QUANTUM MECHANICS I

PHYS 811

Spring 2020

Instructor, Office: Uwe Thumm, Cardwell Hall, Room 212
Class: Tuesday, Thursday 2:30-3:45, Cardwell Hall, room 146
Office hours: by appointment
e-mail, Web page: thumm@phys.ksu.edu, www.phys.ksu.edu/personal/thumm

Prerequisites: Working knowledge of

- Classical Mechanics (PHYS 522 or equivalent)
- Classical Electrodynamics (PHYS 532 or e.)
- Mathematical Methods of Physics (PHYS 801 or e.)

Text book (your choice): **"Quantum Mechanics", E. Merzbacher**, 3rd ed., Wiley & Sons (1998) or **"Modern Quantum Mechanics", J.J. Sakurai and J. Napolitano**, 2nd ed., Pearson (2010)

Supplementary books (not required):

Undergraduate level

- Introduction to Quantum Mechanics, D. J. Griffiths
- Quantum Physics: Atoms, Molecules, Solids, Nuclei, and Particles, R. Eisberg and R. Resnick

Graduate level

- Quantum Mechanics, C. Cohen-Tannoudji, B. Diu, and F. Laloe
- Principles of Quantum Mechanics, R. Shanker
- Quantum Mechanics: Nonrelativistic Theory, L. D. Landau and E. M. Lifshitz
- Feynman Lectures in Physics – Vol. III, R. Feynman
- Quantum Mechanics: Fundamentals, K. Gottfried and T.-M. Yan
- Foundations of Quantum Mech.: From Photons to Quantum Computers, R. Blümel

Mathematical and computational

- Mathematical Methods for Physicists, G. B. Arfken and H. J. Weber
- Mathematical Methods of Physics, J. Mathews and R. L. Walker
- Tables of Integrals, Series, and Products, L. S. Gradsheyn and I. M. Ryzhik
- Handbook of Mathematical Functions, A. Abramowitz and I. A. Stegun
- NIST Digital Library of Mathematical Functions: https://dlmf.nist.gov
- Angular Momentum: Understanding Spatial Aspects in Chemistry and Physics, R. N. Zare
- Numerical Recipes: The Art of Scientific Computing, W. H. Press et al.
- A first course in computational physics, 2. Ed., P. L. DeVries and J. E. Hasbun

More advanced and specialized

- Physics of Atoms and Molecules, B. H. Bransden and C. J. Joachain
- The Theory of Atomic Structure and Spectra, R. D. Cowan
- Theoretical Atomic Physics, H. Friedrich
- Solid State Physics, N. W. Ashcroft and N. D. Mermin
- Introductory Quantum Optics, C. C. Gerry and P. Knight
- Quantum Optics, D. F. Walls and G. J. Milburn
- Elements of Advanced Quantum Theory, J. M. Ziman
- Advanced Quantum Mechanics: The Classical-Quantum Connection, R. Blümel
- Chaos in Classical and Quantum Mechanics, M. C. Gutzwiller

Homework: You are encouraged to discuss strategies for solving homework assignments in small groups. However, I require that you write and return to me at the assigned due dates your **own detailed solutions**. You will receive no credit for solutions that you have copied. In order to obtain full credit, I also request that you present correct answers in a professional, well organized, and readable manner. In addition to the "for credit" homework assignments, I will frequently ask you to complete simple calculations that we don't have time for in class. It is important that you carefully "fill in" these gaps when reviewing your lecture notes. This is done most efficiently **before** the subsequent lecture.

Credit:	Points
Exam 1	200
Exam 2	200
Final Exam	300
Homework	300
Total	1000

Grades:	Points
A	more than 849
B	700-849
C	550-699
D	450-549
F	less than 450

University policy requires the following statements on this syllabus:

Disabilities: If you have any condition such as a physical or learning disability, which will make it difficult for you to carry out the work as I have outlined it or which will require academic accommodations, please notify me and contact the Disabled Students Office (Holton 202), in the first two weeks of the course.

Plagiarism and cheating are serious offenses and may be punished by failure on the exam, paper or project; failure in the course; and/or expulsion from the University. For more information refer to the "Academic Dishonesty" policy in K-State Undergraduate Catalog and the Undergraduate Honor System Policy on the Provost's web page at www.ksu.edu/honor.

Copyright: Students are prohibited from selling (or being paid from taking) notes during this course to or by any person or commercial firm without the express written permission of the professor teaching this course.

Quantum Mechanics 1 – Topics covered in class

1. Introduction

a. Experiments that reveal the particle character of light (Photo-, Compton-effect, black-body radiation)
b. Experiments that reveal the non-classical character of (quasi) particles (Franck-Hertz, H spectrum, Josephson effect).
c. Diffraction of particle beams (single-, double-slit, coherence).

2. Schrödinger Equation for Free Particles

a. Wave function (time independent) and Fourier transforms.
b. Time-dependent wave functions, (Gaussian) wave packets.

3. Schrödinger Equation for a Particle in a Conservative Force Field

a. Consistency with 2a) for $V(x, t) = \text{const.}$
b. Relation to classical mechanics and geometrical optics (Maxwell, Hamilton-Jakobi, eikonal equations).
c. Continuity equation.
d. Schrödinger equation in momentum space.

4. Linear Operators and Expectation Values

a. Definitions (linear, Hermiteian operatiors, scalar product, algebra, Hilbert space, square integrability).
b. Momentum operator in coordinate representation.
c. Classical and quantum observables, their (not unique) correspondence and time evolution. Ehrenfest theorem, uncertainty relation. Virial theorem. Time inversion
d. Stationary states. Degeneracy.
e. Expansions in terms of stationary states. Analytic functions of operators. Completeness.
f. Normalization of continuum wave functions.
g. Unitary operators (examples: displacement, time-evolution,...)
h. Charged particles in electro-magnetic fields.
i (active and passive) Galilei transformations.

5. Linear Harmonic Oscillator

a. Stationary states and energies.
b. Time-dependent solutions. Expectation values and comparison with classical solution (in phase space).

6. Simple Model Potentials (1 D)

a. Quantum phenomena at step potentials (transmission, reflection, penetration).
b. Quantitative solutions (step, well, barrier).
c. Scattering: M and S matrices in relation to incident, transmitted, reflected flux. Resonances.

7. Spherically Symmetric Potentials

a. Orbital angular momentum operator. Commutation relations. Relation to rotations. Ladder operators.
b. Eigenvalues and vectors of $\vec{L}^2$ and L_z. Properties of (associated) Legendre polynomials and spherical harmonics.
c. Radial Schrödinger equation for a free particle. (Spherical) Bessel and Neumann functions.
d. Examples: Coulomb potential. Hydrogen: spectrum and eigenfunctions. Expectation values. Properties of Laguerre polynomials. Momentum-space representation.

8. Perturbation Theory with Applications

a. Time-independent (degenerate) perturbation theory.
b. Valence spectra of alkali atoms.
c. He atom.
d. Stark effect (linear and quadratic) and polarizability for H. Metastable quenching in weak el. field.
e. Time-dependent perturbation theory. Transition rates. Continuum transitions. Fermi's golden rule.
f. Absorption and induced emission of electro -magnetic radiation. Detailed balancing. Application to hydrogen atoms: Dipole selection rules. Cross sections.
g. Photoelectric effect.

9. Scattering at a Central Potential

a. Scattering amplitude and (angle-differential) cross section. Classical versus quantum interpretation.
b. Green's functions (inclusion of boundary conditions, contour integration technique).
c. Born series. First Born approximation cross section for Yukawa and Coulomb potentials.
d. Partial wave expansion of scattering wavefunction, amplitude, and cross section.
e. Scattering phase shifts and resonances. Example: scattering off radial square-well potential.

(over)

Uwe Thumm Fall 2020

Quantum Mechanics 2 – Topics covered in class

1. **Formal Foundation of QM**
   a. Hilbert space. Definition, examples. Dirac (bra-ket) notation.
   b. Linear operators and their representation. Normal, Hermiteian, unitary, adjoint operators.
   c. Operators with continuous spectra. Completeness, closure relation. Functions of operators. Fourier transformation.
   d. Solving eigenvalue problems by i) matrix diagonalization and ii) variation. Example: He atom.
   e. Observables and measurement. Single & simultaneous measurements, complete sets of commuting observables.

2. **Application of Algebraic Operator Techniques**
   a. Harmonic oscillator. Quantization of classical fields. Quasi particles.
   b. Coherent and squeezed states.
   c. Angular momentum algebra.

3. **Quantum Dynamics**
   a. Time-evolution operator.
   b. Schrödinger,- Heisenberg-, and Interaction Picture
   c. Correspondence principle, quantization.
   d. Canonical quantization.
   e. Forced harmonic oscillator: advanced & retarded Green's functions, time ordering, relation to scattering theory.
   f. Coordinate representation of the time-evolution operator. Examples: free particle, harmonic oscillator.
   g. Path integrals. Classical limit. Semi-classical approximations.
   h. Statistical mixtures of quantum states. Density operator. Shannon and von Neumann entropy.

4. **Spin**
   a. Experimental evidence. Stern-Gerlach, Einstein-de Haas experiments. Zeeman effect.
   b. Mathematical description: Two component spinors. Remarks on Dirac equation.
   c. Spin rotations: Pauli matrices.
   d. Spin dynamics. Spin-orbit coupling. Paramagnetic resonances. Remarks on NMR.
   e. Spin-dependent scattering: angle-differential cross section, spin-polarization of scattered particles.
   f. Information content in spin ensembles: von. Neumann and outcome entropy.
   g. Measurement and reduction of quantum states. Multiple Stern-Gerlach experiments.

5. **Addition of Angular Momenta**
   a. Two spin-1/2 particles: product and total spin basis; hyperfine interaction in H (21cm line).
   b. Addition of two arbitrary angular momenta: Clebsch-Gordon coefficients, spectroscopic notation.
   c. (Irreducible) representations of rotations and tensor operators. Wigner-Eckart theorem.
   d. Applications of Wigner-Eckart theorem: normal & anomalous Zeeman effect, Paschen-Back effect.
   e. Other symmetry operations: parity, time reversal, translation, iso-spin (example: nuclear spectra).

6. **Many-Particle Systems**
   a. Identical particles, Fock space, symmetrization postulate, bosons, fermions, spin-statistics theorem.
   b. System of N independent indistinguishable particles (bosons or fermions).
   c. System of N interacting indistinguishable particles (bosons or fermions), "second quantization".
   d. Direct and exchange interactions. Examples: He atom, scattering.
   e. Hartree-Fock method, Brillouin and Koopman theorem.
   f. Molecules (mainly $H_2^{(+)}$).

7. **Relativistic Quantum Mechanics**
   a. Klein-Gordon equation for spinless particles.
   b. Dirac equation for spin-½ particles.
   c. Electromagnetic interactions of Dirac particles. Pauli equation for small $\langle v \rangle/c$. Gyromagnetic factors.
   d. Approximate solution of Dirac equation for H atom.
   e. Outline of exact solution of Dirac equation for H atom and QED corrections.