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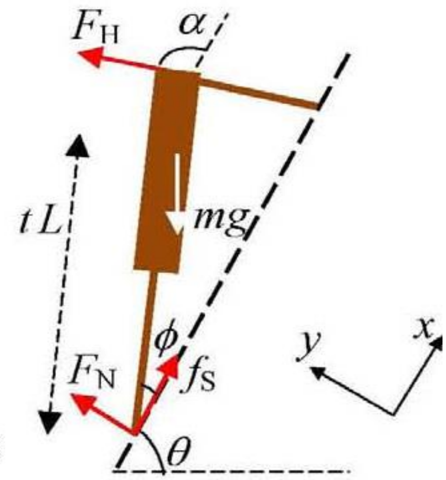
Forces on the climber (extended mass model)



Summing forces along x and y and assuming equilibrium

$$f_S + F_H \cos\alpha - mg\sin\theta = 0 \quad \text{(A)}$$
$$F_N + F_H \sin\alpha - mg\cos\theta = 0 \quad \text{(B)}$$

Taking torque about the climber's feet:

$$\Sigma\tau(\text{feet}):\ F_H L\sin(\alpha-\phi) - mgtL\cos(\phi+\theta) = 0 \quad \text{(C)}$$

Equation (C) yields directly:

$$F_H = \frac{mgt\cos(\theta+\phi)}{\sin(\alpha-\phi)} \quad \text{(6 of manuscript)}$$

From equations (B) and (6)

$$F_N = mg\cos\theta - F_H\sin\alpha = mg\cos\theta - \frac{mgt\cos(\theta+\phi)}{\sin(\alpha-\phi)}\sin\alpha = mg\left(\cos\theta - \frac{t\cos(\theta+\phi)}{\sin(\alpha-\phi)}\sin\alpha\right)$$

(7 of manuscript)

From equations (A) and (6)

$$f_S = mg\sin\theta - F_H\cos\alpha = mg\sin\theta - \frac{mgt\cos(\theta+\phi)}{\sin(\alpha-\phi)}\cos\alpha = mg\left(\sin\theta - \frac{t\cos(\theta+\phi)}{\sin(\alpha-\phi)}\cos\alpha\right)$$

(8 of manuscript)

Stability – Finding $\phi_{\min}$

The maximum value of the static friction force is

$$f_{SM} = \pm\mu F_N \Rightarrow f_{SM} = \pm\mu mg\left(\cos\theta - \frac{t\cos(\theta+\phi)}{\sin(\alpha-\phi)}\sin\alpha\right)$$

When the friction force is at its maximum possible value (at $\phi = \phi_S$), $f_{SM} = f_S$:

$$\Rightarrow \left(\sin\theta - \frac{t\cos(\theta+\phi_S)}{\sin(\alpha-\phi_S)}\cos\alpha\right) = \pm\mu\left(\cos\theta - \frac{t\cos(\theta+\phi_S)}{\sin(\alpha-\phi_S)}\sin\alpha\right)$$

Multiply both sides by $\sin(\alpha-\phi_S)$, expand out sines and cosines, and rearrange:

$$\sin\theta\sin\alpha\cos\phi_S - \sin\theta\cos\alpha\sin\phi_S - t\cos\theta\cos\phi_S\cos\alpha + t\sin\theta\sin\phi_S\cos\alpha$$
$$= \pm(\cos\theta\sin\alpha\cos\phi_S - \cos\theta\cos\alpha\sin\phi_S - t\cos\theta\cos\phi_S\sin\alpha + t\sin\theta\sin\phi_S\sin\alpha)\mu$$

Divide both sides by $\cos\theta\cos\alpha\cos\phi_S$ and rearrange to get the two solutions:

$$\tan\phi_S = \frac{\pm\mu(1-t)\tan\alpha + t - \tan\theta\tan\alpha}{\pm\mu(1-t\tan\theta\tan\alpha) - (1-t)\tan\theta} \quad \text{(the '+' gives equation 9 of manuscript)}$$

The '+' solution provides a minimum value for ϕ while the '–' solution (relevant when $\alpha <$ 90°) provides a maximum value, see figure 6.

Stability – Finding ϕ_H

Maximum hand force $|F_H| = amg$ ($a > 0$). Setting $F_H = \pm\, amg$ in equation (6) of manuscript:

$$\pm\, amg = tmg\cos(\theta + \phi_H)/\sin(\alpha - \phi_H)$$

$$\Rightarrow \pm a\sin\alpha\cos\phi_H \mp a\cos\alpha\sin\phi_H = t\cos\theta\cos\phi_H - t\sin\theta\sin\phi_H$$

Dividing both sides by $\cos\phi_H$ and rearranging yields:

$$\tan\phi_H = \frac{\mp\sin\alpha + (t/a)\cos\theta}{\mp\cos\alpha + (t/a)\sin\theta}$$

(the '+' is equation 10 of manuscript).

The '+' solution is the only one that provides limits on ϕ for the examples of this work.

Maximum value of θ for stability when $\alpha = 90°$

Maximum hand force $|F_H| = amg$ ($a > 0$). At the largest stable value of θ, $\phi_S = \phi_H$ (see figure 6, middle panel on left). Selecting the positive signs in equations (9) and (10):

$$\tan\phi_S\big|_{\alpha\to 90} = \left.\frac{\mu(1-t)\tan\alpha + t - \tan\theta\tan\alpha}{\mu(1 - t\tan\theta\tan\alpha) - (1-t)\tan\theta}\right|_{\alpha\to 90^\circ} = \frac{\mu(1-t) - \tan\theta}{-\mu t\tan\theta}$$

$$\tan\phi_H\big|_{\alpha\to 90} = \left.\frac{\sin\alpha + t\cos\theta}{\cos\alpha + t\sin\theta}\right|_{\alpha\to 90^\circ} = \frac{1 + t\cos\theta}{t\sin\theta}$$

Setting $\phi_S = \phi_H$: $$\frac{\mu(1-t) - \tan\theta_{\max}}{-\mu t\tan\theta_{\max}} = \frac{1 + t\cos\theta_{\max}}{t\sin\theta_{\max}}$$

$$\Rightarrow \mu(\cos\theta_{\max} + 1) = \sin\theta_{\max} \Rightarrow \mu = \frac{\sin\theta_{\max}}{\cos\theta_{\max} + 1} \equiv \tan(\theta_{\max}/2)$$

<u>For $\mu = 2$,</u> $\theta_{\max} = 2\arctan\mu = 127^0$

Minimum value of α for stability when $\theta = 90°$

Maximum hand force $|F_H| = amg$ ($a > 0$). At the smallest stable value of α, $\phi_S = \phi_H$:

$$\tan\phi_S\big|_{\theta\to 90} = \left.\frac{\mu(1-t)\tan\alpha + t - \tan\theta\tan\alpha}{\mu(1 - t\tan\theta\tan\alpha) - (1-t)\tan\theta}\right|_{\theta\to 90^\circ} = \frac{-\tan\alpha}{(t-1) - \mu t\tan\alpha}$$

$$\tan\phi_H\big|_{\theta\to 90} = \left.\frac{\sin\alpha + t\cos\theta}{\cos\alpha + t\sin\theta}\right|_{\theta\to 90^\circ} = \frac{\sin\alpha}{\cos\alpha + t}$$

Setting $\phi_S = \phi_H$: $$\frac{-\tan\alpha_{\min}}{(t-1) - \mu t\tan\alpha_{\min}} = \frac{\sin\alpha_{\min}}{\cos\alpha_{\min} + t}$$

$$\Rightarrow \cos\alpha_{\min} + 1 = \mu\sin\alpha_{\min} \Rightarrow \mu = \frac{\cos\alpha_{\min} + 1}{\sin\alpha_{\min}} \equiv \frac{1}{\tan\alpha_{\min}/2}.$$

<u>For $\mu = 2$,</u> $\alpha_{\min} = 2\arctan(1/\mu) = 53.1^0$ and at this value of α, $\phi = \sin\alpha/(\cos\alpha + t) = 32.3^0$