Homework 3, Kansas State University PHYS522, Mechanics   Due: Monday, February 5, 2007

1. Coffee Filter Problem: Part III

In Part II, we derived the differential equation for the free fall of coffee filters under a retardation force of $F_r = \eta |v|^\alpha$. We now want to solve for the velocity as a function of time of the filter under this force. We need to solve the differential equation that was derived in Part II:

$$\dot{v}(t) = -g + \frac{\eta}{nm}|v(t)|^\alpha$$

where m is the mass of one filter (1 g) and n is the number of filters. Since α is not integer (α should be between 2 and 3, and $\eta \sim 0.015$), it will be necessary to solve this differential equation numerically for $v(t)$.

a) Solve the differential equation for $v(t)$ numerically using your experimental values of α and η for n=1 filter. Assume the condition that $v(0)=0$. Choose your solution for times t=0 to 2 seconds. Plot $v(t)$ from t=0 to 2. Show explicitly that your solution $v(0)=0$.
b) Solve the differential equation for $v(t)$ numerically n=10 filters. Assume the condition that $v(0)=0$. Choose your solution for times t=0 to 2 seconds. Plot $v(t)$ from t=0 to 2. Show explicitly that your solution $v(0)=0$.
c) Explain, using the results from your numerical solution, whether it was a good assumption that the filters reach terminal velocity instantaneously.
d) For n=1 and n=10, at approximately what time did the filters hit terminal velocity?
e) For n=1 and n=10, what was the terminal velocity?
f) For n=1 and n=10, how does the computed terminal velocity compare with what you measured?

2. Problem 3-7
3. Problem 3-16.

   Also, sketch out $x(t)$ for the case $\omega^2 > b^2$ for $b<0$. In your discussion, describe any possible violations of physical laws if $b<0$. Can you think of a physical system where $b<0$?
4. Problem 3-24
5. Problem 3-41

6. A particle of mass m= 1 kg undergoes simple harmonic motion in two dimensions. Assuming that $3\omega_x = 5\omega_y = 2\pi$ rad/s.

a) Plot the trajectory for a least one full period for the initial conditions $x(0) = 1$ m, $\dot{x}(0) = 0$ m/s, $y(0) = 1$ m, and $\dot{y}(0) = 1$ m/s
b) Repeat for $x(0) = 0$ m, $\dot{x}(0) = -1$ m/s, $y(0) = 0$ m, and $\dot{y}(0) = 1$ m/s
c) Discuss whether your results in a) and b) make sense. Did the results agree with your expectations?
d) Now, say the particle experiences a retardation force of $\vec{F}_r = b v_y \hat{y}$, a force that acts only in the y direction and is proportional to the y component of the velocity. Plot the trajectory for a least one full "period" for the initial conditions of part a). Let b= 8 kg/s.

7. Trapezoid integration method. The trapezoid method of integration is given by

$$\int_a^b f(x) \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{j=1}^{n-1} f(a+jh) \right] \quad \text{where } h = \frac{b-a}{n} \text{ and } n \text{ is the number of steps}$$

a) Using any software program, write a program to integrate the function $\int_0^{2\pi} x \sin x \, dx$ for n=100 steps. Print out your code.
b) Compute the value of the integral analytically.
c) Find the percent error between your numeric answer and the analytic answer.

8. A periodic triangular wave is represented by

$$f(t) = \begin{cases} t & \text{for } 0 < t < T/2 \\ -t & \text{for } -T/2 < t < 0, \text{ repeat over all t} \end{cases}$$

a) Sketch the function $f(t)$ by hand
b) Is $f(t)$ an even or odd function?
c) Represent $f(t)$ by a Fourier series
d) Create three plots of the Fourier series using 5, 10, and 50 terms. Set T=1 second.

9. Fourier series of a measured sound wave

I measured a sound wave produced by plucking one string on an electric guitar. Go to K-state online and download the data in the file FS_data.txt in the Homework directory. We want to approximate this periodic data with a Fourier series.

a) Plot the data using a computer and show the graph.
b) Estimate the period T of the data in seconds.
c) Write the first five harmonic frequencies in Hz, where the frequency $f_n = \omega_n / 2\pi$.
d) Compute the first five coefficients a_n and b_n for the cosine and sine terms.
e) Sketch two bar graphs, one for the components a_n versus ω_n, and the other for the components b_n versus ω_n
f) Using a computer, plot the data and the Fourier series approximation of the data on the same graph. You will need about 50 terms in the Fourier series to get a good approximation to the data.
g) Extra credit: What note did I play on the guitar?