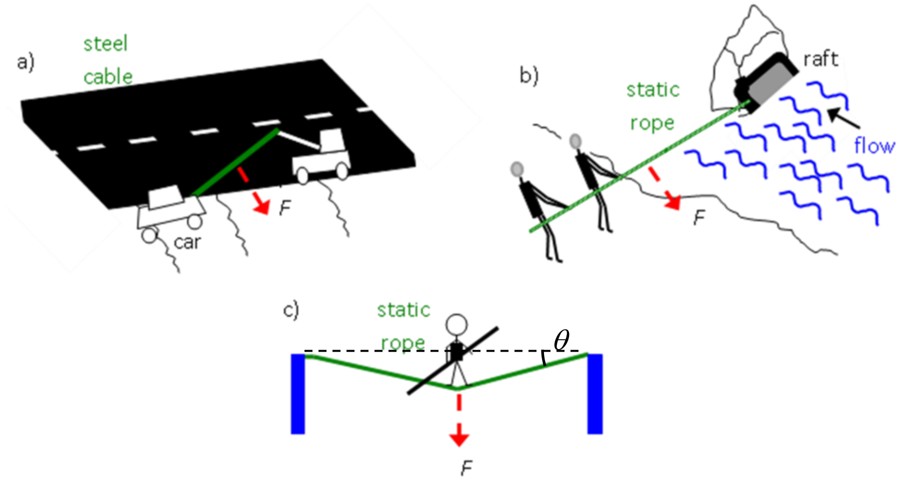
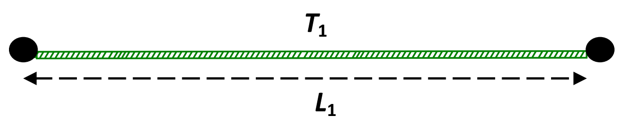
'**Elasticity and mechanical advantage in cables and ropes**', M. J. O'Shea, Eur. J. Phys. **28** (2007) 715-727.

There are many situations where a sideways pull on a fixed (attached at both ends) rope to increase tension is advantageous such as cases where an object needs to be pulled. There are other cases where a 'sideways' force happens to be present just because of the activity as is case for a tightrope walker. The tension in the rope can be determined straightforwardly if the 'sideways' force F *and* the angle θ the rope deviates from a straight line is measured. In this work we find the tension in the rope from 'first principles' using the elasticity of the rope.

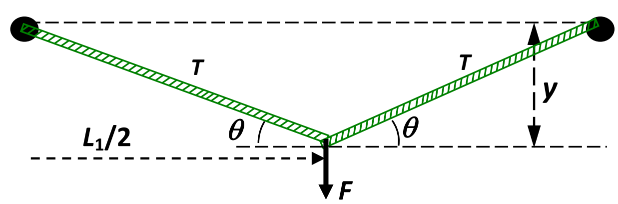


In a) and b) it is advantageous to apply a force to increase the tension in the rope. In c) a sideways force is already present and so the bend in the rope is shown.

We first consider the case where the force F is applied perpendicular to the mid-point of the rope. The rope has initial length L_o and length L_1 when under tension T_1. Applying a force F perpendicular to the rope at its mid-point bends the rope as shown and application of Newton's laws leads to:



$$T = \frac{F(T+\kappa)}{2[(T+\kappa)^2 - (T_1+\kappa)^2]^{1/2}}$$



which can be rearranged to a quartic equation in T allowing T to be found. In the case of ropes, for reasonable value of applied forces it's found that pulling sideways on the rope almost never leads to significant mechanical advantage, i.e. T_1 is almost never significantly larger than T. In the case of steel cables and other lines that can be much stiffer than ropes, significant mechanical advantage can be obtained.

An excel file that provides a numerical solution of this equation is here.

Applying the perpendicular forces at a point away from the mid-point of the rope leads to different tensions in the two sections of rope. The resulting equations are somewhat more complex (they are two quartic equations with two unknowns) but can be solved numerically without too much difficulty.

A derivation of these two equations can be found here.
A numerical method of solution of these two equations can be found here.
An excel sheet that implements this solution can be found here.