Two Dimensional Dielectric Cavities

Scattering and Resonance

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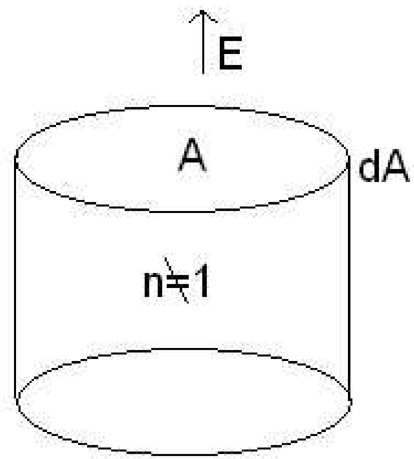
The Problem

Physics: What happens when light enters a dielectric material? We consider the case of translational symmetry along one axis and transverse magnetic polarization.



The Problem

Math:

$$\psi : \mathbb{R}^2 \to \mathbb{C} \quad (1)$$

$$0 = (k^2 + \nabla^2)\psi \quad (2)$$

where ψ is the electric field in the direction of symmetry and $k = \omega\sqrt{\epsilon\mu}$, where ϵ, μ are piecewise constant and depend on the material.

Weber's Theorem

$$\text{(3)} \quad \psi(r) = -\frac{i}{4}\oint_{\partial A}\left[\psi\frac{\partial}{\partial\nu}H_0^{(1)}(k|l-r|) - H_0^{(1)}(k|l-r|)\frac{\partial\psi}{\partial\nu}\right]dl$$

- ∂A is the boundary of the dielectric
- ν is the outward unit normal of ∂A
- l goes along ∂A
- $H_0^{(1)}$ is the zeroth Hankel function of the first kind

Mathematische Annalen **1** (1869), 1-36

Boundary Element Method

Using a delta function to absorb ψ into the integral:

$$0 = \oint_{\partial A} B(l)\frac{\partial \psi}{\partial \nu} + C(l)\psi dl \tag{4}$$

Boundary Element Method

- We form a matrix M
- Elements B_{ij} and C_{ik}
- N is the number of discretization points
- $i \in [1, 2N]$ (integrate along the both sides of the boundary)
- $j \in [1, N]$ and $k \in [N + 1, 2N]$

Thus M is a $2N \times 2N$ matrix.

Boundary Element Method

The eigenvector X has $2N$ elements.
The top N elements form $\frac{\partial\psi}{\partial\nu}$, the rest from ψ.

(5) $$\vec{0} = MX$$

Scattering

Suppose there is an incoming wave X_{in}.
Extend ∂A to include a boundary at ∞.
The boundary at ∞ has d discrete points.

$$M_0 X_{\text{in}} = MX \tag{6}$$

where M_0 is M, with $B_{ij}, C_{ik} = 0$ for $i < 2N - d$.

Scattering

The total scattering cross section

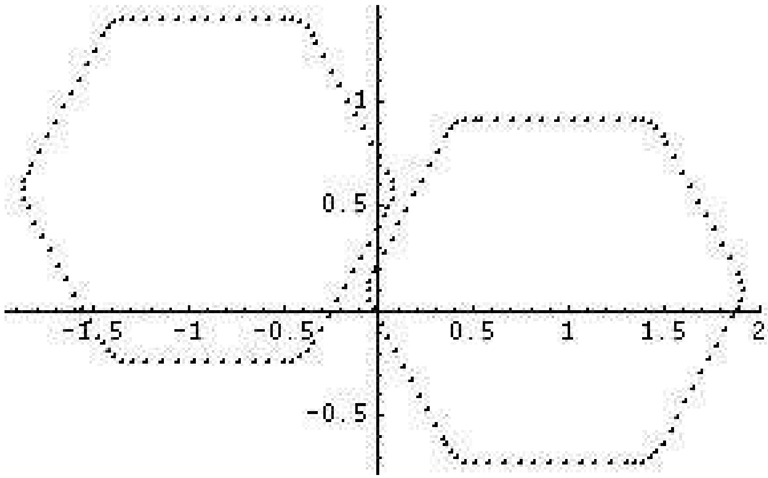
$$(7)\quad \sigma = \frac{1}{|k|}\text{Im}\left[\oint_{\delta\infty} e^{-i\vec{k}\cdot\vec{l}}\left(i\vec{k}\cdot\vec{\nu}(\psi-\psi_{\text{in}})+\frac{\partial\psi}{\partial\nu}-\frac{\partial\psi_{\text{in}}}{\partial\nu}\right)dl\right]$$

gives the effective area in which collisions take place

Results

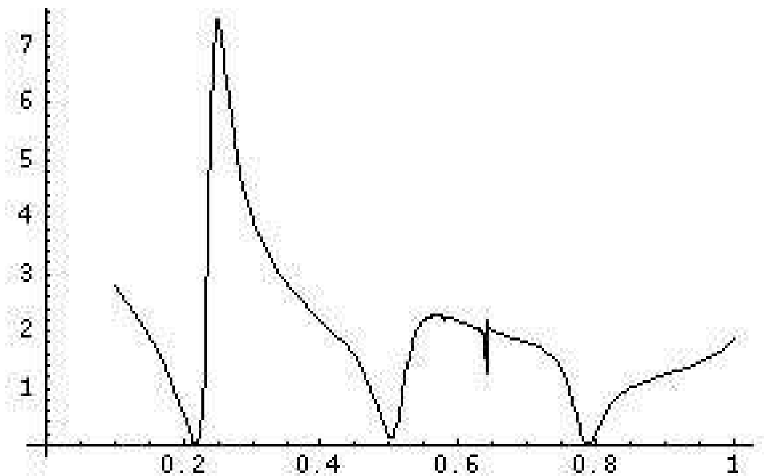
- Descritization of the boundary with Bézier Curves

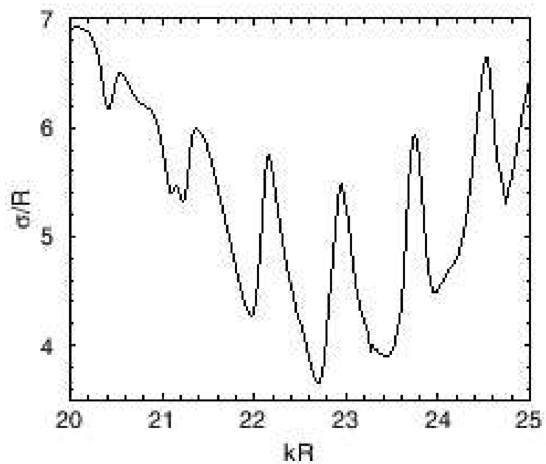
$$B_2(t) = (1-t)^2\vec{x}_0 + 2t(1-t)\vec{x}_1 + t^2\vec{x}_2, t \in [0,1] \tag{8}$$



Results

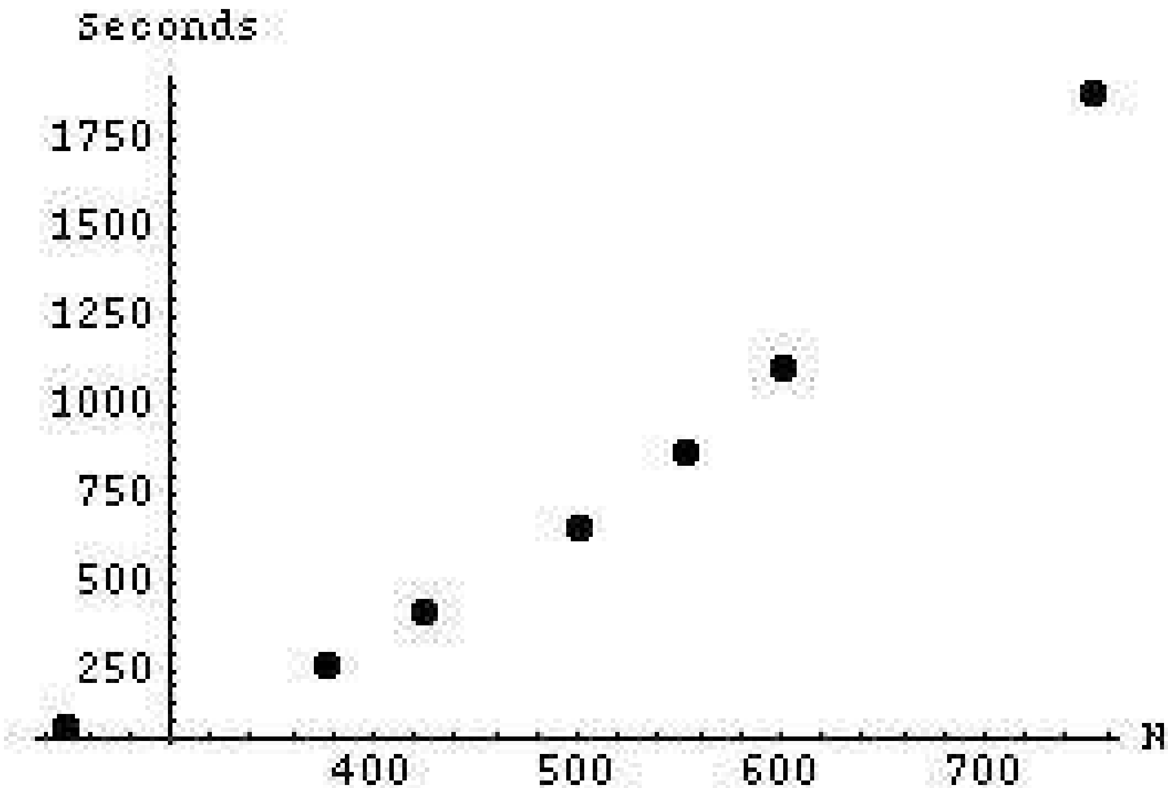
- Abnormally large values for the first few elements of X
- Backscattering





Left: My Plot. Right: Jan Wiersig's Plot.

Quadratic Time



Resonances

Newton's Method:

$$k_{n+1} = k_n - \frac{q}{\text{Tr}[M^{-1}(k_n)\frac{\partial M}{\partial k}]} \tag{9}$$

converges to q-fold degenerate resonances.
Estimate resonant k using peaks in the scattering cross section:

$$k_0 = \text{height} - \frac{i}{2}\text{width} \tag{10}$$

Off-Boundary Wave

Using M's null eigenvector $\psi(l)$

$$
\begin{aligned}
(11) \quad r &\in A \\
(12) \quad l &\in \partial A \\
(13) \quad \psi(r) &= \oint \left[\psi(l) \frac{i}{2} \frac{\partial}{\partial \nu} H_0^{(1)}(k|r-l|) - \frac{\partial \psi}{\partial \nu} \frac{i}{2} H_0^{(1)}(k|r-l|) \right] dl
\end{aligned}
$$

References

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