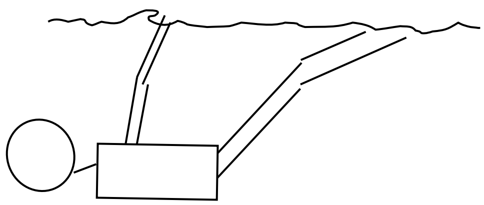
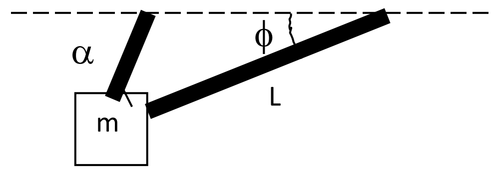
Problem – an unusual special case

Consider a climber on the underside of a roof. The roof is flat except for some good handholds. The feet of the climber are able to friction against the roof.



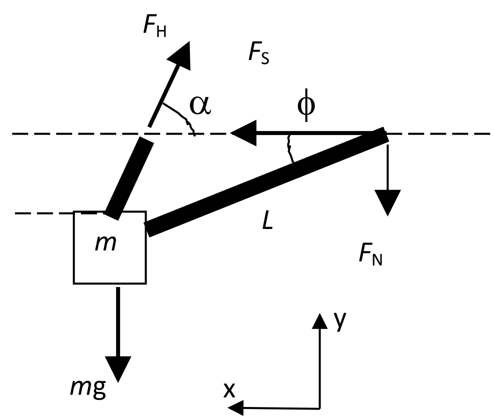


a) Construct a free body diagram and argue that this situation is possible. It is easiest to take the hand force as some value F_H inclined at angle α to the horizontal, and to break up the force exerted on the foot into a normal force F_N and a static friction force F_S. The feet can be assumed to be at some angle ϕ to the horizontal.

b) Find expressions for F_N and F_S in terms of the above angle and the mass *m*.

c) Find the condition on the angles that the climber is stable assuming a static coefficient of friction of value μ.

Solution: a) The vertical forces can be balanced by making F_H large enough. The torque about the point mass can balanced by making F_S large enough



b) We have three unknowns so we should look for 3 equations.

$$\Sigma F_x: \ F_S - F_H \cos\alpha = 0 \qquad \text{(A)}$$

$$\Sigma F_y: \ -F_N - mg + F_H \sin\alpha = 0 \qquad \text{(B)}$$

The third equation will be a torque equation. It is easiest to take torque τ about the feet since this eliminates two unknowns F_S and F_N.

$$\Sigma\tau\text{(point mass)}: \ -F_H L \sin(\alpha-\phi) + mgL\cos\phi = 0 \qquad \text{(C)}$$

Eq.(C) (one equation with one unknown) yields directly a value for F_H as:

$$F_H = \frac{mg\cos\phi}{\sin(\alpha-\phi)} \qquad \text{(D)}$$

From Eq.(A)

$$F_S = F_H\cos\alpha = \frac{mg\cos\phi}{\sin(\alpha-\phi)}\cos\alpha$$

From Eq.(B)

$$F_N = -mg + F_H\sin\alpha = -mg + \frac{mg\cos\phi}{\sin(\alpha-\phi)}\sin\alpha = mg(-\sin(\alpha-\phi)+\cos\phi\sin\alpha)\frac{1}{\sin(\alpha-\phi)}$$

$$= mg(-\sin\alpha\cos\phi + \cos\alpha\sin\phi + \cos\phi\sin\alpha)\frac{1}{\sin(\alpha-\phi)} = mg\frac{\cos\alpha\sin\phi}{\sin(\alpha-\phi)}$$

Then we require that $F_S < F_{S,\max} = \mu F_N$ yielding: $\dfrac{1}{\tan\phi} < \mu$