Experimental Determination of the Speed of Light by the Foucault Method

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The speed of light was measured using the Foucault method of reflecting a beam of light from a rotating mirror to a fixed mirror and back creating two separate reflected beams with an angular displacement that is related to the time that was required for the light beam to travel a given distance to the fixed mirror. By taking measurements relating the displacement of the two light beams and the angular speed of the rotating mirror, the speed of light was found to be $(3.09\pm0.204)\times10^{8}$ m/s, which is within 2.7% of the defined value for the speed of light.

1 Introduction

The goal of the experiment was to experimentally measure the speed of light, c, in a vacuum by using the Foucault method for measuring the speed of light. Although there are many experimental methods available to measure the speed of light, the underlying principle behind all methods on the simple kinematic relationship between constant velocity, distance and time given below:

$$c = \frac{D}{t} \tag{1}$$

In all forms of the experiment, the objective is to measure the time required for the light to travel a given distance. The large magnitude of the speed of light prevents any direct measurements of the time a light beam going across a given distance similar to kinematic experiments. Galileo himself attempted such an experiment by having two people hold lights across a distance. One of the experiments would put out their light and when the second observer saw the light cease, they would put out theirs. The first observer would time how long it took for the second light to go out, giving an estimate on the speed of light. However, it was found that the time lapse between the two events was near instantaneous suggesting that such a method was not precise enough, and a more interesting result at the time, that the speed of light was much larger in magnitude than thought[1].

To correct for these problems, Foucault devised a method which was able to avoid such issues by indirectly measuring the time the light traveled. Foucault's method used a light source and rotating mirror together to derive the speed of light[2]. The Foucault method uses the light source to produce a focused beam on the rotating mirror. The light from the rotating mirror is then reflected at an angle to a fixed mirror which is aligned to face perpendicular to the reflected light beam. Therefore the light is reflected directly back to the rotating mirror where it was first reflected. During the time the light had traveled the distance between the two mirrors, the rotating mirror had changed its orientation to the beam of light, thus the

[1],[2] Hecht, Eugene. Optics, 4th Edition. San Francisco: Addison Wesley, 2002.

returning beam of light will be reflected off at a separate angle. The difference in the angle between the light source to the rotating mirror and the rotating mirror the second reflected beam is related to the time that was required by the light to travel the distance between the fixed and rotating mirrors. Using the relations of the experimental setup, Equation 1 was used to determine the speed of light.

1.1 Setup

For the experiment the methods were updated to use modern equipment to provide more accurate and precise results. A diagram of the experimental setup is shown below in Figure 1:

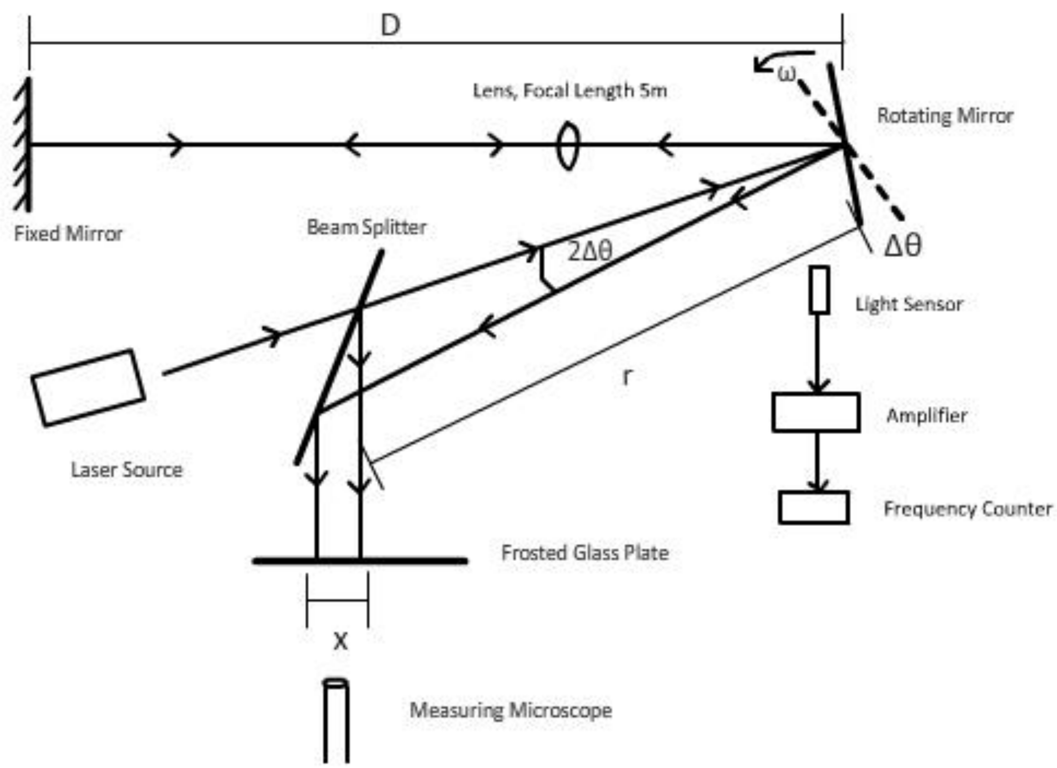

Figure 1 - Diagram of Experimental Setup

The experiment used a laser to provide as the light beam because it creates a focused beam of light to travel between all components. The rotating mirror used was a double sided plane mirror attached to a motor apparatus that allowed a variable control of the motor rotation speed. The rate of rotation was measured using a light sensor in conjunction with a frequency counter. As the mirror rotated with the laser positioned on it, there was an angle of the mirror that reflected the laser light beam toward the light sensor. When the laser beam was directed at the light sensor, it created a voltage which could be then measured by a connected frequency counter.

A lens with a 5 meter focal length was placed between the rotating and fixed mirrors because the laser beam had an angular divergent property associated with the beam. Over large distances, the laser beam became too dispersed to be targeted precisely onto the two mirrors. The lens focused the laser beam directly to a point source on both mirrors making the beams more precisely optically aligned. A beam splitter was used in order to redirect the returning light beam onto a frosted glass plate. The frosted glass plate allowed a simplification in viewing because it disperses the incoming laser beams in all direction so that a 'head-on' viewing angle with the microscope is not required. The difference between these two points was measured by using the measuring microscope placed in front of the frosted glass screen and recording the value of displacement of the two beams.

1.2 Derivation

From simple kinematics, the speed of light is related to the time t it takes a beam of light to travel a distance D from Equation 1. In the experiment, the distance and time measured were for a beam of light to travel between the rotating and fixed mirrors and back in Figure 1. Therefore by the setup the time to travel the distance was:

$$t = \frac{2D}{c} \tag{2}$$

However, during the time period t, the rotating mirror is held at a fixed angular velocity of ω. Because the angular velocity is constant, then

$$\omega_{mirror} = \frac{\Delta\theta}{t} \tag{3}$$

The angular dependence $\Delta\theta$ is also related to the return of the light beam to the beam splitter and the displacement Δx of the light beam on the frosted glass plate. The angle between the two laser beams striking the beam splitter is determined from the law of reflection:

$$\theta_i = \theta_r \tag{4}$$

For a beam of light striking a stationary mirror, the total angle between the reflected and incident light beams is given as:

$$\theta_i + \theta_r = 2\theta_i \tag{5}$$

Therefore for every angular change in the rotating mirror by $\Delta\theta$, the incident angle of the laser beam striking it changes by $\Delta\theta$ and the new incident angle becomes $\theta_i + \Delta\theta$. The increase in the angle results in the total angle increasing as $2\theta_i + 2\Delta\theta$, thus the total distance between the two light beams striking the beam splitter is given by a change of $2\Delta\theta$. By the relationships from the geometry in the two return paths and from small angle approximations the displacement and angle are related by:

$$\mathrm{Tan}(2\Delta\theta) = 2\Delta\theta = \frac{\Delta x}{r}$$

$$\Delta\theta = \frac{\Delta x}{2r} \tag{6}$$

Therefore by relating Equations 3 and 6 for the angle based on time and the displacement on the glass screen together yields the relationship:

$$\omega_{mirror} = \frac{\Delta\theta}{t} = \frac{\Delta x}{2rt} \tag{7}$$

However, the time of rotation can be related to the time traveled by the light:

$$t = \frac{\Delta x}{2r\omega_{mirror}} \tag{8}$$

Equation 2 and Equation 8 can be related so that:

$$\frac{2D}{c} = \frac{\Delta x}{2r\omega_{mirror}} \tag{9}$$

$$c = \frac{4rd\omega_{mirror}}{\Delta x} \tag{10}$$

Within the experiment, it is important to know c in terms of the frequency instead of angular frequency because the frequency counter used returns the measurement in units of Hertz. Therefore by definition,

$$\omega_{mirror} = 2\pi f_{mirror} \tag{11}$$

Then by substituting Equation 11 into 10:

$$c = \frac{8\pi rdf_{mirror}}{\Delta x} \tag{12}$$

Another correction is required for the experiment in terms of frequency. The frequency measured on the frequency counter is twice that of the rotation frequency because the mirror rotated is doubled sided so then the laser beam is reflected into the light sensor twice per each rotation of the mirror. Therefore,

$$f_{mirror} = \frac{1}{2} f_{measured} \tag{13}$$

Substituting Equation 13 into Equation 12 results in:

$$c = \frac{4\pi rdf_{measured}}{\Delta x} \tag{14}$$

Because the experiment is performed within air and not a vacuum, the result from Equation 14 above would give c within air. Therefore by multiplying by the index of refraction of air, we get c within a vacuum:

$$c = \frac{4\pi rdfn_{air}}{\Delta x} \tag{15}$$

Where:

- r is path from the rotating mirror to the beam splitter to the frosted screen
- D is the distance between the fixed mirror and the rotating mirror
- n_{air} is the index of refraction for air, given as 1.0002926[3]
- f is the frequency as given by the frequency counter used within the experimental setup

[3] Hecht, Eugene. Optics, 4th Edition. San Francisco: Addison Wesley, 2002.

- Δx is the displacement of the laser beam across the frosted glass screen

2 Experimental Methods

From Equation 15, the objective of the experimental measurements was to gain the relationship between the two observable, alterable quantities: the frequency of the mirror rotation and the observation of the laser beam displacement across the frosted glass plate. By rearranging Equation 15, a more useful relationship can be obtained:

$$x = \frac{4\pi r d n_{air}}{c} f \tag{16}$$

Equation 16 reveals a linear relation between the displacement on the glass screen and the frequency of the mirror. Therefore the method of collecting data to find the speed of light was to take data points relating the frequency and displacements and to perform a least-square fit in order to determine experimental value of the relationship and compare it to the known value.

To setup the experiment, the optical systems involved had to be aligned in order to create the beam path displayed in Figure 1. First, the laser was aimed at the rotating mirror and slowly adjusted so that the laser beam was directed toward the center of the mirror. Next the rotating mirror was slowly aimed at the center of the fixed mirror. Next the lens was placed between the fixed and rotating mirrors in order to focus the return beam from the fixed mirror to the rotating mirror as otherwise the angular dispersion of the laser beam created difficulty in detecting where the return beam was directed to. The fixed mirror was very carefully adjusted so that the return path through the lens reflected back onto exactly the same location on the rotating mirror where the first reflection occurred. With all the optics aligned, the beam appeared on the frosted glass screen as a point. The setup was then tested by covering the lens between the mirrors. When this was performed, the beam on the glass plate disappeared, verifying the beam was being reflected from the fixed mirror. Next, the position of the light beam on the frosted glass plate with no rotation was recorded by using the measuring microscope by aligning the crosshair of the microscope to the center of the laser beam as it appeared within the scope.

To measure the mirror rotation frequency, the light sensor was placed next to the rotating mirror so that during the operation of the rotating mirror, the laser beam would strike the light sensor. The sensor and frequency counter setup was tested by covering and uncovering the light sensor to verify that it was connected. With all of the optics and equipment in place, the distances between the two mirrors as well as the mirror to the glass screen were recorded as precisely as possible. Next, the motor that rotated the mirror was adjusted to a rotational frequency that caused a displacement in the laser beam. The displacement in the laser beam was measured by the microscope by centering the laser beam in the scope and recording the displacement value. Once the frequency versus displacement data point was recorded, the mirror was set to a new rotation frequency in order to record another data point.

3 Data Set 1

For the first experimental data set, the data in Table 1 displays the values of the displacement versus the frequency, the uncertainty in the values as well as the values of the distances between the mirrors and the glass plate.

Mirror Frequency (Hz, σ = 1 Hz)	σ Frequency (Hz)	ΔX (m, σ= 2.54e-4 m)	σ ΔX (m)
33.3	1	0.0005588	0.000254
50.3	1	0.0006096	0.000254
83.1	1	0.0006096	0.000254
84	1	0.0008128	0.000254
126	1	0.000889	0.000254
150	1	0.0010668	0.000254
179.1	1	0.0010414	0.000254
196.5	1	0.0012954	0.000254
217.1	1	0.0011938	0.000254
224.1	1	0.001397	0.000254
266	1	0.0014478	0.000254
283.1	1	0.001397	0.000254
331.5	1	0.0011684	0.000254
335.7	1	0.0017018	0.000254
351.4	1	0.0017272	0.000254
377.1	1	0.0017272	0.000254
394	1	0.001778	0.000254
407.2	1	0.0018288	0.000254
432.1	1	0.0018034	0.000254
444	1	0.0019304	0.000254
511	1	0.0023368	0.000254
576.5	1	0.0021844	0.000254

Least Squares Data-Fit:	
$\Delta x = x_0 + \frac{4\pi r D}{c} f$	
a (x_0)	5.03282E-04
b (4πrD/c)	3.21045E-06
σ_a	1.13037E-04
σ_b	3.60617E-07

Error Propagation:	
$\sigma_c^2 = \frac{16\pi^2}{b^2}\left(\sigma_r^2 D^2 + \sigma_D^2 r^2 + \frac{2\sigma_b^2 r^2 D^2}{b^2}\right)$	
r	5.11982376
d	15.0114
σ_r	0.0254
σ_D	0.0254

Resulting Values:	
c	3.00829E+08
σ_c	3.38277E+07

Table 1 - Data Set 1 Results and Error

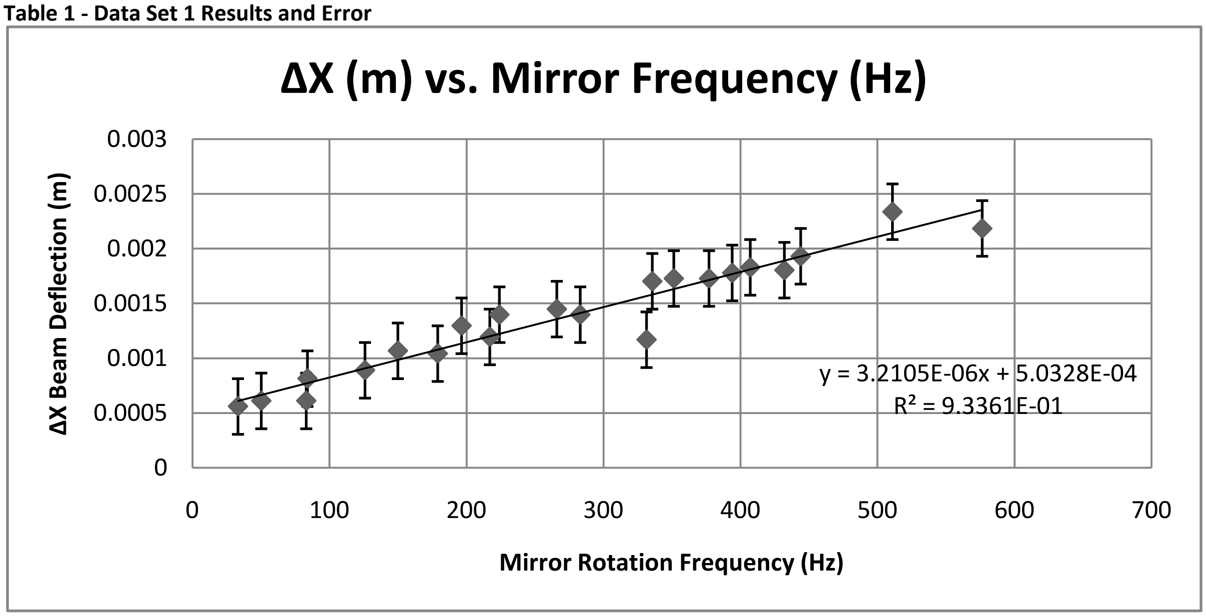

Figure 2 - Data Set 1 Data Points

The data collected was analyzed using a least-square fit method to obtain the value of b for the equation:

$$x = bf + a \tag{17}$$

However, from Equation 16 it can be said then that:

$$b = \frac{4\pi r d n_{air}}{c}$$

Rearranging gives:

$$c = \frac{4\pi r d n_{air}}{b} \tag{18}$$

Therefore the speed of light can be found by using the known distances in the experimental setup as well as the least-square fit of the data points collected. The value of b, along with the uncertainty in the value, was calculated from the least-squares fit method using the data collected from the experiment. The values of r and d are listed in Table 1 along with their respective uncertainties in measurement.

To find the uncertainty in the value of the speed of light Equation 18 was used in the error propagation derivational formula:

$$\sigma_c^2 = \sigma_r^2 \left(\frac{\partial c}{\partial r}\bigg|_{\bar{r}}\right)^2 + \sigma_b^2 \left(\frac{\partial c}{\partial b}\bigg|_{\bar{b}}\right)^2 + \sigma_D^2 \left(\frac{\partial c}{\partial D}\bigg|_{\bar{D}}\right)^2$$
$$\sigma_c^2 = \frac{16\pi^2}{B^2}\left[\sigma_r^2 D^2 + \frac{\sigma_b^2 r^2 D^2}{B^2} + \sigma_D^2 r^2\right] \tag{19}$$

Where c is the relation in Equation 18. By using the least squares analysis along with the error propagation methods, the result for the speed of light for the first data set was $(3.008 \pm 0.34)x10^{8}$ m/s.

From the first experimental set, there were extra methods that would have allowed an increase in the precision of the measurement. With the data analyzed, a second experimental set was recorded in order to reach a more accurate and precise value for the speed of light by taking a larger data set.

4 Data Set 2

For the second experiment, more data points were taken so that the uncertainty in the experiment would be reduced. For each adjustment of the rotating mirror frequency, ten measurements of the position and the frequency were taken so that an experimental uncertainty and average could be extracted, yielding a more precise and accurate result.

Mirror Frequency (Hz, σ = 1 Hz)	σ Frequency (Hz)	ΔX (m, σ= 2.54e-4 m)	σ dx (m)
58.6	0.362222222	0.00031496	0.000169418
97.25	0.647222222	0.00030988	0.000177921
126.02	0.910666667	0.00026162	0.000219497
167.31	0.814333333	0.00049276	0.000167032
194.02	0.806222222	0.00067056	0.00019682
222.88	3.744	0.0005969	0.000205393
244.34	1.211555556	0.00078232	0.000176708
290.78	2.399555556	0.0009779	0.000154664
304.39	0.178777778	0.00109982	0.000165067
369.86	0.173777778	0.00132842	0.000154293
391.91	0.063222222	0.00162306	0.000170115
412.78	0.455111111	0.00126238	0.000214208
440.92	1.855111111	0.00150114	0.000171207
490.26	0.331555556	0.00146558	0.00015522
524.75	0.262777778	0.00187524	0.000160197
527.99	0.452111111	0.00173736	0.000147232
543.3	0.124444444	0.0016002	0.000172687
632.8	0.568888889	0.00191516	0.000190904
686.46	0.336	0.00206502	0.000192717
714.78	0.426222222	0.00229362	0.000162882

Table 2 - Data Set 2 Data Points

Least Squares Data-Fit:

$$\Delta x = x_0 + \frac{4\pi r D}{c} f$$

a (x_0)	6.31805E-05
b (4πrD/c)	3.11399E-06
σ_a	8.74874E-05
σ_b	2.05909E-07

Error Propagation:

$$\sigma_c^2 = \frac{16\pi^2}{b^2}\left(\sigma_r^2 D^2 + \sigma_D^2 r^2 + \frac{2\sigma_b^2 r^2 D^2}{b^2}\right)$$

r	5.10192
d	15.01298
σ_r	0.002
σ_D	0.001587

Resulting Values:	
c	3.09096E+08
σ_c	2.04390E+07

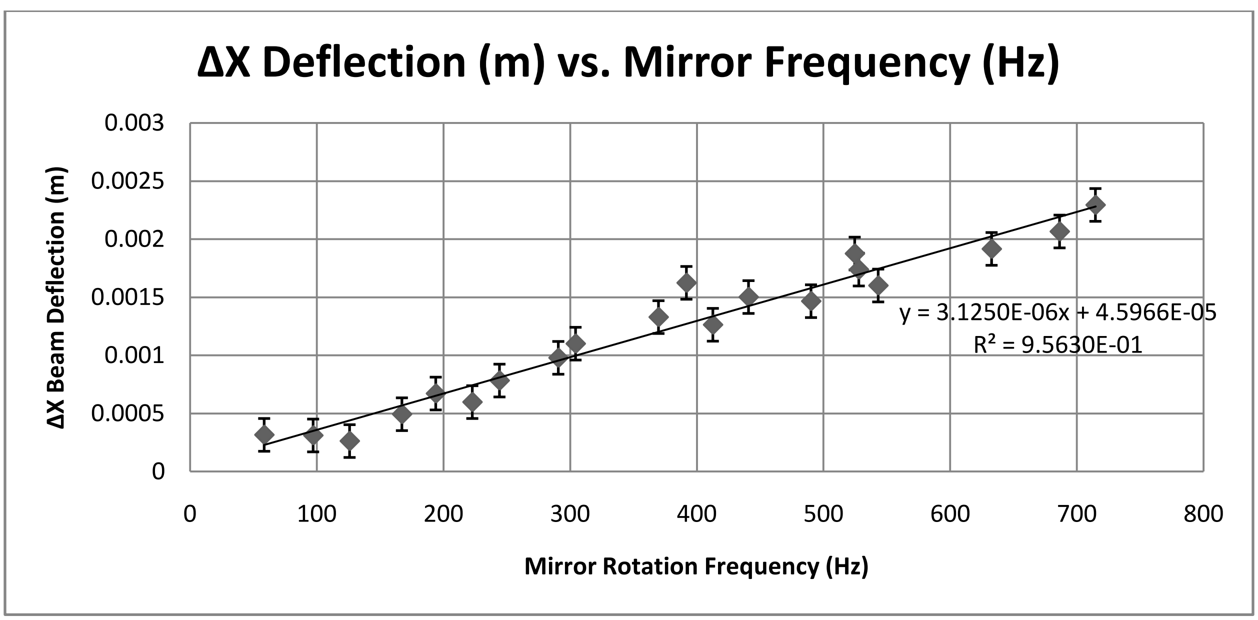

Figure 3 - Data Set 2 Data Plot

Using the process of error analysis and least-squares data fitting from the first data set, the second data set had the result for the speed of light to be $(3.091 \pm 0.204)\text{x}10^8$ m/s.

5 Results

From the second data set, it was found that the speed of light is approximately $(3.09 \pm 0.204) \times 10^8$ m/s. The experimental measurement is approximately within 2.7% compared to the actual value of c defined as c = 299,792,458 m/s. Also it should be noted that the defined value of c is within the bounds of the uncertainty of the experimental value, providing a sense of validity to the experimental method.

The first data set produced a value of $(3.00 \pm 0.34) \times 10^8$ m/s, a 0.34% difference. Although surprisingly accurate, the uncertainty is higher for the first data set in comparison to the second data set is therefore is considered less precise and less reliable. The second data set had ten times the amount of data points taken, which for the experiment would indicate it was closer to the defined value of c. Although the value of c is defined, for an experiment where the value in question is not known a priori it would be prudent to have trust in the value with the least uncertainty and more measurements.

Although uncertainties of around 10% of the value exist from the experiment, it would be hard using the existing methods to further reduce the values. It would require extremely precise measurements to reach an uncertainty of 1%. For example, it would require taking distance measurements with an uncertainty of 0.1 mm and displacement measurements on the frosted screen of around 25 μm. Considering the average uncertainty of measurements using the microscope measurement device was approximately 200 μm from random uncertainty, reaching around 25 μm becomes a difficult task. Such assumptions also assume that no other sources of error or experimental setup issues interfere with the experiment itself. Eventually a newer method would have to be used in order to measure c more precisely past three significant figures. One of the ways such an experiment could be improved was if the microscope measurements were replaced by a photo detector that was used to measure the intensity of the light as a function of the distance across the glass plate and determine the position of the maximum intensity. The photo detector would have to be able to measure intensity over small sections of the glass plate, on the order of a few microns, in order to decrease the uncertainty in the resulting value for the speed of light.