
A classic problem in introductory physics involves crossing a river of width \( d \) in a canoe. One can paddle to the opposite side of the river with no downstream motion provided \( v_c (\text{canoe speed wrt water}) > v_R (\text{water speed}) \) and if one chooses the angle \( \phi \) correctly. If \( v_c < v_R \) this is no longer possible but one can choose \( \phi \) such that downstream motion is minimized. All of these paths are straight line paths.

Here we consider a canoe that chooses a point \( P \) on the opposite shore and keeps the canoe pointed at that point as the canoe crosses the river. As the canoe is pushed downstream the paddler must change the canoe direction \( \phi \) to keep pointed at the same point and so follows a curved path across the river. Analysis of this problem yields a non-linear differential equation that can be solved for a variety of flow profiles, \( v_R(x) \), (assuming downstream streamline flow) to find the path of the canoe, \( y(x) \):

\[
y(x) = (d - x) \sinh\left( \int f(q) dq + C \right) + b, \quad q = \ln(d - x), \quad f(x) = v_R(x)/v_C.
\]

One of the cases we consider is a linear flow profile \( f(x) = v_R(x)/v_C = sx/d \). Here the water by the left shore is not moving while the water by the right shore is moving with a speed \( s \). The path the canoe followed is shown in the diagram for selected values of \( s \) from 0.5 up to 2.0. For \( s > 1 \), the canoe does not make it to the right shore. Although the path corresponding to \( s = 1.0 \) makes it to the right shore, the canoe cannot travel this path in a finite time and again does not make.

It is possible to solve the inverse problem: given a canoe path, what river flow profile will yield this path?

Finally it’s interesting to note that this problem, for the case of uniform flow, can be transformed to a classic predator-prey problem. This can be done by shifting to a frame of reference that moves with the water.

See here for numerical integration of flow data to find a canoe path.

See here for several other flow profiles not discussed in the paper.

See here for transformation of this problem (uniform flow case) to a predator-prey problem.

See here for a simple problem involving straight line paths.

See here for a special case of a curved path assuming uniform flow.

See here for a special case of a curved path assuming a selected non-uniform flow.