

Numerical method of solution of equations (11) and (12).

To solve equations (11) and (12) numerically we first recast them in a modified form. We start with:

$$t_L + 1 = \frac{t_L(t_L + 1)(f^2 + t_R^2 - t_L^2)}{\alpha f \sqrt{2(t_L^2 + t_R^2)f^2 - (t_R^2 - t_L^2)^2} - f^4} \quad (\text{B1})$$

[equation (11) of paper]

$$t_R + 1 = \frac{t_R(t_L + 1)(f^2 + t_L^2 - t_R^2)}{(1 - \alpha)f \sqrt{2(t_L^2 + t_R^2)f^2 - (t_R^2 - t_L^2)^2} - f^4} \quad (\text{B2})$$

[equation (12) of paper]

Now defining $\beta = t_L^2 - t_R^2$ we rewrite (B1) and (B2) as

$$t_L + 1 = \frac{t_L(t_L + 1)(f^2 - \beta)}{\alpha f \sqrt{2(t_L^2 - \beta)f^2 - \beta^2} - f^4} \quad (\text{B3})$$

$$t_R + 1 = \frac{t_R(t_L + 1)(f^2 + \beta)}{(1 - \alpha)f \sqrt{2(t_R^2 + \beta)f^2 - \beta^2} - f^4} \quad (\text{B4})$$

Equation (B3) now depends on t_L and β while equation (B4) depends on t_R and β and only one variable, β , links (B3) and (B4).

To solve (B3) and (B4) for t_L and t_R , the quantities f , t_1 and α are first fixed. A value of β is now selected. (B3) is solved to find t_L^{cal} ('cal' indicates a calculated value), and (B4) is solved for t_R^{cal} for this same value of β . The quantity $\beta^{cal} = (t_L^{cal})^2 - (t_R^{cal})^2$ is then calculated for this selected value of β . Finally the quantity $[\beta - \beta^{cal}]^2$ is calculated. The quantity $[\beta - \beta^{cal}]^2$ is then minimized by varying β and repeating the above process, an exact solution should give zero for $[\beta - \beta^{cal}]^2$. The solutions for t_L and t_R are those values of t_L^{cal} and t_R^{cal} obtained at the end of this minimization.

Once t_L and t_R are determined, the quantities y and d are determined from

$$\frac{y}{L_1} = \frac{(f^2 + t_R^2 - t_L^2)(f^2 - t_R^2 + t_L^2)}{2f^2 \sqrt{2(t_L^2 + t_R^2)f^2 - (t_R^2 - t_L^2)^2} - f^4}$$

[equation (9) of paper in reduced form]

and

$$\frac{d}{L_1} = \frac{f^2 + t_R^2 - t_L^2}{2f^2}$$

[equation (8) of paper in reduced form]

and the angles θ_L and θ_R are calculated from these values of y and d . Finally d_{shift} is calculated from $d_{\text{shift}}/L_1 = (d - c)/L_1$.

The above calculation is repeated for selected values of α between 0 and 0.5.

Then symmetry may then be used to extract the values of t_L , t_R , θ_L , θ_R , y/L_1 and d_{shift}/L_1 between 0.5 and 1 using:

$$\begin{aligned} t_L(\alpha) &= t_R(1 - \alpha) \\ t_R(\alpha) &= t_L(1 - \alpha) \\ \theta_R(\alpha) &= \theta_L(1 - \alpha) \\ \theta_L(\alpha) &= \theta_R(1 - \alpha) \\ y(\alpha) &= y(1 - \alpha) \\ d_{\text{shift}}(\alpha) &= -d_{\text{shift}}(1 - \alpha) \end{aligned}$$

(The α in brackets denotes the functional dependence)