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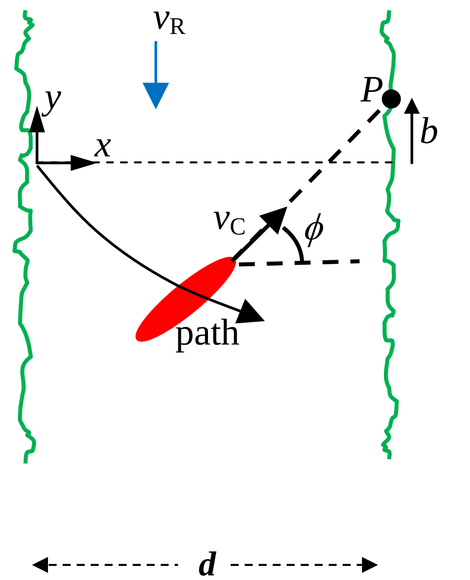
'Crossing a river in a canoe—how complicated can it get?', M J O'Shea, Eur. J. Phys. 31, 857 (2010).

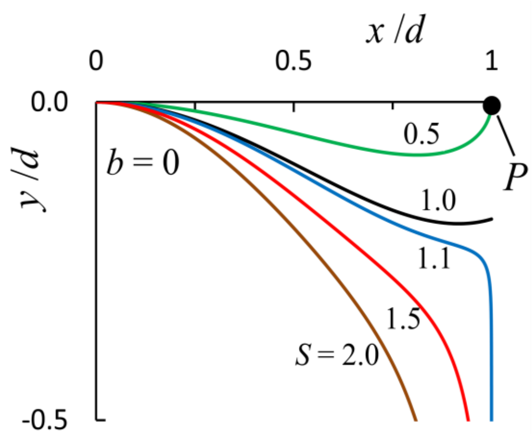
A classic problem in introductory physics involves crossing a river of width d in a canoe. One can paddle to the opposite side of the river with no downstream motion provided v_c(canoe speed wrt water) > v_R(water speed) and if one chooses the angle ϕ correctly. If $v_c < v_R$ this is no longer possible but one can choose ϕ such that downstream motion is minimized. All of these paths are straight line paths.



Here we consider a canoe that chooses a point P on the opposite shore and keeps the canoe pointed at that point as the canoe crosses the river. As the canoe is pushed downstream the paddler must change the canoe direction ϕ to keep pointed at the same point and so follows a curved path across the river. Analysis of this problem yields a non-linear differential equation that can be solved for a variety of flow profiles, $v_R(x)$, (assuming downstream streamline flow) to find the path of the canoe, $y(x)$:

$$y(x) = (d - x)\sinh\left(\int f(q)dq + C\right) + b, \;\; q = \ln(d - x), \;\; f(x) = v_R(x)/v_C.$$

One of the cases we consider is a linear flow profile $f(x) = v_R(x)/v_C = sx/d$. Here the water by the left shore is not moving while the water by the right shore is moving with a speed s. The path the canoe followed is shown in the diagram for selected values of s from 0.5 up to 2.0. For $s > 1$, the canoe does not make it to the right shore. Although the path corresponding to $s = 1.0$ makes it to the right shore, the canoe cannot travel this path in a finite time and again does not make.



It is possible to solve the inverse problem: given a canoe path, what river flow profile will yield this path?

Finally it's interesting to note that this problem, for the case of uniform flow, can be transformed to a classic predator-prey problem. This can be done by shifting to a frame of reference that moves with the water.

See here for numerical integration of flow data to find a canoe path.

See here for several other flow profiles not discussed in the paper.

See here for transformation of this problem (uniform flow case) to a predator-prey problem.

See here for a simple problem involving straight line paths.

See here for a special case of a curved path assuming uniform flow.

See here for a special case of a curved path assuming a selected non-uniform flow.