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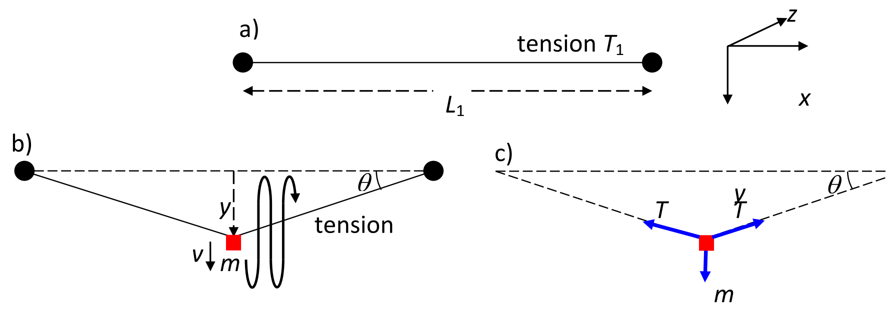
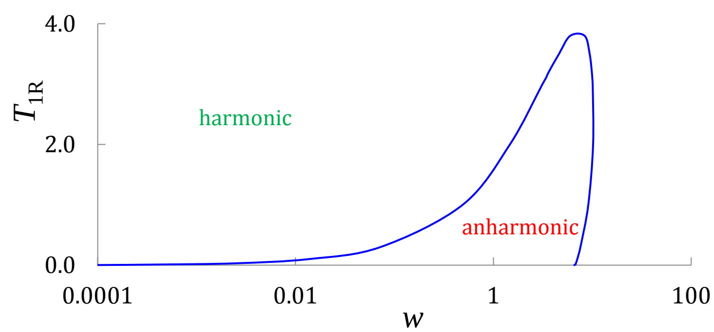
'Harmonic and anharmonic behaviour of a simple oscillator', M. J. O'Shea, *Eur. J. Phys.* **30** No 3, 549-558 (May 2009).

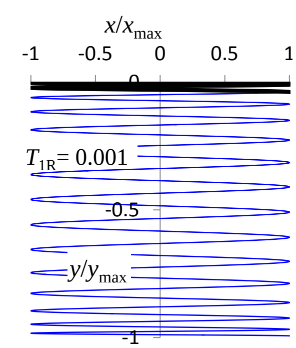


There are few anharmonic systems that can be easily analyzed. When a mass is placed on at the mid-point of a horizontal tensioned line and is displaced it will undergo oscillations. We show that this oscillator has interesting anharmonic behavior that lends itself to analytical solutions when the mass *m* is dropped from rest.

For very small amplitudes and very large amplitudes these oscillations turn out to be harmonic albeit with different frequencies. In the intermediate regime where the oscillator crosses over between these two harmonic limits, its behavior is anharmonic. Analytical expressions can be obtained for the equilibrium position and amplitude over the complete range from small to large oscillations.



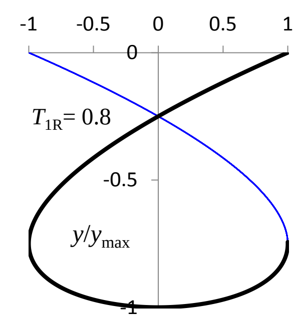
A 'phase diagram' of regions of harmonic and anharmonic behavior can be obtained as shown. The vertical and horizontal axes are proportional to T_1, the tension in the unweighted line, and the mass *m* respectively. More precisely $T_{1R} = T_1/\kappa$ and $w = mg/\kappa$. The limiting cases of small *w* (i.e. small *m* and small oscillation) and large *w* (i.e. large *m* and large oscillation) correspond to harmonic regimes.



If the mass is pulled to one side before it's dropped there is a vertical force and a horizontal force exerted on it. For small oscillations the vertical and horizontal motions are not coupled. The equations of motion can be solved and the ratio of horizontal to vertical frequencies is:

$$\omega_x/\omega_y = \sqrt{(T_{1R}+1)/T_{1R}}.$$

Examples of the path followed by the dropped mass are shown for two selected values of T_{1R}. The first full horizontal oscillation is shown in black for each case. For the case T_{1R} of 0.001 the ratio of frequencies is an irrational number and the path is shown over 15 horizontal oscillations. If the path were continued, it would eventually fill all of the 2-d space. For the case T_{1R} of 0.8 the ratio of frequencies is a rational number, ω_x/ω_y is 3/2. In this case the path retraces itself and will not fill all space.



See here for a problem on small harmonic oscillations in this system.

See here for a problem on velocity of the mass for during small harmonic oscillations

See here for a problem on small anharmonic oscillations in this system.

See here for a problem on the equilibrium positon of a mass on a horizontal line.