1. Two pendulums of equal mass *m* but unequal lengths *a* and *b* are coupled by a spring of spring constant *k*. The spring is attached a distance *a*/2 from the base as shown. Let *a*= 1 m, *b*=1.1 m, *m*=1 kg, and *k*=1 N/m.
   a) Find an expression for the kinetic and potential energy without any small angle approximation.
   b) For simple harmonic motion, find an approximate expression for the potential energy keeping terms up to φ_j^2 as done in class and Example 12.4.
   c) Find the matrix **m**. (Hint, write out the terms by hand but input the matrix into *Mathematica*)
   d) Find the matrix **A**. (Hint, write out the terms by hand but input the matrix into *Mathematica*)
   e) Find the numerical value of the eigenfrequencies. How many eigenfrequencies do you expect? (Hint: try solving for the coefficients a_{jr} by hand, then plug in numbers in *Mathematica*.)
   f) Find the normal modes $\eta_1(t)$ and $\eta_2(t)$ assuming the normal modes are orthogonal to each other. (Hint: use my *Mathematica* notes giving in class and online on kstate online).
   g) Plot $\varphi_1(t)$ and $\varphi_2(t)$ for $\varphi_1(0) = 10°$ and $\varphi_2(0) = 0°$ for *t*=0 to 50 seconds.
   h) Plot $\varphi_1(t)$ and $\varphi_2(t)$ for $\varphi_1(0) = 10°$ and $\varphi_2(0) = 10°$.
   i) Plot $\varphi_1(t)$ and $\varphi_2(t)$ for $\varphi_1(0) = 10°$ and $\varphi_2(0) = -10°$.

   Extra: What does this problem have to do with neutrino oscillations? (Hint: see the High Energy Physics display in the basement of Cardwell Hall.)

Fig. 1

2. Three pendulums of equal mass *m* are attached to the same semi-rigid rod (Fig. 2). Pendulum 1 and Pendulum 3 have lengths *a*= 0.5 m, and Pendulum 2 has a length $b = a/\sqrt{3}$. The semi-rigid rod does not bend but allows mechanical energy to be transferred between the pendulums, contributing to the total potential energy of the system:

   $$U(\varphi_1, \varphi_2, \varphi_3) = mga(1-\cos\varphi_1) + mgb(1-\cos\varphi_2) + mga(1-\cos\varphi_3) + \varepsilon(\varphi_1-\varphi_2)^2 + \varepsilon(\varphi_2-\varphi_3)^2 + \varepsilon(\varphi_3-\varphi_1)^2$$

   where ε is proportional to the energy transfer in the rod. Let *m*=1 kg, *k*=1 N/m, and ε=0.1 J.
   a) Find an expression for the kinetic energy.
   b) Find an approximate expression for the potential energy keeping terms up to φ_j^2.
   c) Find the matrix **m**.
   d) Find the matrix **A**.
   e) Find the numerical value of the eigenfrequencies. (Hint: use *Mathematica*)
   f) Find the normal modes assuming the normal modes are orthogonal to each other.
   g) Plot $\varphi_1(t), \varphi_2(t),$ and $\varphi_3(t)$ for $\varphi_1(0) = 10°$, $\varphi_2(0) = 0°$, and $\varphi_3(0) = 0°$ for *t*=0 to 50 seconds.
   h) Plot $\varphi_1(t), \varphi_2(t),$ and $\varphi_3(t)$ for $\varphi_1(0) = 0$, $\varphi_2(0) = 10°$, and $\varphi_3(0) = 0°$ for *t*=0 to 50 seconds.
   i) What would you expect to happen for the case $\varphi_1(0) = 0$, $\varphi_2(0) = 0°$, and $\varphi_3(0) = 10°$?
   j) Explain the results of parts f) thru h) in terms of resonance between the three pendulums.

Fig. 2

Extra Credit: “Stairway to Heaven” Problem

For stage performances of the song “Stairway to Heaven”, Jimmy Page would use a two-necked “double electric guitar” (see below). One of the necks had six strings and the other had twelve strings. The twelve string guitar had standard tuning and was in tune with the six string neck. Page used is monster guitar so he would not have to change guitars half-way through the song, which required both a six and twelve string guitar.

Page noticed that if he kept the volume of the twelve string pickups on high while playing the six string neck, the twelve string pickups would produce an echo of the melody he played on the six string neck. This effect was described by Davis*:

> “Jimmy had discovered that if he left the upper twelve-string neck of his red Gibson guitar wide open, the strings would vibrate in sympathy with the melodies played on the six-string neck, similar to the sympathetic overtones of the sitar and adding to the overall tonal display.”

Write a paragraph giving a physical explanation of the observed coupling between six and twelve neck sections in terms of coupled oscillations and resonance. In your paragraph, describe the setup of the guitar, the tuning of the strings, the function of electric pickups, and how this leads to any coupling or resonance between the two necks. You will need to do a bit of external research on the guitar to answer this question properly. Also, explain how this observation is related to Problem #2.

Gibson EDS 1275 Double-Neck Electric Guitar, courtesy of Gibson.

* S. Davis, “Hammer of the Gods”, pg 169-170