Phys 971 Stat Mech: Midterm

9/26/2013

1

(40 points) An ideal gas of N particles is contained in a vessel of height H and cross-sectional area A. The gas is subjected to a gravitational field so that the potential energy of a particle at height h is mgh. Compute the heat capacity C_v of the system.

2

(30 points) Consider a system of six distinguishable, non-interacting spins (see figure). Each spin can only occupy two states: 'up' and 'down' relative to an external field. For the first five spins the energy levels are $-\varepsilon$ for an up spin and $+\varepsilon$ for down spin. However, the sixth spin has twice the magnetic moment and, therefore, it's energy levels are -2ε and 2ε. If the total energy is -3ε, calculate the entropy and the average number of 'up' spins, $\langle N_+ \rangle$.

-ε   -ε   -ε   +ε   +ε   -2ε

$E_{total}=-3\varepsilon$
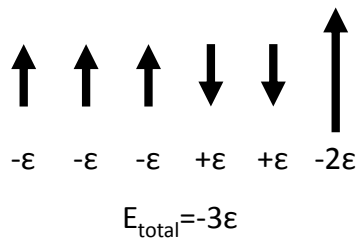
3

(30 points) Two oppositely charged ions can move freely on a lattice of N sites. The separation between lattice sites is sufficient that the ions are non-interacting if the ions are on different sites, however, when they occupy the same site there is an attractive energy $-\delta$. Compute the average energy of the system. Also, calculate and sketch the probability that the ions occupy the same site as a function of temperature (clearly label both limits).