Phys 971 Stat Mech: Homework 5

due 11/7/2013

1

Pathria 6.1 Show that the entropy of an ideal gas in thermal equilibrium is given by the formula

$$S = k \sum_{\epsilon} [\langle n_\epsilon + 1 \rangle \ln \langle n_\epsilon + 1 \rangle - \langle n_\epsilon \rangle \ln \langle n_\epsilon \rangle]$$

in the case of *bosons* and by the formula

$$S = k \sum_{\epsilon} [-\langle 1 - n_\epsilon \rangle \ln \langle 1 - n_\epsilon \rangle - \langle n_\epsilon \rangle \ln \langle n_\epsilon \rangle]$$

in the case of *fermions*. Verify that these results are consistent with the general formula

$$S = -k \sum_{\epsilon} \left(\sum_{n} p_\epsilon(n) \ln p_\epsilon(n) \right)$$

where $p_\epsilon(n)$ is the probability that there are exactly n particles in the energy state ϵ.

2

Pathria 7.13 Consider an ideal Bose gas confined to a region of area A in *two* dimensions. Express the number of particles in the excited states, N_e, and the number of particles in the ground state, N_0, in terms of z, T, and A, and show that the system does not exhibit Bose-Einstein condensation unless $T \to 0\text{K}$.

Refine your argument to show that, if the area A and the total number of particles N are held fixed and we require both N_e and N_0 to be of order N, we do achieve condensation when

$$T \sim \frac{h^2}{mk\ell^2} \frac{1}{\ln N} \tag{1}$$

where $\ell [\sim \sqrt{A/N}]$ is the mean interparticle distance in the system. Of course, if both A and $N \to \infty$, keeping ℓ fixed, then the desired T does go to zero.

3

Consider blackbody radiation in a d-dimensional universe. The dispersion relationship for photons is $\epsilon = \hbar\omega = \hbar ck$. Compute the energy density of a blackbody cavity of volume L^d as a function of temperature. Express any remaining integrals in dimensionless form and compare your answer to the T^4 dependence of Stefan's law.

4

Consider an n-dimensional Bose gas whose single-particle energy spectrum is given by $\epsilon \propto p^s$, where s is some positive number. Show that

$$\begin{aligned} P &= \frac{s}{n}\frac{U}{V} \\ C_V(T \to \infty) &= \frac{n}{s} N k_B \\ C_P(T \to \infty) &= \left(\frac{n}{s} + 1\right) N k_B \end{aligned} \tag{2}$$

5

Pathria 8.7: Show that for an ideal Fermi gas

$$\langle u \rangle \left\langle \frac{1}{u} \right\rangle = \frac{4}{\pi} \frac{f_1(z) f_2(z)}{\{f_{3/2}(z)\}^2},$$

u being the speed of a particle. Further show that at low temperatures

$$\langle u \rangle \left\langle \frac{1}{u} \right\rangle \simeq \frac{9}{8} \left[1 + \frac{\pi^2}{12} \left(\frac{kT}{\epsilon_F} \right)^2 \right];$$

compare with Problem 6.6.