Homework 8. Kansas State University PHYS522, Mechanics Due: Tuesday March 27, 2007 at beginning of class

The problems we will be doing are becoming more difficult, so to do well you will need to

1. Read the book before starting the homework and before the lecture.
2. Start the homework early. Ask questions early.
3. Make sure you know how to do all the problems. Never surrender, giving up is not an option!
4. The total points for this homework is 100.

1. Determine which of the following forces are conservative, and find the potential energy for the forces that are conservative. (pick 4 out of the six)

   a. $\vec{F} = -\frac{F_0}{r^2}\hat{r}$ b. $\vec{F} = F_0 x\hat{z}$ c. $\vec{F} = F_0\cos(\omega t)\hat{z}$ d. $\vec{F} = -k(\dot{x}\hat{x} + \dot{y}\hat{y} + \dot{z}\hat{z})$; $k > 0$

   e. $\vec{F} = -k(x\hat{x} + y\hat{y} + z\hat{z})$; $k > 0$ f. $\vec{F} = F_0\sinh(\alpha\rho)\hat{\rho}$

2. Problem 7-4 but use Hamiltonian dynamics. Write the force as $\vec{F} = -Ar^{\alpha-1}\hat{r}$ in spherical coordinates.
   a. Prove that the force is conservative.
   b. Define the proper generalized coordinates and momenta for the system.
   c. Write the Hamiltonian. Is it equal to the total energy? Is the total energy conserved?
   d. Find the Hamilton equations of motion (for three dimensional space).
   e. Is angular momentum conserved about the origin conserved?
   f. Justify your answer of e) based on arguments of the symmetry of $\vec{F}$.

3. Two masses m and M are connected by a string of length l. Mass m is on a frictionless table, and the string connected to mass M passes through a hole in the center of the table so that mass M is hanging from the string below the table. Mass m is free to move anywhere on the table, and the string slides without friction through the hole. Assume that mass M can only move vertically. Choose polar coordinates for the motion with the origin located at the hole. Let ρ be the distance from the hole to mass m on the table top, φ be the angle of mass m on the table top, and z be the length of string that hangs down from the table top to mass M.
   a. Draw a picture of the system and label the coordinates.
   b. Identify any constraints between ρ, φ, and z.
   c. Write down the Lagrangian for the system.
   d. Write down the Hamiltonian for the system.
   e. Obtain the Hamilton equations of motion.
   f. Identify any conserved quantities.
   g. Using e), write a second order differential equation for ρ.
   h. Using e), write down a differential equation for φ.
   i. Identify an effective potential and sketch it. Discuss it physically. Under which conditions will M remain stationary?

4. Solve the differential equations of Problem ~~X~~ 3 Parts g) and h) numerically for $\rho(t)$ and $\varphi(t)$. Use the parameters m=0.1 kg, M= 0.5 kg, and l =2 m.
   a. Solve for $\rho(t)$ and $\varphi(t)$ using initial conditions $\varphi(0) = 0$, $\dot{\varphi}(0) = 1$ rad/s, $\rho(0) = l$, $\dot{\rho}(0) = 0$.
   b. What is the constant angular momentum p_φ for the above initial conditions?
   c. Plot $\rho(t)$ and $\varphi(t)$ from time t=0 to 10 seconds.
   d. Plot the trajectory $y(t)$ versus $x(t)$ time t=0 to 10 seconds. This is the motion of mass m on the table top.
   e. Describe the motion in words. Is the motion periodic? Does it ever stop?
   f. Describe the motion in words for $\varphi(0) = 0$, $\dot{\varphi}(0) = 0$ and $\rho(0) = l$, $\dot{\rho}(0) = 0$.
   g. Extra credit: Plot a movie of the motion on the table top for a).

5. Artificial gravity and the centrifugal "force". The movie, <u>2001: A Space Odyssey</u> showed a space station that looked like two parallel rings connected to a common axis (Fig. 1). Artificial gravity was generated by the station when it rotated with angular velocity ω about its axis. The rotation axis is perpendicular to the plane of one of the rings. Inside the station, a person could stand on the side of a ring and experience a "force" like gravity although there is no real force acting on the person (Fig 2.). This fictitious force has a name, the centrifugal force.

A person standing on the "bottom" one ring throws a ball of mass m=0.1 kg with velocity v_0= 5 m/s at an angle θ_0 with respect to the "horizontal" (Fig. 3). The particle starts at an initial distance a= 1 m from the edge of the station, where the edge is a distance R from the center of the station. $\theta_0 = 45°$.

a. Express the initial conditions in polar coordinates. (Hint: set the origin at the center of the center of the station and express the position of the ball in terms of polar coordinates ρ and φ.)
b. Write the Lagrangian of the particle using the generalized coordinates of a).
c. Write the Hamiltonian of the particle
d. Is energy conserved?
e. What is the angular momentum p_φ for the above initial conditions? Is it constant?
f. Identify an effective force. This will be the centrifugal "force".
g. Identify an effective potential $U_{eff}(\rho)$ and sketch it.
h. Find two second order differential equation for the position of the ball as a function of time (i.e. find a differential equation for ρ and φ).
i. If the radius of the station is 100 m, what would be the angular velocity ω needed to produce an "acceleration" of 9.81 m/s^2?
j. Explain why this effective potential $U_{eff}(\rho)$ "looks" like the gravitational potential near the earth $U_{eff}(\rho)$. Will $U_{eff}(\rho)$ "look" like the gravitational potential for all ρ?

Extra credit: Numerically solve the differential equations of part h) using the angular velocity from e). Express your solution in Cartesian coordinates $x(t)$ and $y(t)$ as shown below. Plot the trajectory y vs. x and compare to the trajectory one would get from solving projectile motion on the earth.

Fig. 1

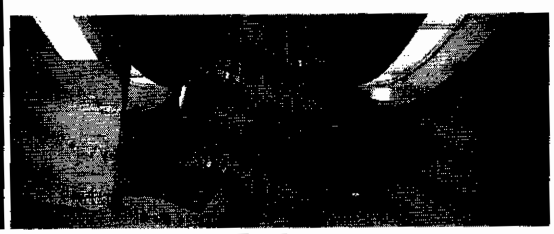

Fig 2

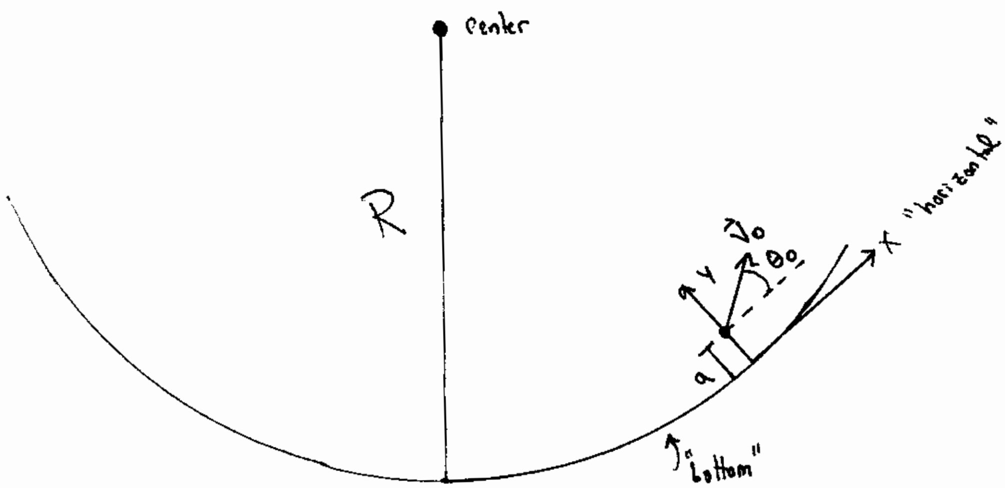

Fig 3

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