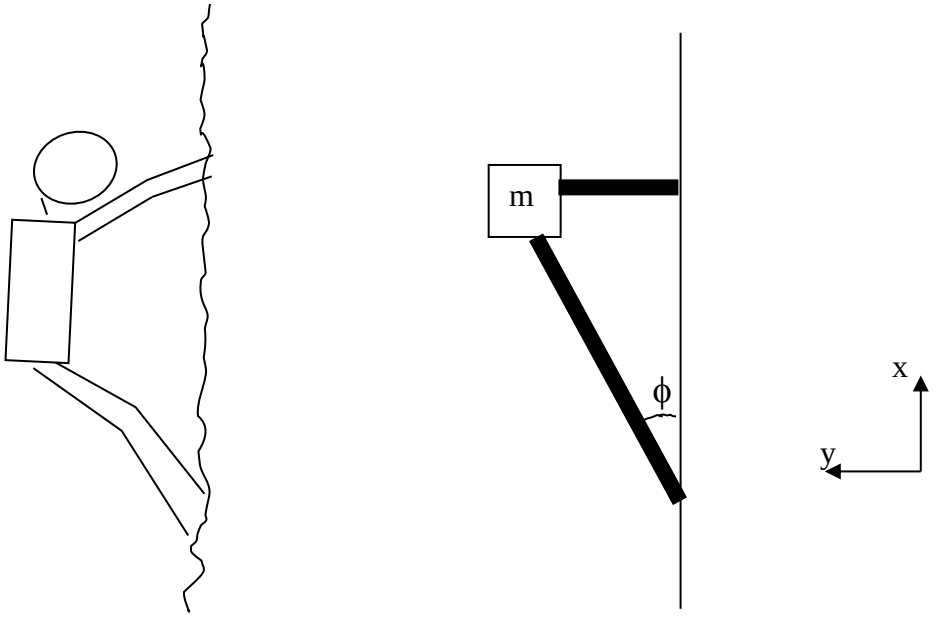
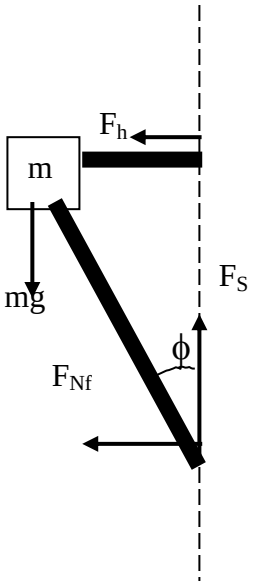
Special case of climber on vertical rock

Consider the special case where a climber is on vertical rock and their arms are assumed to be horizontal. The climbers hands are assumed to grip the rock the the feet are held in place by friction, static coefficient of friction μ. Model the climber as a point mass and the arms and legs of the climber as massless struts as shown. Show that the condition that the climber to be stable on the rock is $\tan\phi > \mu$.



Hint: Start by drawing a free body diagram of the climber and apply Newton's laws.

Solution: Free body diagram (system is point mass and struts)



We have three unknowns so we should look for 3 equations.

$$\Sigma F_x: \; F_S - mg = 0 \qquad (1)$$

$$\Sigma F_y: \; F_N + F_h = 0 \qquad (2)$$

The third equation will be a torque equation. It is easiest to take torque τ about the feet since this eliminates two unknowns F_f and F_N.

$$\Sigma\tau(\text{feet}): \; F_H L\cos\phi - mgL\sin\phi = 0 \qquad (3)$$

Eq.(3) yields directly a value for F_h as:

$$F_H = \frac{mg\sin\phi}{\cos\phi} = mg\tan\phi \qquad (4)$$

From Eq.(1)

$$F_S = mg$$

$$F_N = F_H = mg\tan\phi$$

Then $F_S < F_{S,m} = \mu F_F$ yields the required condition.