Phys 971 Stat Mech: Homework 2

due 9/24/13

1

In this problem you will find an expression for entropy in terms of the canonical distribution function $\rho_s = \frac{e^{-\beta E_s}}{Z}$ where the partition function Z is given by $Z = \sum_s e^{-E_s/kT}$. Start from $S = -\left(\frac{\partial F}{\partial T}\right)_{N,V}$ and show that $S = -k\sum_s \rho_s \ln \rho_s$. This formula (sometimes called the Shannon Entropy) is accepted as the general definition of entropy, even in situations like information theory where there is no underlying free energy.

2

An ideal gas of N particles is confined to a two-dimensional disk of radius R. Each particle is attracted to the center by a force that increases proportional to its distance from the center, so the Hamiltonian is

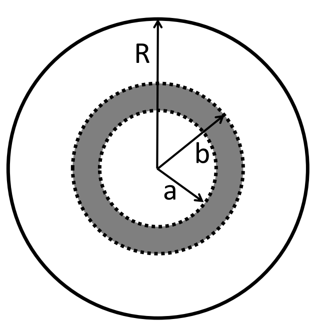
$$H = \sum_{i=1}^{N}\left(\frac{p_i^2}{2m} + \frac{1}{2}kr_i^2\right)$$

where k is the effective spring constant of the central force and m is the mass of the particles.

(a) Write down the partition function for the full system.

(b) Calculate the average energy, pressure, and heat capacity of the system.

(c) Using the single particle partition function, compute the probability that a given particle is in the shaded region of the figure (greater than a distance a from the center but less than b).



3

Pathria 3.15: Show that the partition function $Q_N(V,T)$ of an extreme relativistic gas consisting of N monatomic molecules with energy-momentum relationship $\epsilon = pc$, c being the speed of light, is given by

$$Q_N(V,T) = \frac{1}{N!}\left\{8\pi V\left(\frac{kT}{hc}\right)^3\right\}^N.$$

Study the thermodynamics of the system, checking in particular that

$$PV = U/3, \quad U/N = 3kT, \text{ and } \quad \gamma = 4/3.$$

Next, using the inversion formula (3.4.7), derive an expression for the density of states $g(E)$ of this system.

4

Consider a system of N magnetic dipoles, each of which has an energy $+\epsilon$ in the 'up' state and $-\epsilon$ in the 'down' state.

a) Enumerate the number of microstates $\Omega(N, E)$ accessible to the system at energy E and show that the entropy is given by Eq. (3.10.9).

$$\frac{S}{Nk} = -\frac{N\epsilon + E}{2N\epsilon}\ln\left(\frac{N\epsilon + E}{2N\epsilon}\right) - \frac{N\epsilon - E}{2N\epsilon}\ln\left(\frac{N\epsilon - E}{2N\epsilon}\right)$$

b) Now consider the same system in the canonical ensemble. Evaluate the partition function for the N particle system and show that $Q_N = Q_1^N$.

5

Consider a system of N non-interacting spins of angular momentum J in the presence of an external field H. The energy levels for each spin are $g\mu_B Hm$, where $m = -J, -J+1 \ldots J$. a) Compute the partition function for this system (evaluate all sums). b) Compute the average magnetic moment for the system. c) Compare your results to the $J = 1/2$ example from lecture and the $J = \infty$ classical case (take the limit $J \to \infty$ and $g \to 0$ such that $\mu_B gJ$ retains the constant value μ_0). d) Evaluate the Curie constant for the generic spin J case.