\ \varphi_2(0) = \pi,\ \dot{\varphi}_2(0) = 0,$

      Plot $\varphi_1(t)$ and $\varphi_2(t)$ from t=0 to 40 seconds for each case.
   c. Is the motion chaotic for any of the initial conditions of part b)?
   d. Extra credit: Plot the trajectory (x,y) for mass 1 and mass 2 as a function of time from time t=0.01 to 10 seconds as a movie. An easy way to do this in Mathematica is using the command

      ```
      <<Graphics`Animation`
      Animate[ListPlot[{{x1[t],y1[t]},{x2[t],y2[t]}}, Frame->True],{t,0.01,10,0.05}]
      ```

      Where x1[t] and y1[t] are the x and y positions of mass 1, the same for mass 2 at time t.

2. Problem 7-21, The Spinning Hoop of Doom. Let the radius of the hoop be 0.3 m. In addition to the problem, answer the following questions.
   a. Find the equilibrium angle θ_0 of the mass as a function of ω. Plot $\theta_0(\omega)$.
   b. Find frequencies of small oscillation ω_0 about θ_0 as a function ω. Plot $\omega_0(\omega)$.
   c. From the plot and the expression for $\theta_0(\omega)$, define a logical critical frequency ω_c of the hoop rotation that separates the mass' motion into two categories.
   d. What happens to $\theta_0(\omega)$ and $\omega_0(\omega)$ as $\omega \to \infty$.
   e. Solve the differential equation for the cases $\omega<\omega_c$ and $\omega>\omega_c$. Pick a proper $\theta(0)$ and $\dot{\theta}(0)$ for each case.
   f. Plot the phase space diagrams for cases $\omega<\omega_c$ and $\omega>\omega_c$. Pick a proper $\theta(0)$ and $\dot{\theta}(0)$ for each case.
   g. In words, describe the motion of the mass for the two cases $\omega<\omega_c$ and $\omega>\omega_c$.
   h. Solve the differential equation for $\theta(0) = \pi,\ \dot{\theta}(0) = 0$. Plot $\theta(t)$ for t=0 to 40 seconds. Describe the motion in words. Does the particle move? If it does move, why? This is a point of unstable equilibrium so one would expect the particle to be stationary for all time. Is the motion real? If not, what causes the apparent motion?

3. Problem 7-6
4. Problem 7-12

5. Problem 7-16. Instead., let the support move horizontally by $x(t) = a\cos\omega t$
   a. Find the pendulum's equation of motion.
   b. Is there an equilibrium angle φ_0?
   c. In words describe the motion for initial conditions $\varphi(0) = 0$, $\dot{\varphi}(0) = 0$ with the length b= 1 m, a=0.3 m, and $\omega=\sqrt{g/b}$. Plot $\varphi(t)$ and a phase space diagram for t=0 to 20 seconds.
   d. For c) is the motion periodic or chaotic? Does the motion repeat itself after a time τ? If so what is τ?

6. Problem 7-37