Homework 2, Kansas State University PHYS522, Mechanics Due: Friday, January 26, 2007

Note: if you cannot do any of the integrals by hand analytically, you can try the Mathematica integrator you used for Homework 1 at http://integrals.wolfram.com. If you find that you cannot do an integral analytically (or a differential equation) by any means, you can always do it numerically and show the resulting plots. This can be done using any software you like (*Mathematica*, *Mathcad*, *Matlab*, *Maple*, *Excel*, etc.), but to get full credit your result must cover all cases discussed in the problem. There is a tutorial on how to do analytic and numerical integrals and differential equations in *Mathematica* on Kstate online.

1. Problem 2-23
2. Problem 2-39
3. Problem 2-54
4. Problem 3-1
5. Appendix C-1, (a)-(d)
6. Appendix C-2, (b)-(d)
7. Repeat Example 2.5 from the text using a retarding force whose magnitude is kmv^2.
   a) Find $v(t)$. Is there a terminal velocity? If so, what is it?
   b) Plot $v(t)$ similar to Fig. 2.6, using the same initial conditions, and discuss the plot physically.
   c) Find and plot $x(t)$ for the same initial conditions as (b). Discuss the plot physically.

8. Coffee Filter Problem, Part I
   Consider a coffee filter of mass m dropped from a height h. The retardation force due to air resistance F_r is given by

$$F_r = \eta |v|^\alpha \text{ where } \eta \text{ is a constant independent of mass.}$$

   Experimentally determine the value of α. To determine α, start by dropping one filter at some height $h >$ 3 meters and measure the time of flight with a stopwatch. Compute the velocity. Repeat by stacking two, three, four, etc. filters together and re-measure the time and compute the velocity. You do not need to measure the mass of the coffee filter.
   Complete a table with at least 10 values of n (including n=1 and n=10):

# of Filters	Time of flight (s)	Velocity (m/s)
n=1	…	…
n=2	…	…
etc	…..	

   How to find α? You need derive an expression for the freefall using the force of gravity and F_r. Assuming that the coffee filter(s) hits terminal velocity instantaneously (*i.e.* once released), one can write

$$nmg = \eta |v|^\alpha \tag{1}$$

   where n is the number of filters and m is the mass of one filter. Using this expression take the natural log of both sides to derive the equation (put $\ln(n)$ and $\ln(|v|)$ in your table)

$$\ln(n) = \alpha \ln(|v|) + \beta \text{ where } \beta \text{ is a constant} \tag{2}$$

   This is an equation for a line with a slope of α and intercept of β! Put $\ln(n)$ and $\ln(|v|)$ in your table. To find α, produce a plot of $\ln(n)$ versus $\ln(|v|)$ and determine α from the slope.

   In a short paragraph, discuss if the outcome of the experiment. Also, answer the following questions:
   a) Derive Eq. (2) and show $\eta = mg/e^\beta$. Find the value of η from your data. The mass of one filter is 1 g.
   b) We made the initial assumption that η is **not** a function of the total mass of filters. Is this a good assumption? Come up with a physical explanation to justify this assumption.
   c) Is it a good assumption that the filters attained terminal velocity instantaneously?
   d) Did all the points fit on the line? Which ones did not?
   I strongly encourage you to do the experiment in a group, but you should do your analysis/discussion independently. You can get coffee filters and may borrow a stopwatch from Dr. Washburn.

9. Coffee Filter Problem, Part II

In Part I, we experimentally determined α and β assuming a retardation force of $F_r = \eta |v|^{\alpha}$. We now want to solve find $v(t)$ by solving the differential equation with our experimental α and β. We want to answer the following questions:

1. Was the assumption that the filters reached terminal velocity "instantaneously" valid?
2. According to the equations of motion, at which time did the filters hit terminal velocity?
3. What are the terminal velocities predicted by equations of motion as a function of number of filters?

In this part we will derive the differential equation to solve. In Part III, we will solve this differential equation (to be done in Homework 3).

Complete the following:

a) Draw a picture of the forces action on the coffee filter and derive the differential equation to be solved (using $F_r = \eta |v|^{\alpha}$) in terms of η, g, n, and α. Write as $\dot{v}(t) = ?$

b) In your differential equation, what is the sign (+ or -) is in front of F_r. Explain why did you picked that sign?

In the next homework we will solve this differential equation.